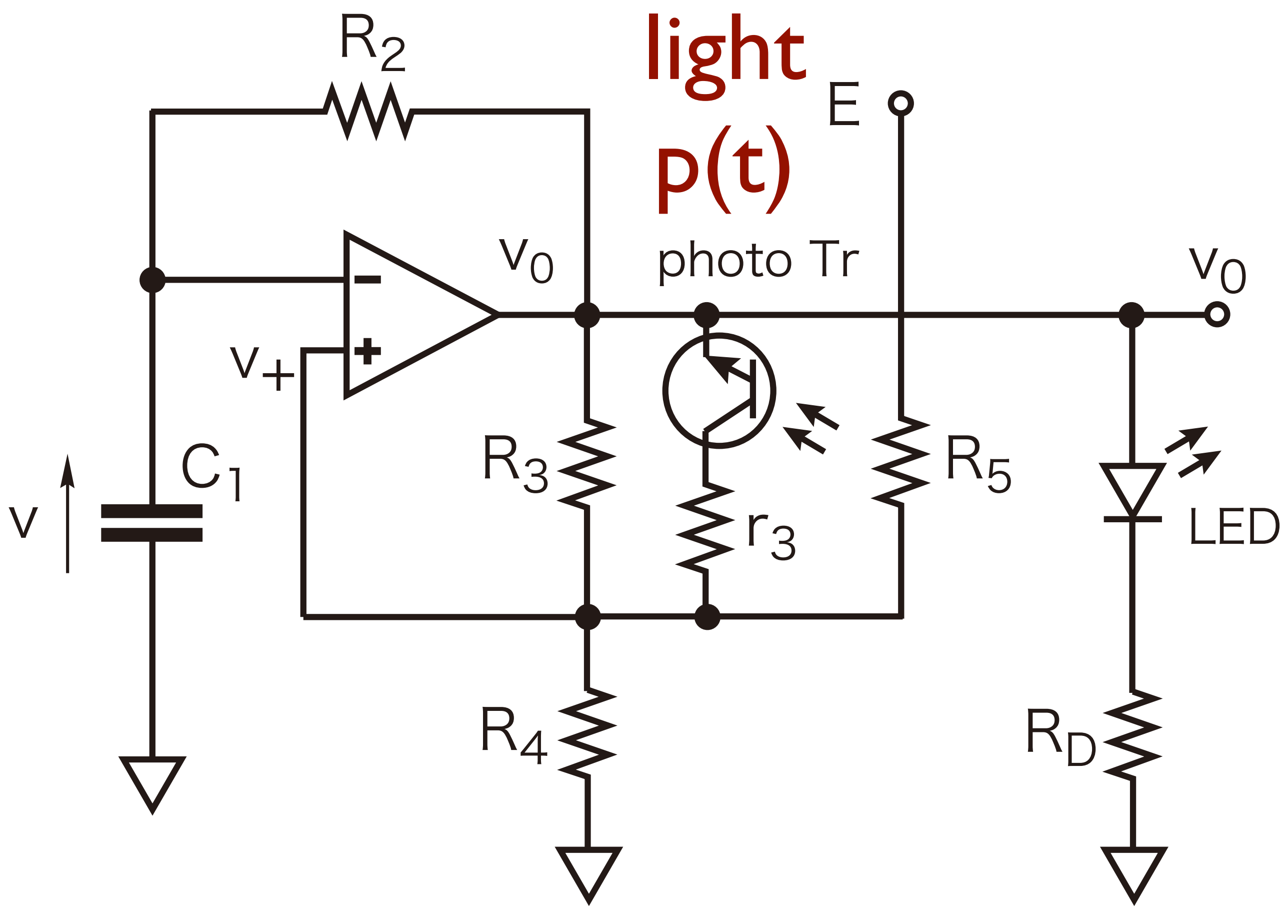
上田研　ゼミ

LEDホタルA1の強制振動 (I)



川上　博

2014(H26).10.20

話の流れ

1．力学系の導出

◎ ホタルの状態空間：貼り合わせ多様体

◎ イベントと状態の運動則

◎ 状態の時間発展

2．波形の持つ情報

◎ 波形の型：符号数付き波形

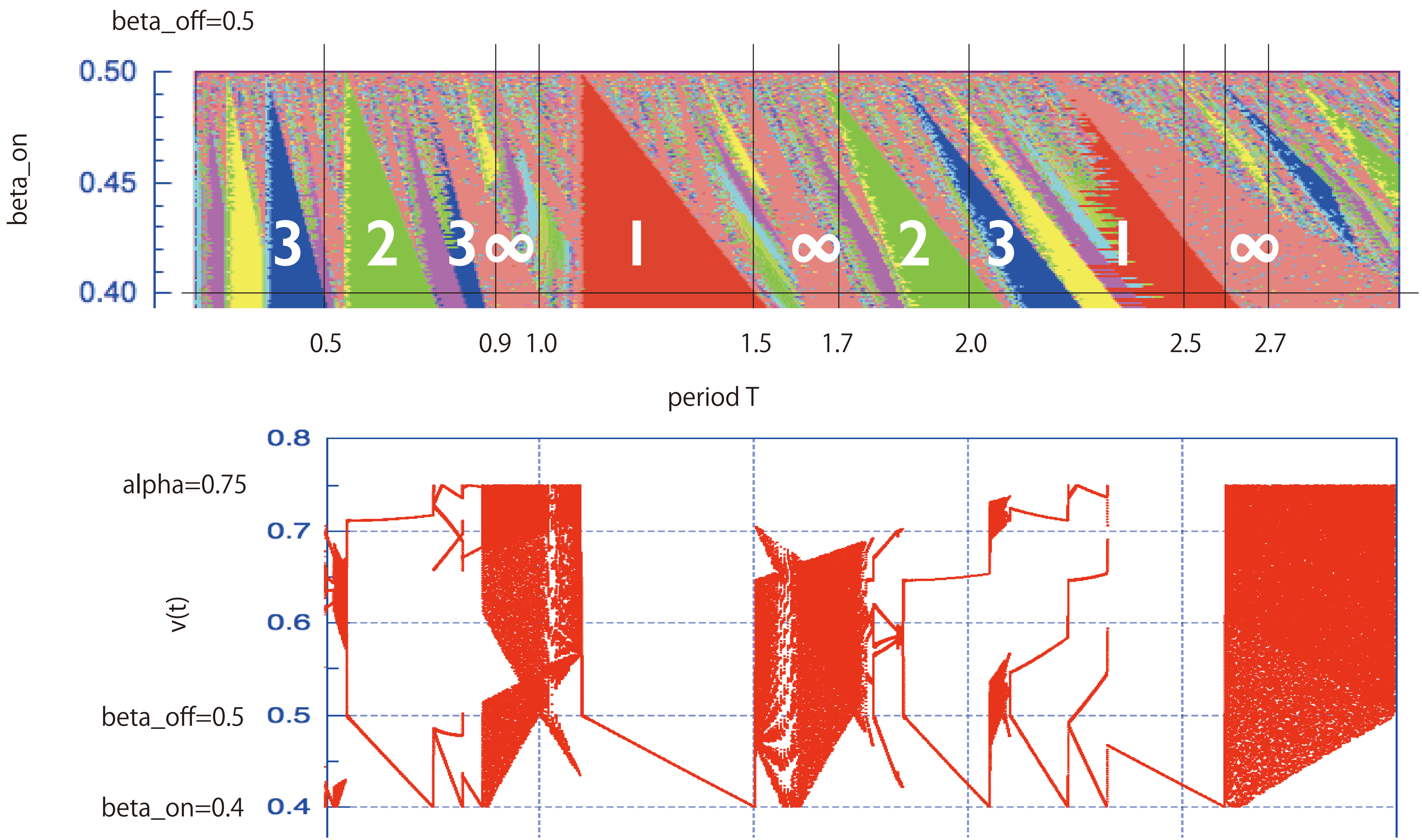
3．周期波形とその分岐

◎ 周期波形の分類と分岐

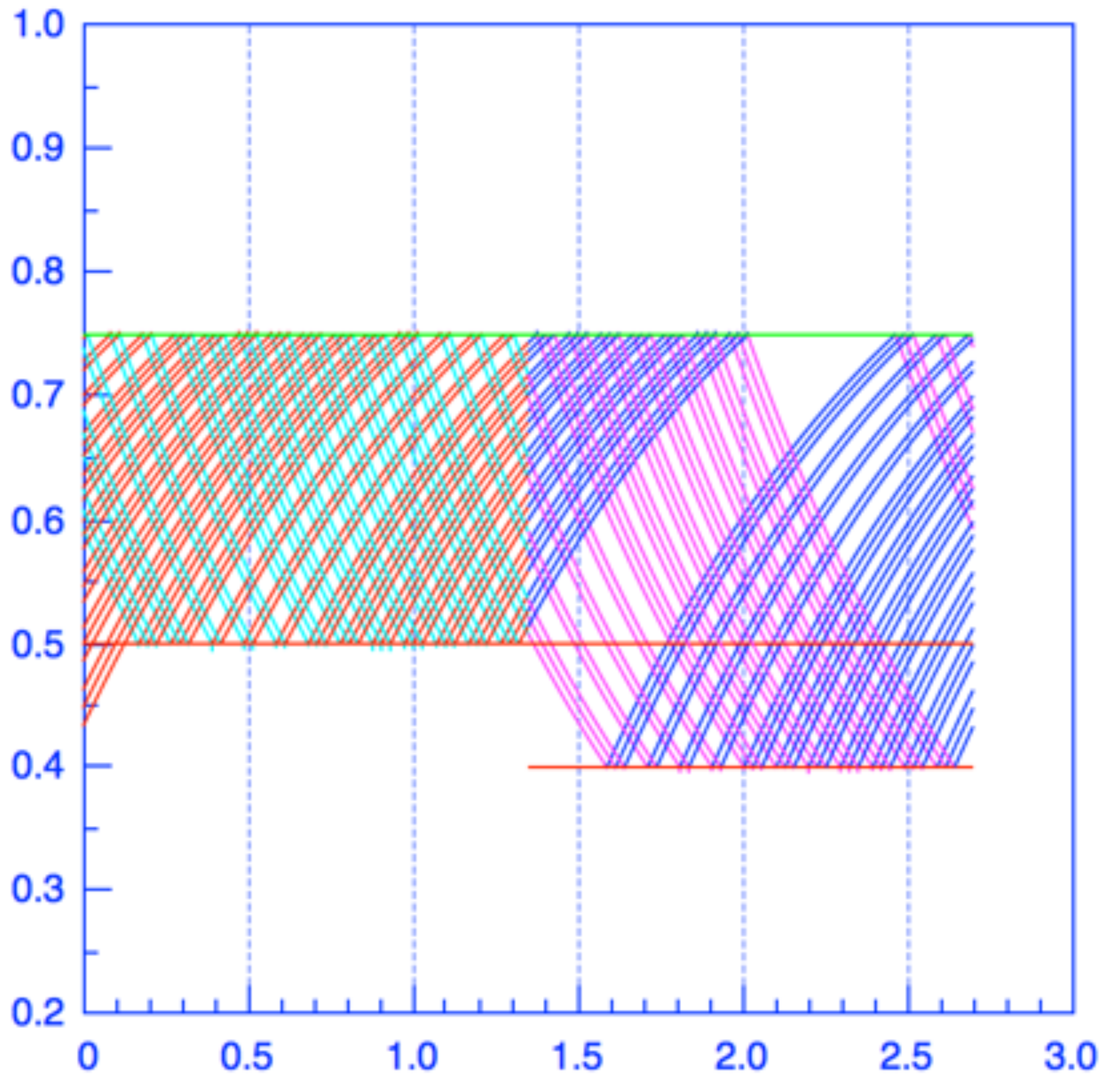
4．非周期波形の存在

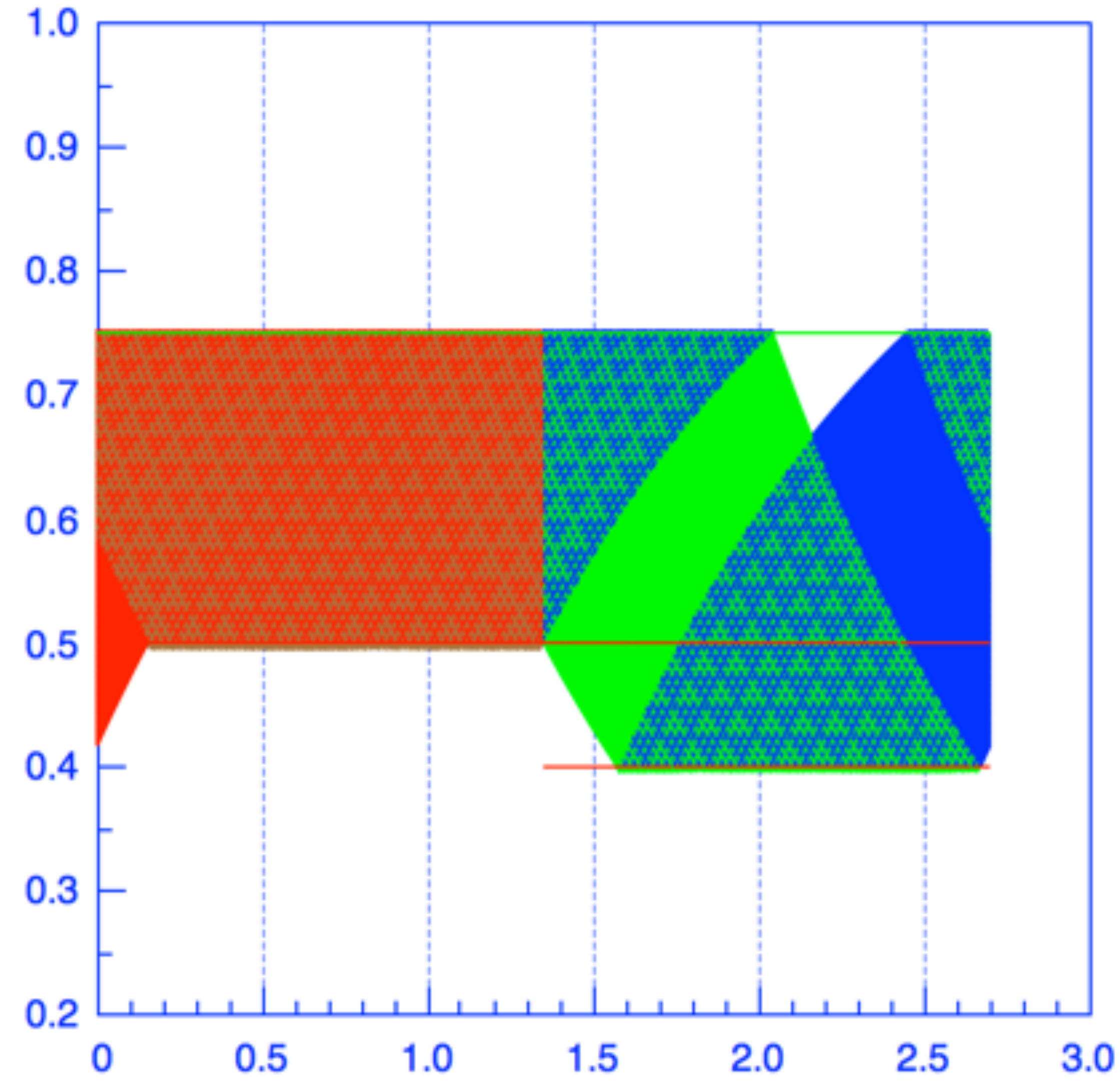
◎ 準周期解かカオス解か

分岐図：duty cycle=0.5

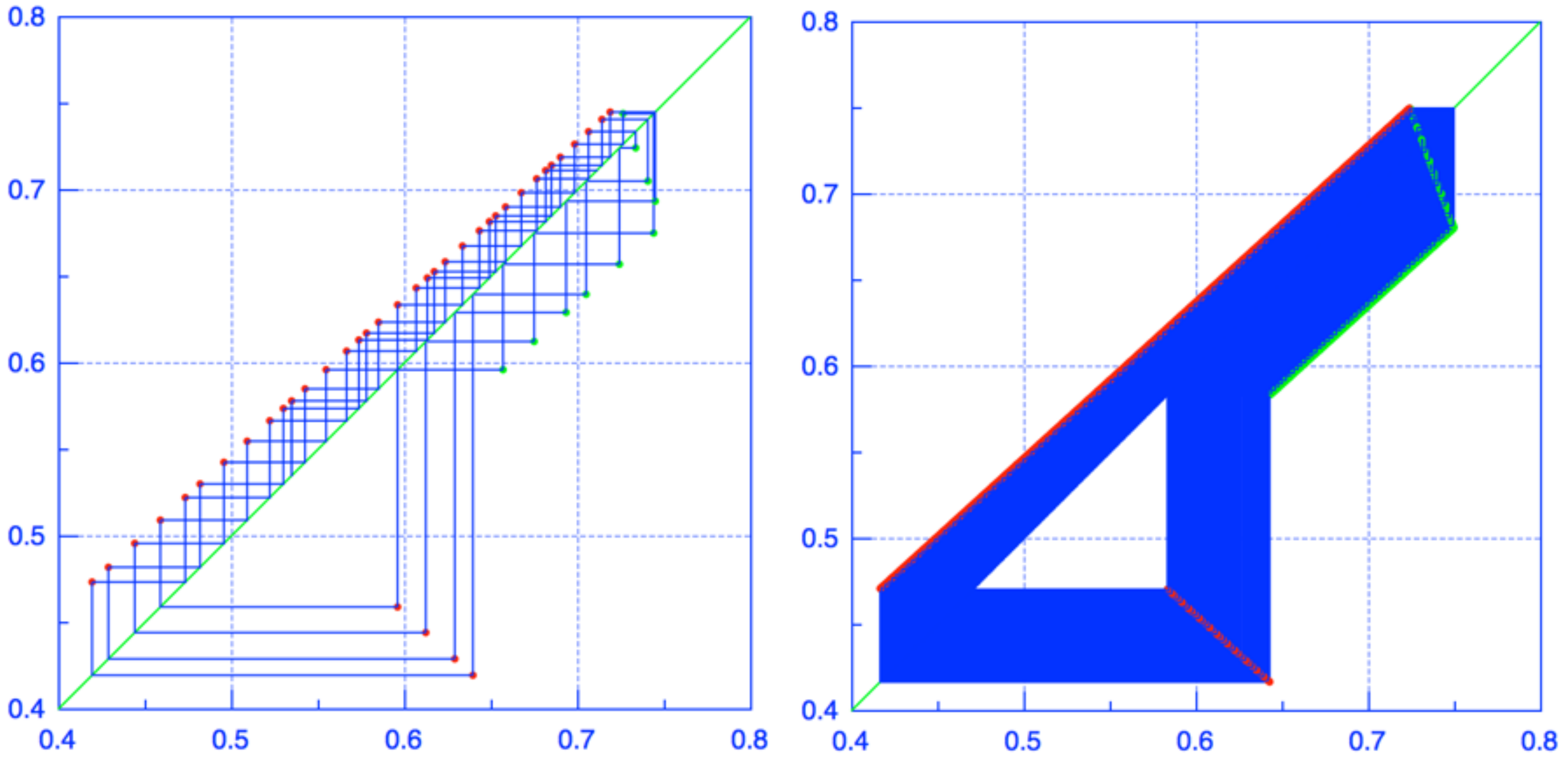


waveform for T=2.7



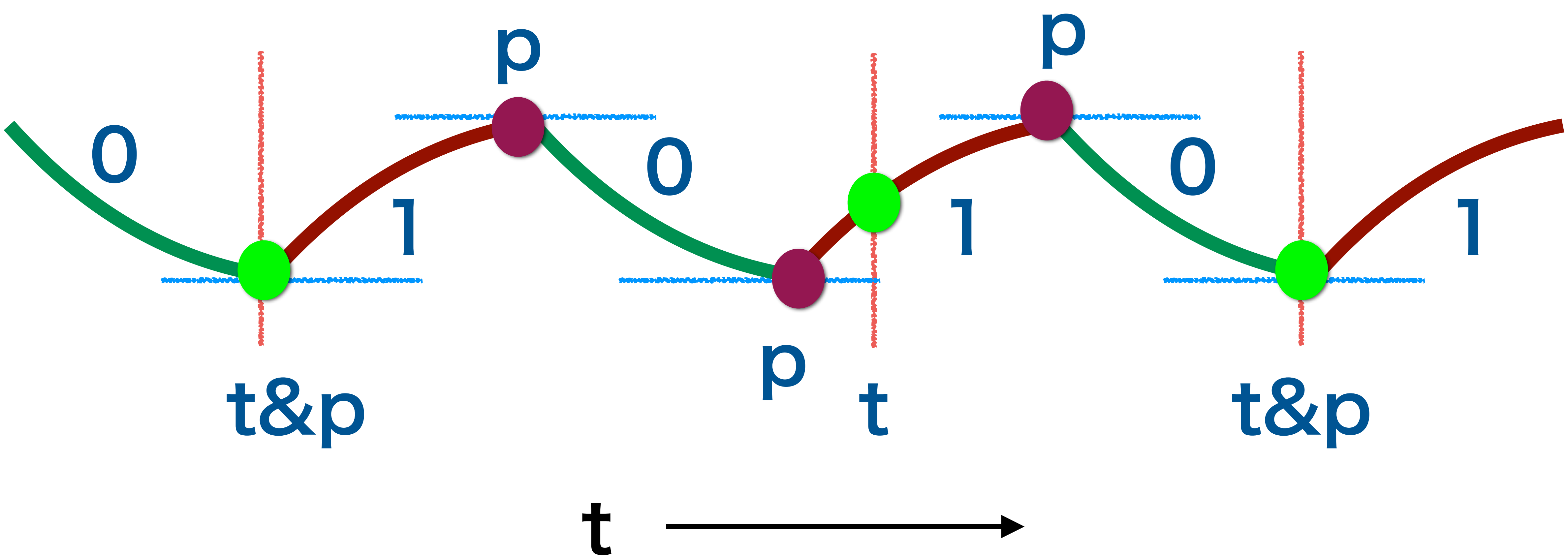


time 1 map trajectories for T=2.7



解析の要点

◎ 波形情報の活用：分類と線形性



◎ Poincaré 写像の活用

LEDホタルの強制振動

LEDホタルの回路

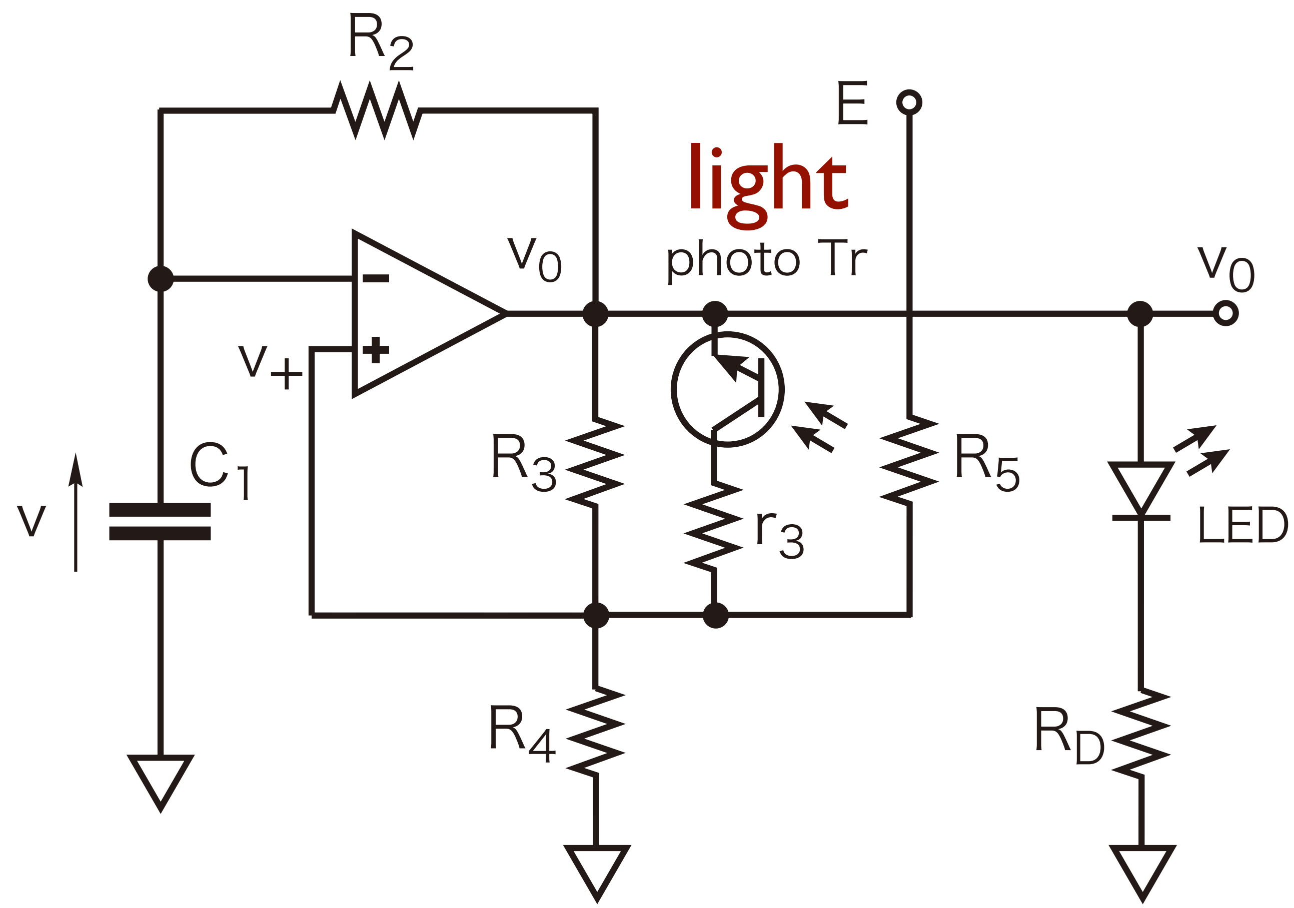
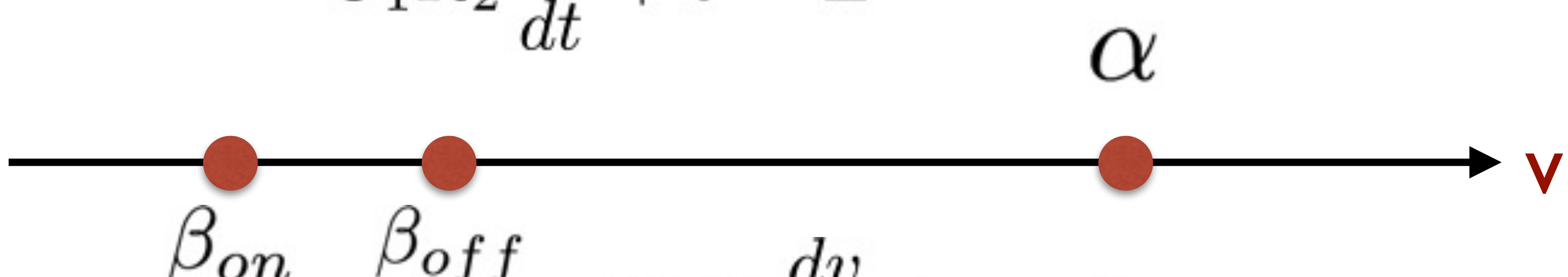


photo Tr	light on	light off
v0 = 0	on	off
v0 = E	off	off

$$C_1 R_2 \frac{dv}{dt} + v = E$$



$$C_1 R_2 \frac{dv}{dt} + v = 0$$

LED FF with SW forcing term

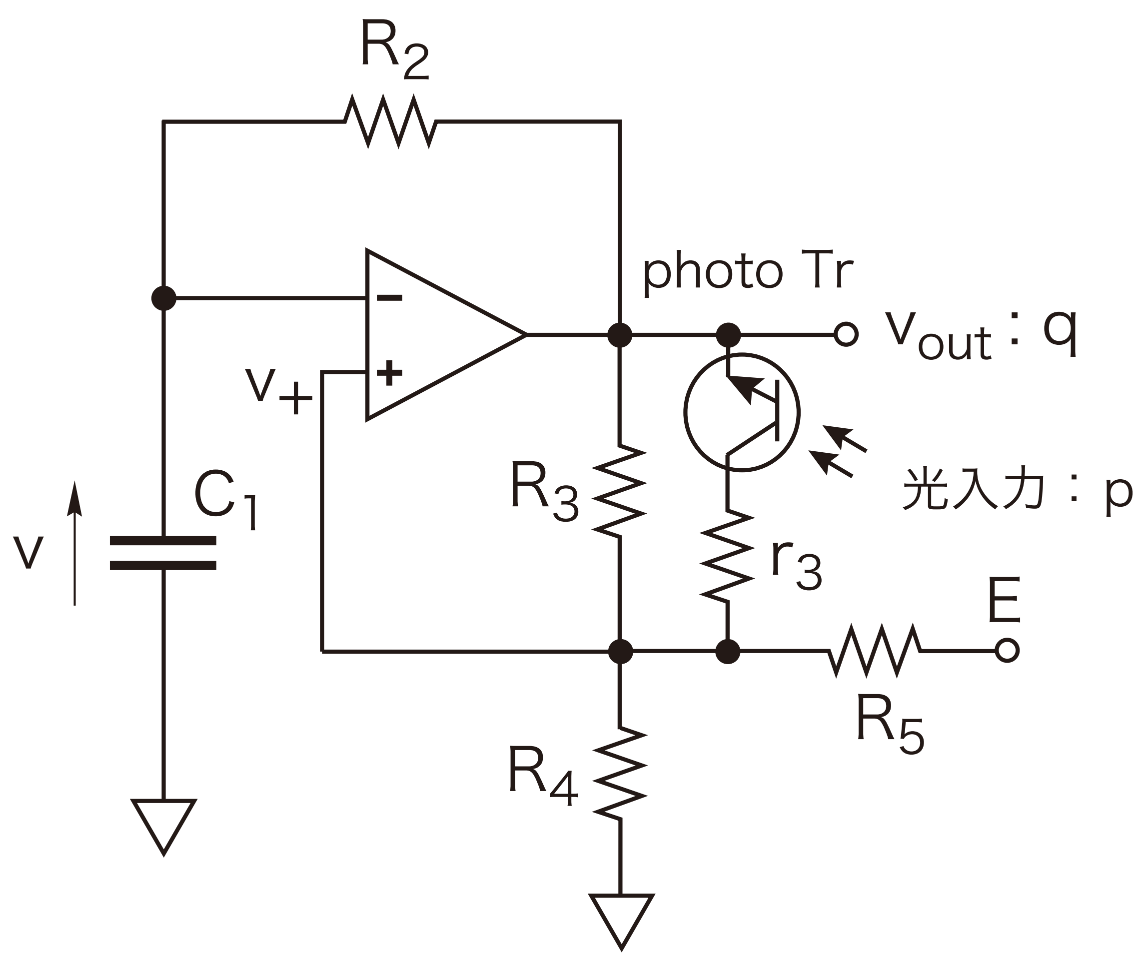


photo Tr		comparator : q	
		on:1	off:0
光入力 : p	on:1	off:0	on:1
	off:0	off:0	off:0

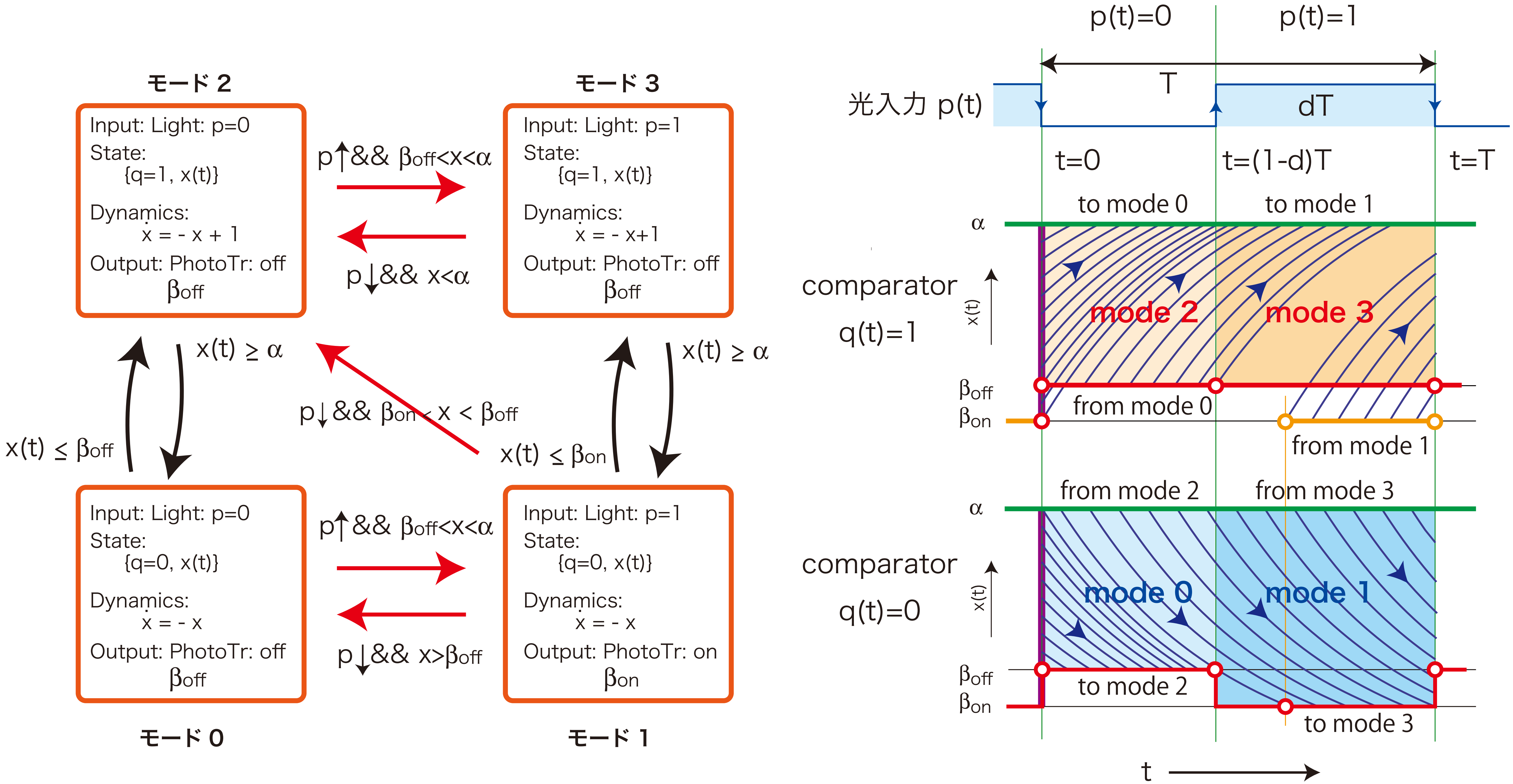
mode0 : (p, q) = (0, 0), dx/dt+x = 0

mode1 : (p, q) = (1, 0), dx/dt+x = 0, \beta_on

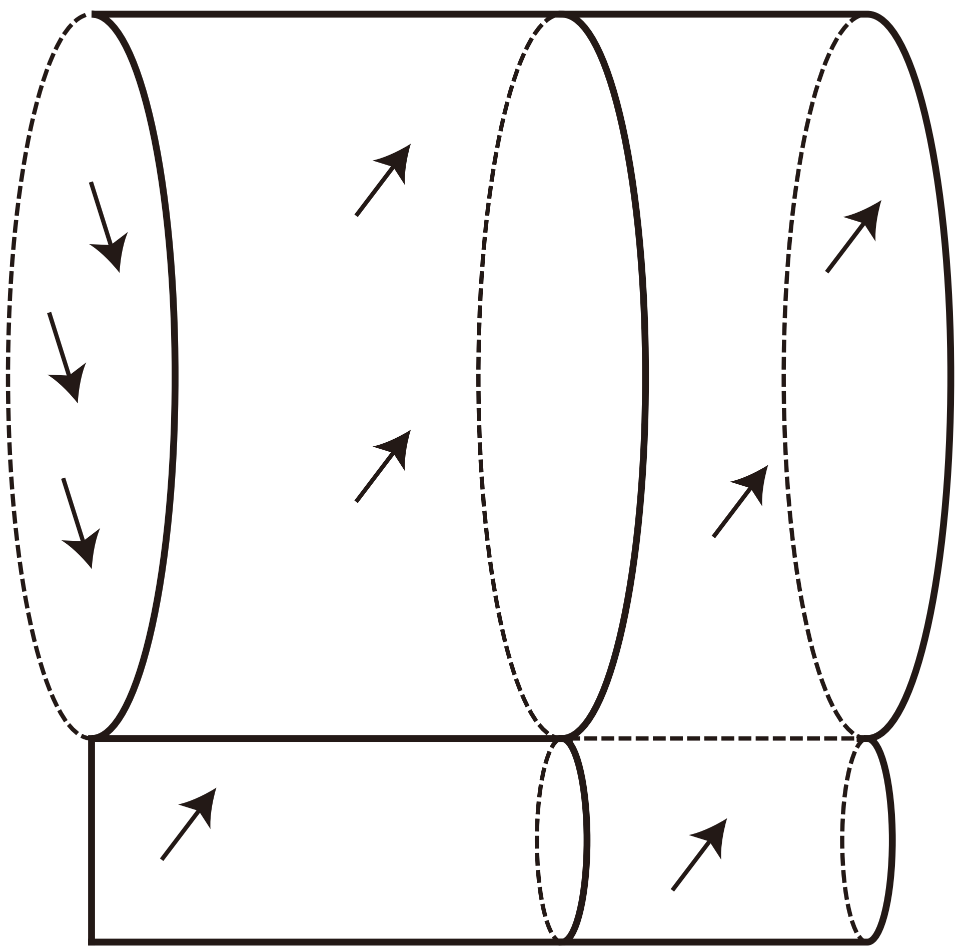
mode2 : (p, q) = (0, 1), dx/dt+x = 1

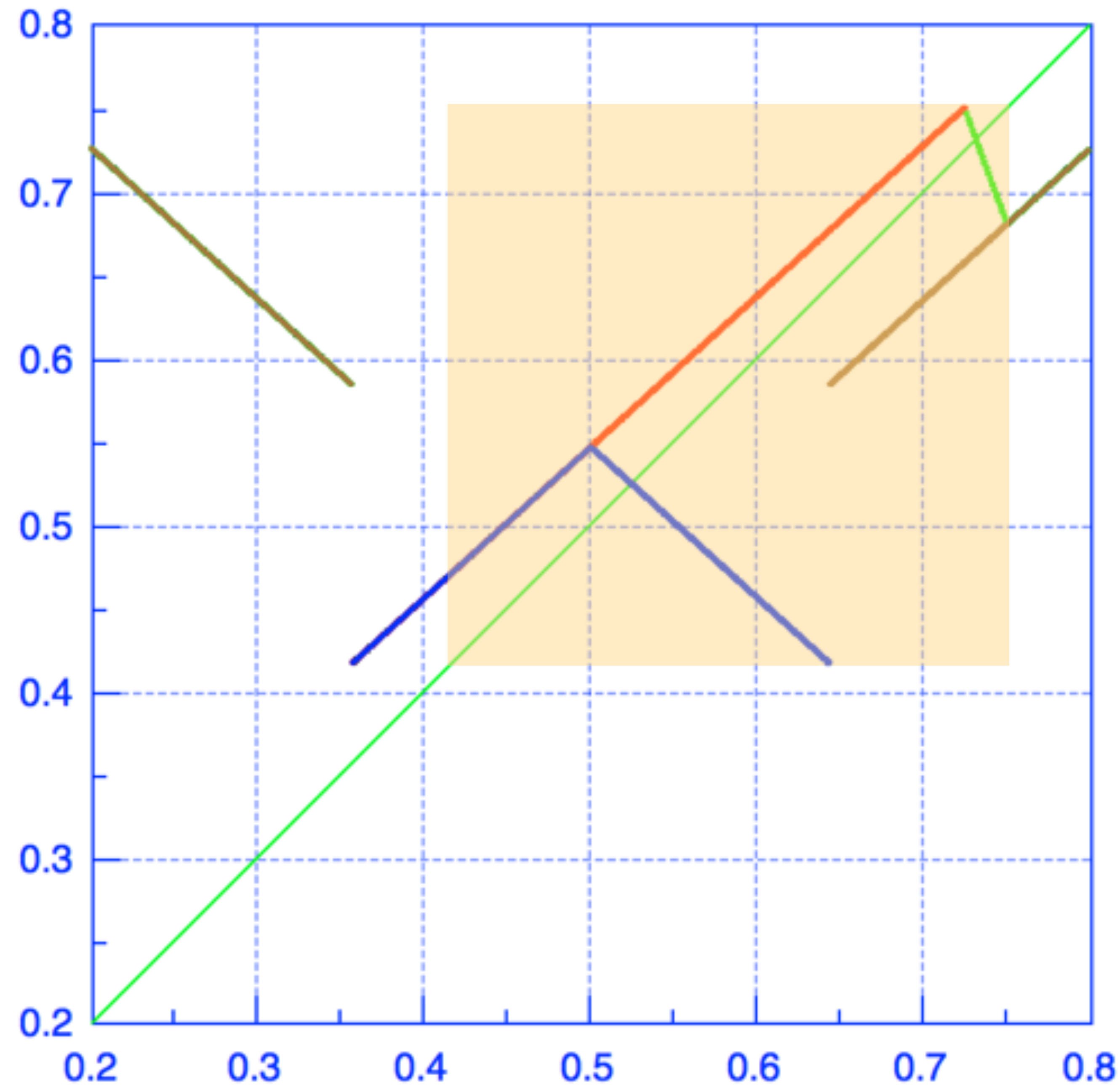
mode3 : (p, q) = (1, 1), dx/dt+x = 1

各モードのflowの様子



貼り合わせトーラス上のPoincaré 写像

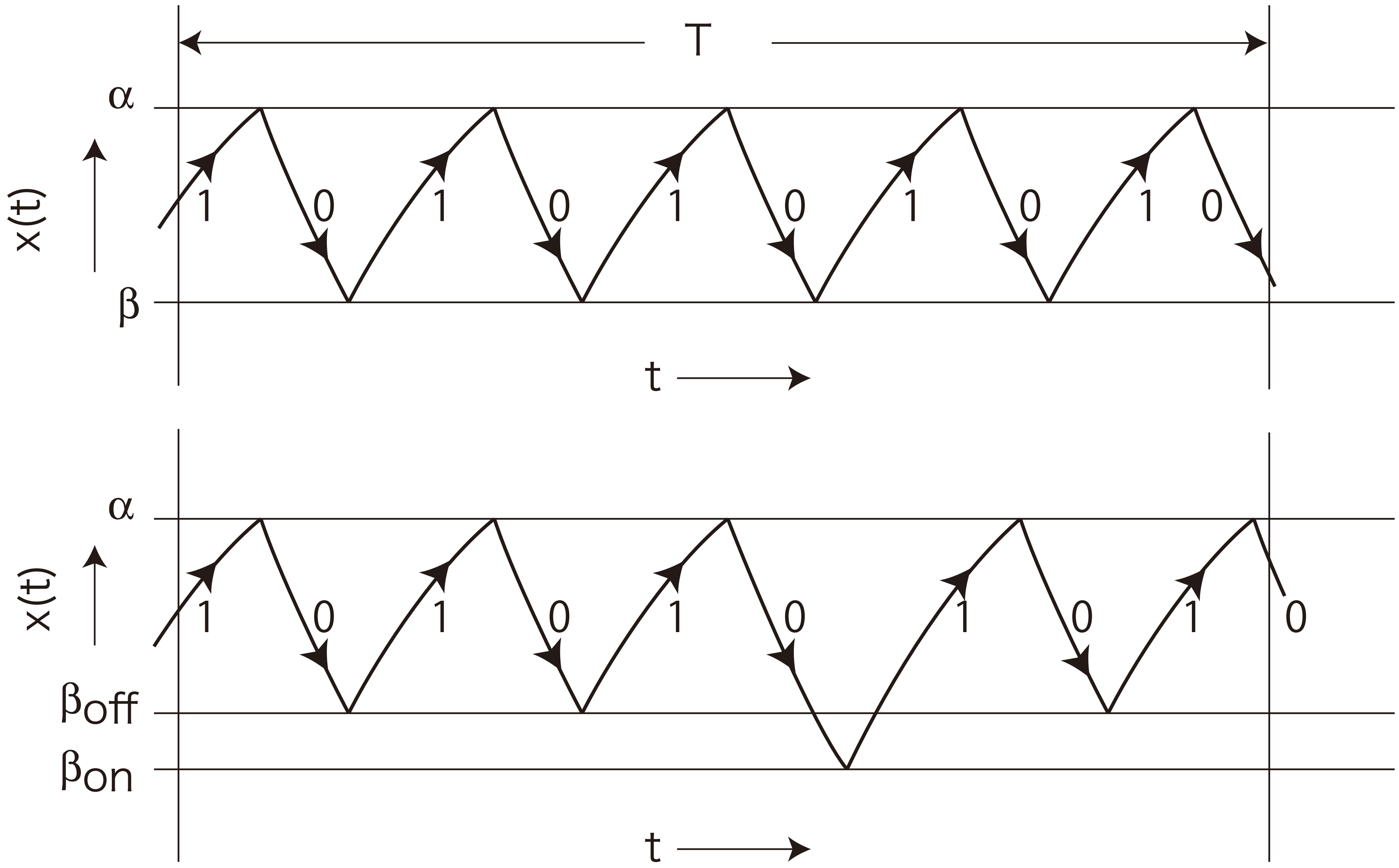




LEDホタルの波形情報

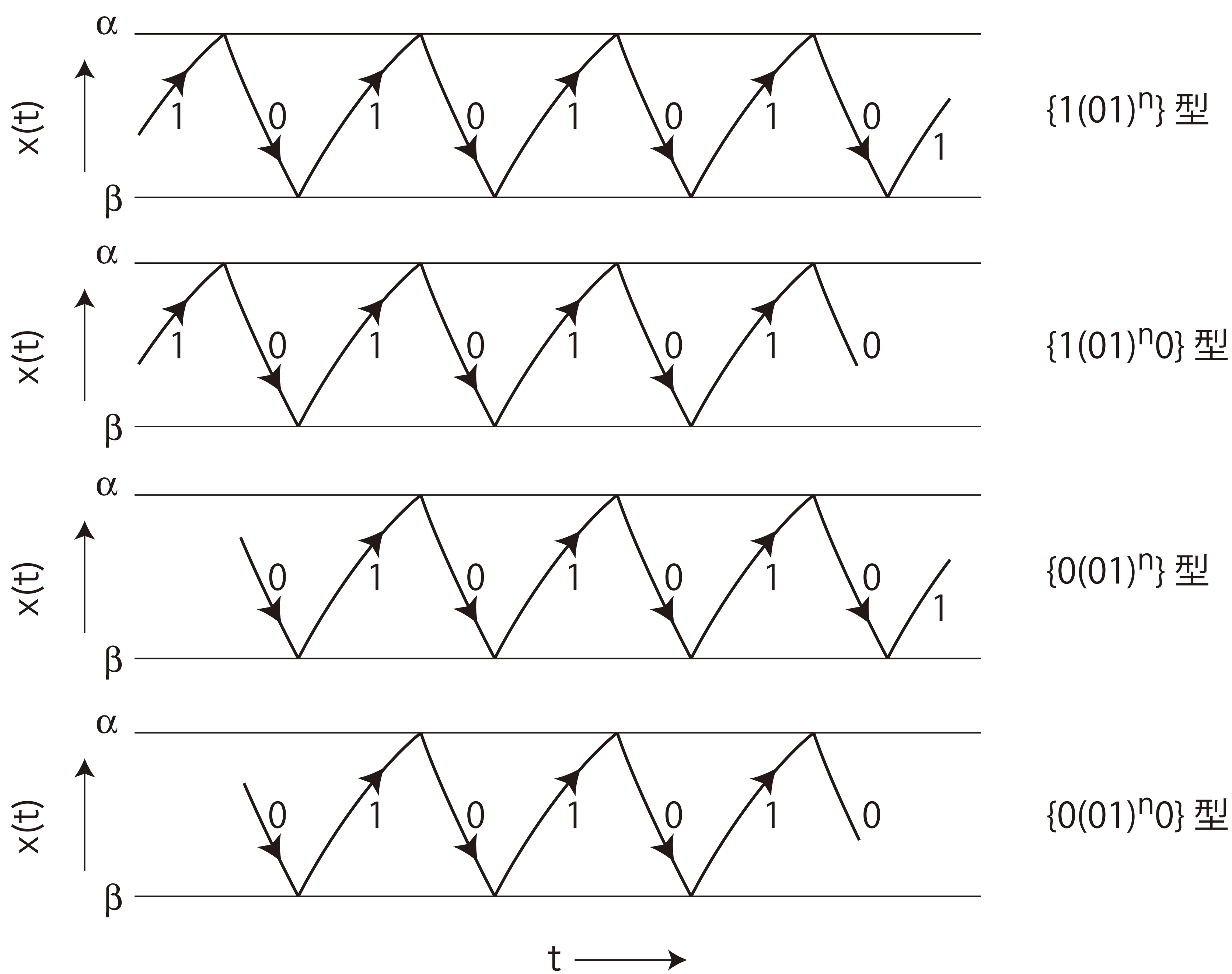
状態 x(t) の波形

◎ 波形の符号数：1010101010=(10)^5

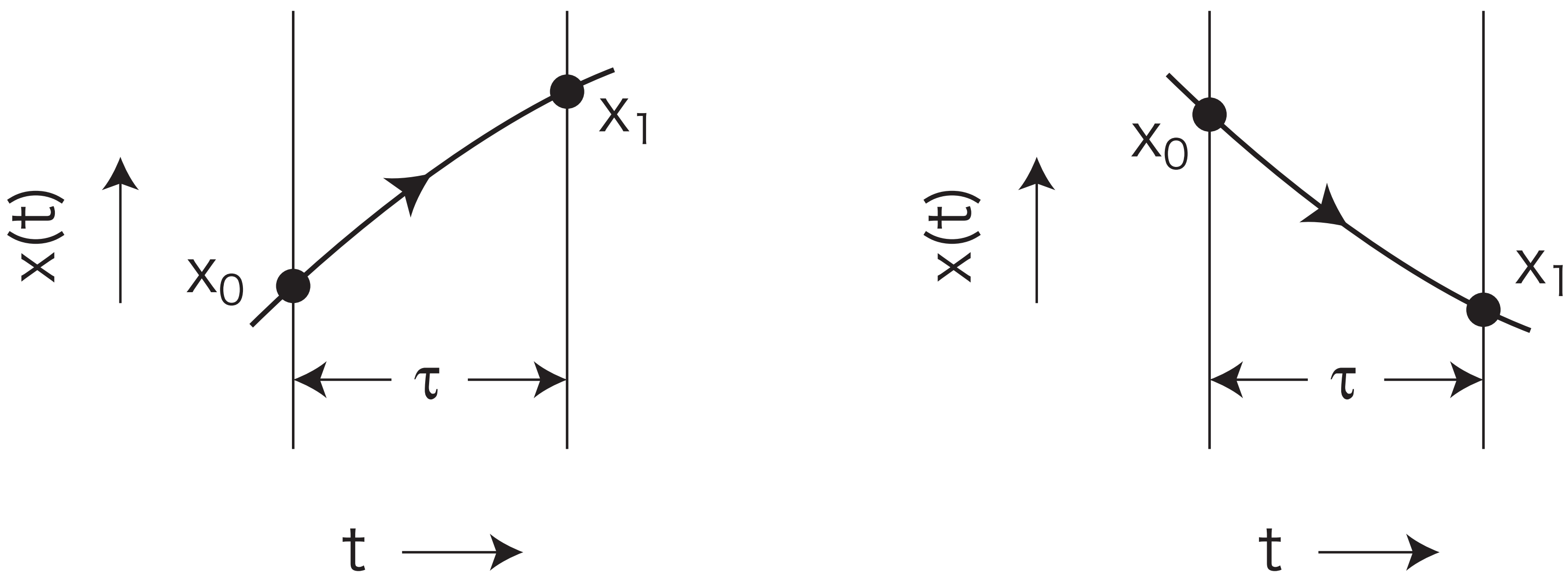


状態 x(t) の波形

◎ 波形の型：11, 10, 00, 01 type



波形と経過時間



(a)

(b)

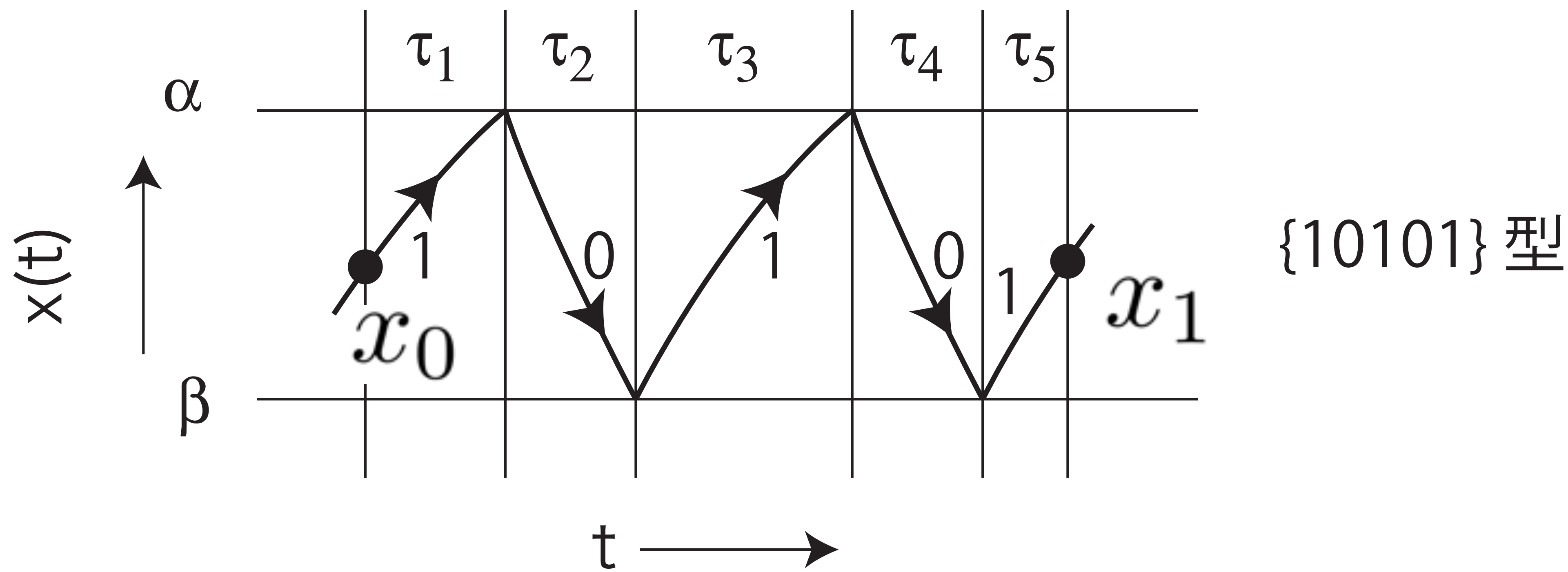
$$x_1 - 1 = e^{-\tau}(x_0 - 1)$$

$$\tau = \ln \frac{x_0 - 1}{x_1 - 1}$$

$$x_1 = e^{-\tau} x_0$$

$$\tau = \ln \frac{x_0}{x_1}$$

{10101}型波形と経過時間



$$\tau_1 = \ln \frac{x_0 - 1}{\alpha - 1}, \tau_2 = \tau_4 = \ln \frac{\alpha}{\beta}, \tau_3 = \ln \frac{\beta - 1}{\alpha - 1}, \tau_5 = \ln \frac{\beta - 1}{x_1 - 1}$$

$$e^{\tau_1 + \cdots + \tau_5} = e^T = \frac{(\beta - 1)^2 \alpha^2 (x_0 - 1)}{(x_1 - 1)(\alpha - 1)^2 \beta^2}$$

$$x_1 - 1 = e^{-T} \left(\frac{\alpha(\beta - 1)}{\beta(\alpha - 1)} \right)^2 (x_0 - 1)$$

波形型と経過時間

{1(01)^n}型

$$x_1 - 1 = e^{-T}\left(\frac{\alpha(\beta-1)}{\beta(\alpha-1)}\right)^n (x_0 - 1)$$

{1(01)^n0}型

$$x_1 = e^{-T}\left(\frac{\alpha(\beta-1)}{\beta(\alpha-1)}\right)^n \frac{\alpha}{\alpha-1}(x_0 - 1)$$

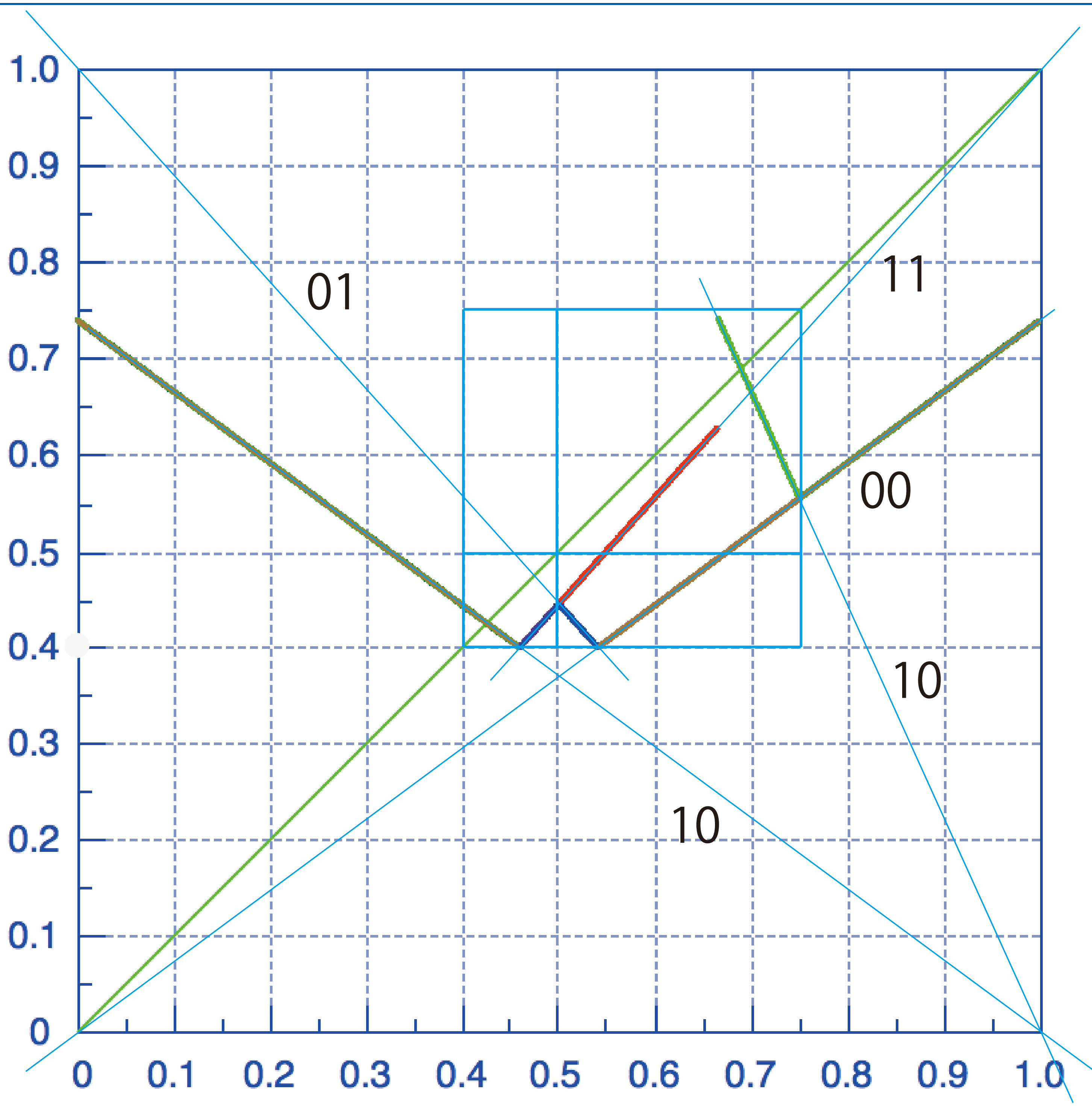
{0(10)^n}型

$$x_1 = e^{-T}\left(\frac{\alpha(\beta-1)}{\beta(\alpha-1)}\right)^n x_0$$

{0(10)^n1}型

$$x_1 - 1 = e^{-T}\left(\frac{\alpha(\beta-1)}{\beta(\alpha-1)}\right)^n \frac{\beta-1}{\beta} x_0$$

Time one map への応用



周期波形と周期解

周期波形の型

◎ m個のフェーズイベントをもつ周期波形の符号数：

$$\{1(01)^{m-1}0\}$$

◎ \beta eventの数：

$$\#\beta_{off} + \#\beta_{on} + \#\beta = \#\beta_{off} + \#\beta_{on} + 1 = m$$

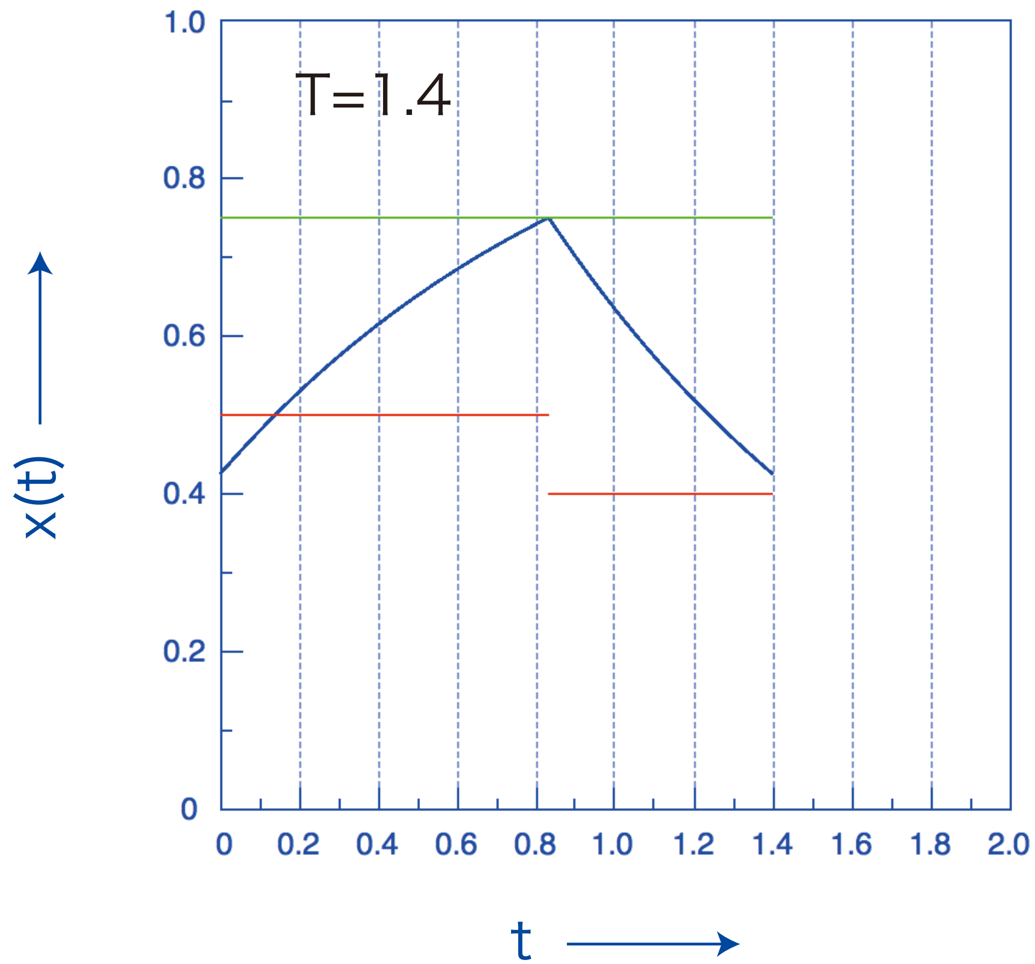
◎ n周期時間（タイマーイベントがn回）に埋め込むと

$$\{\{1(01)^{m-1}0\}, (\#\beta_{off}, \#\beta_{on}, \#\beta), n\}$$

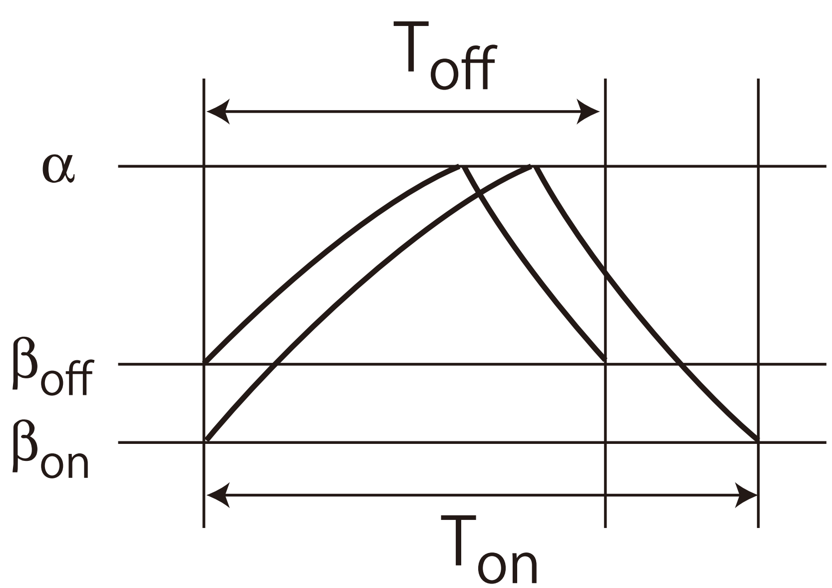
周期波形の型

m	{1(01)^(m-1)0}	(#boff,#bon,1)
1	10	(0,0,1)
2	1010	(1,0,1) (0,1,1)
3	(10)^3	(2,0,1) (1,1,1) (0,2,1)

{{10},(0,0,1),1}型周期解の例



$$\alpha = 0.75, \beta_{off} = 0.5, \beta_{on} = 0.4$$

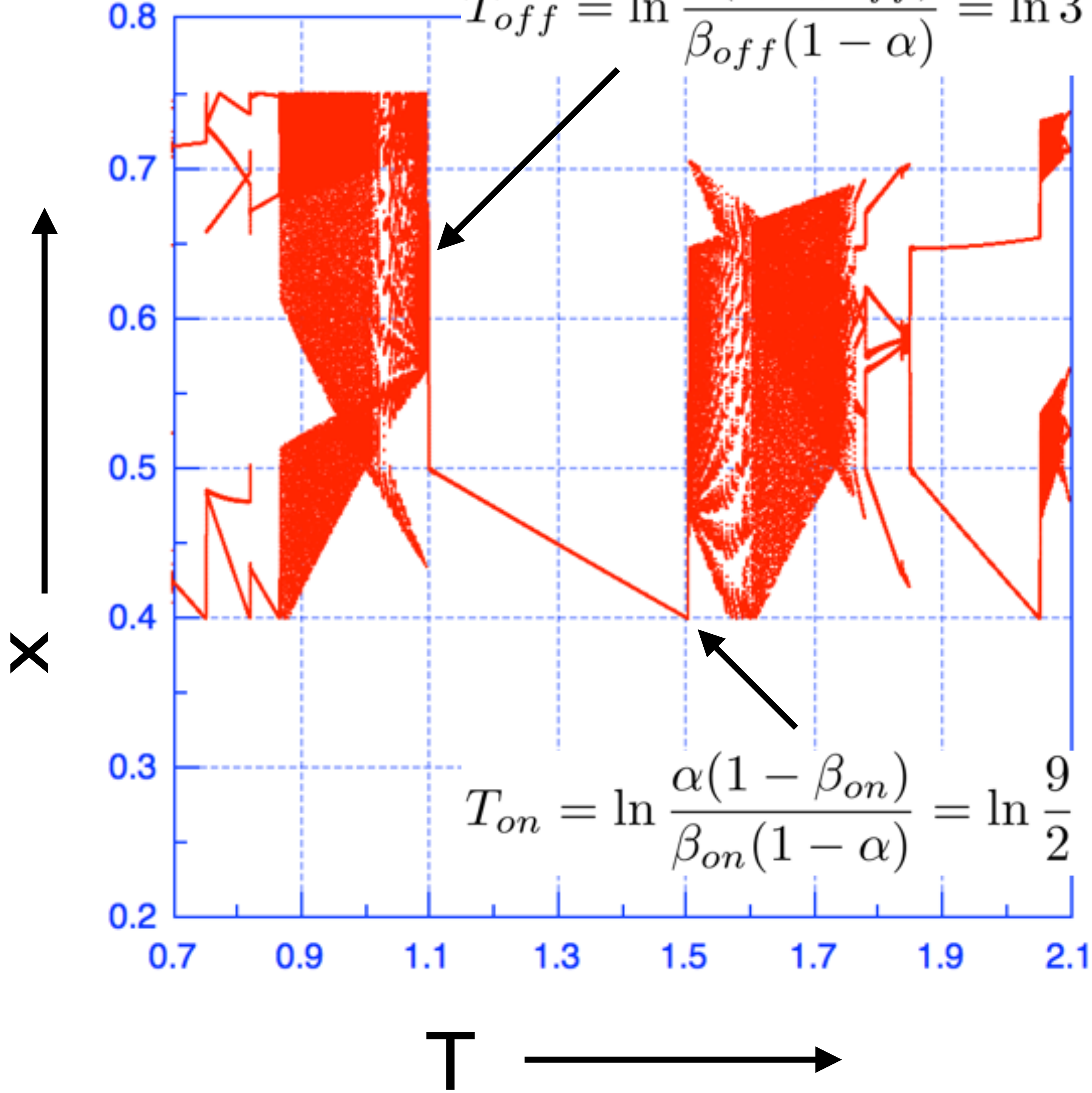


$$T_{off} = \ln \frac{\alpha(1-\beta_{off})}{\beta_{off}(1-\alpha)} = \ln 3 = 1.0986$$

$$T_{on} = \ln \frac{\alpha(1-\beta_{on})}{\beta_{on}(1-\alpha)} = \ln \frac{9}{2} = 1.5041$$

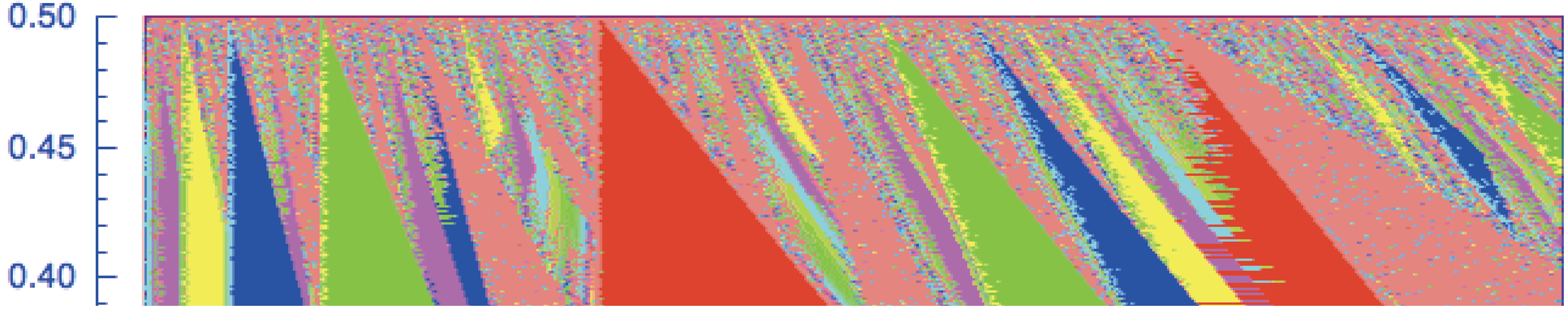
外力の周期を変化させた場合の分岐図

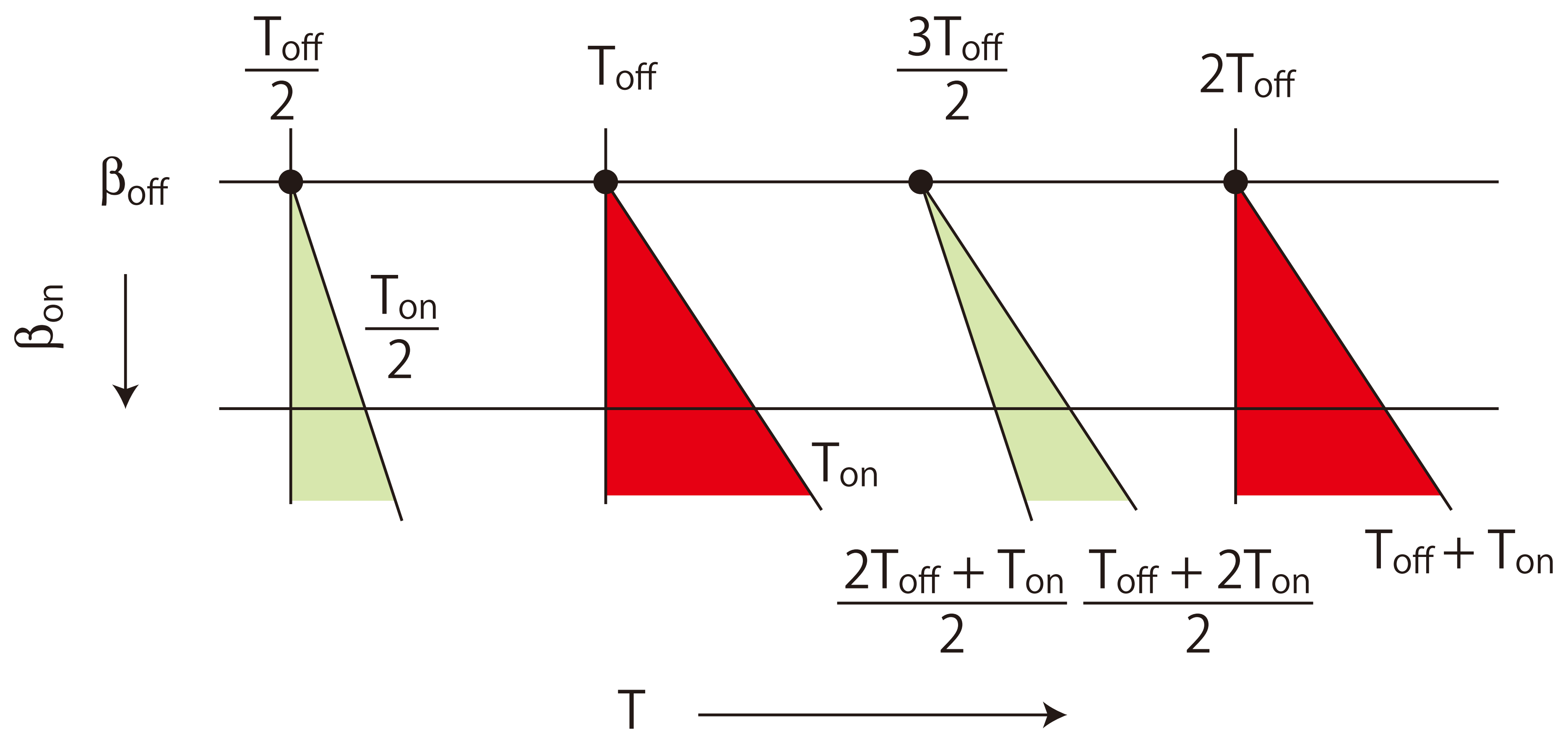
$$T_{off} = \ln \frac{\alpha(1-\beta_{off})}{\beta_{off}(1-\alpha)} = \ln 3 = 1.0986$$



$$T_{on} = \ln \frac{\alpha(1-\beta_{on})}{\beta_{on}(1-\alpha)} = \ln \frac{9}{2} = 1.5041$$

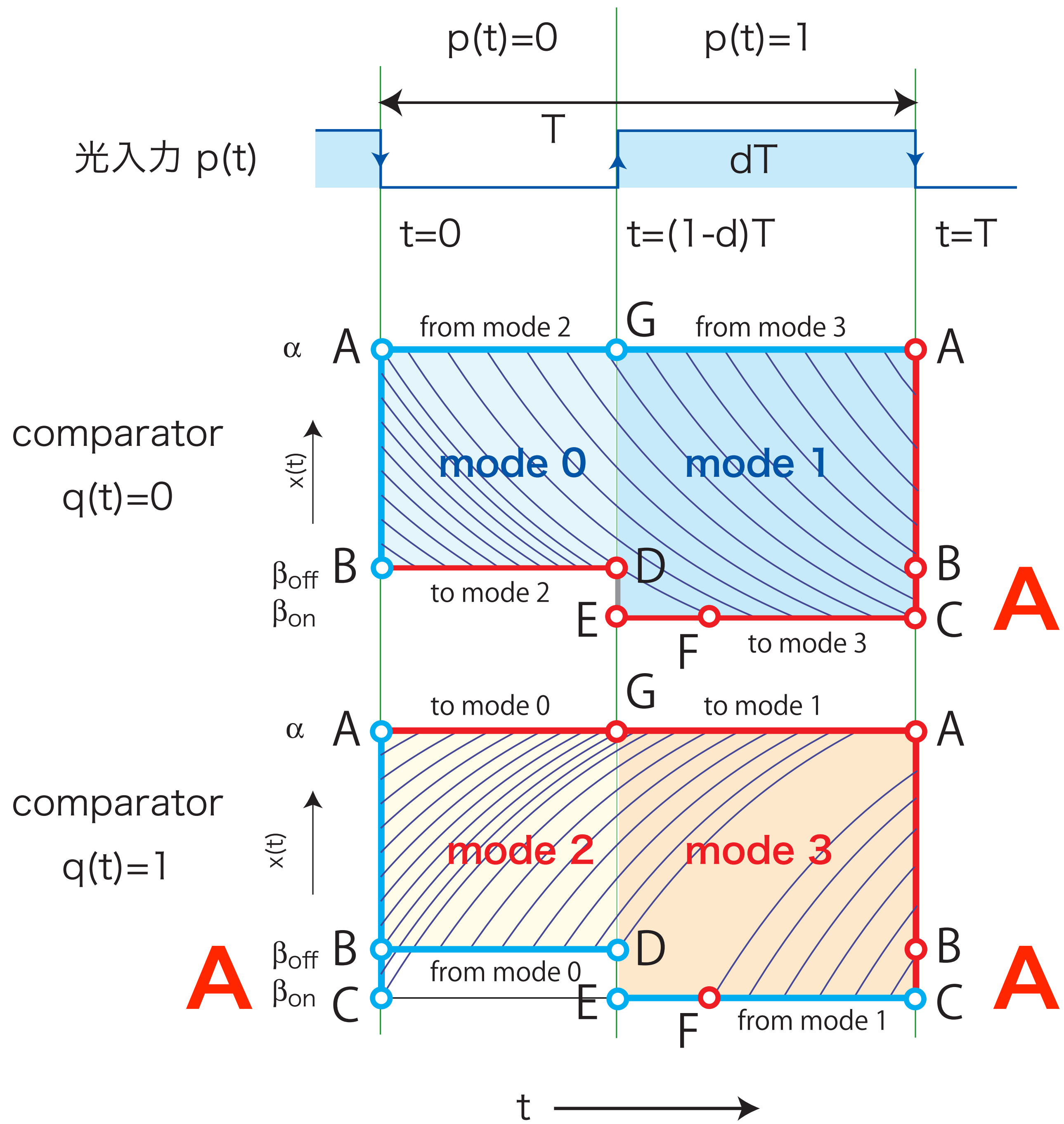
外力の周期とbeta_on変化させた場合の分岐図



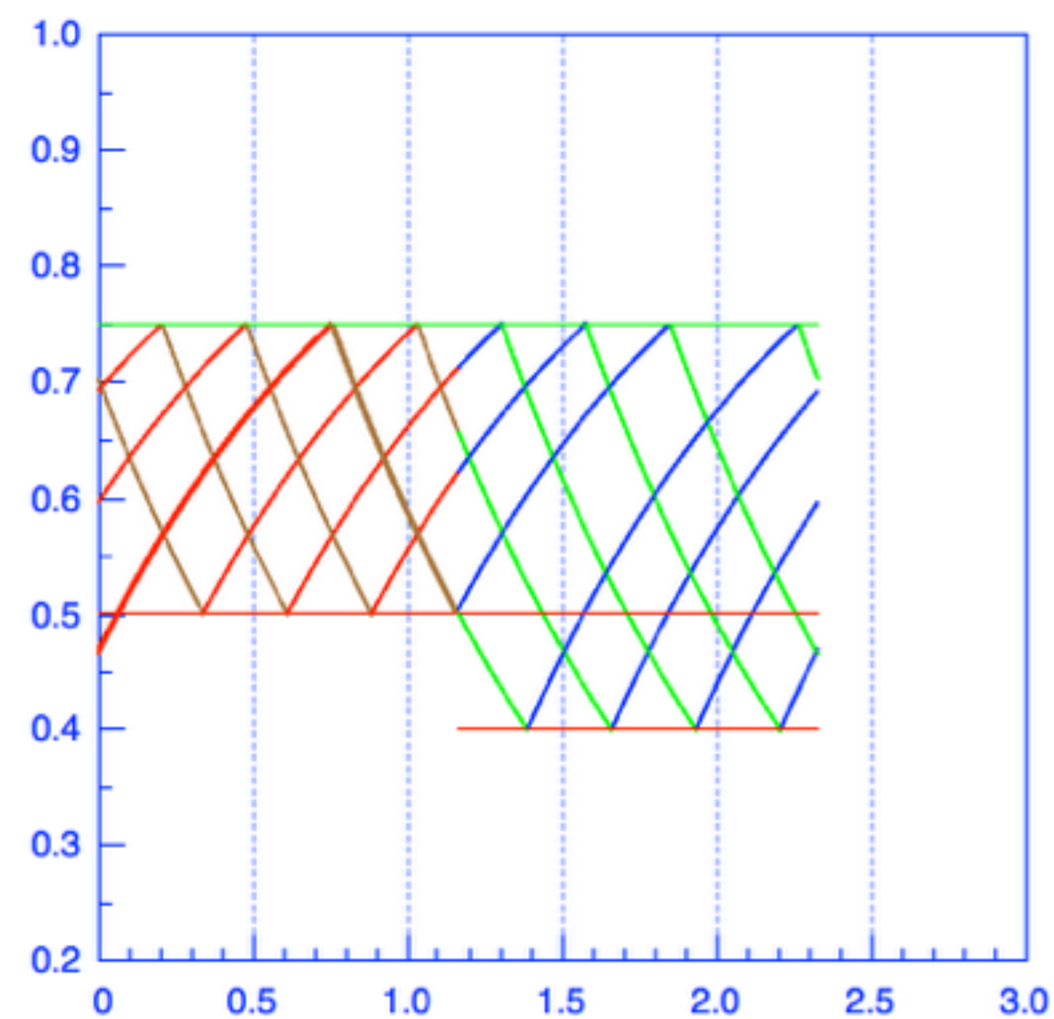


貼り合わせトーラス

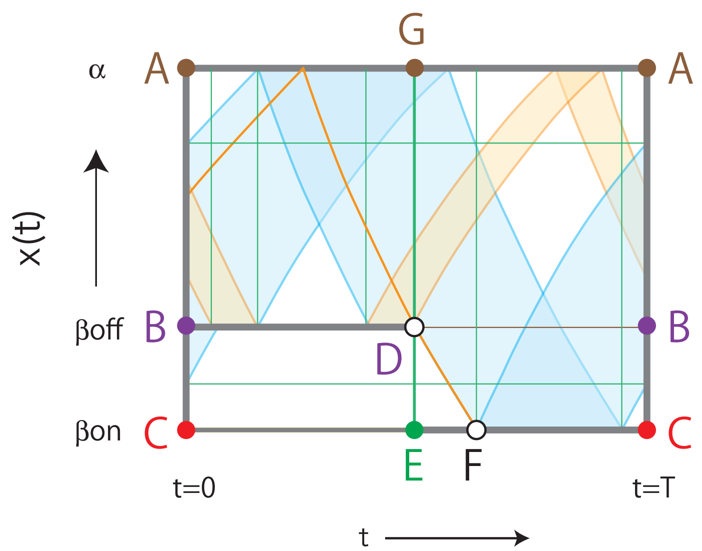
1 周期分の波形

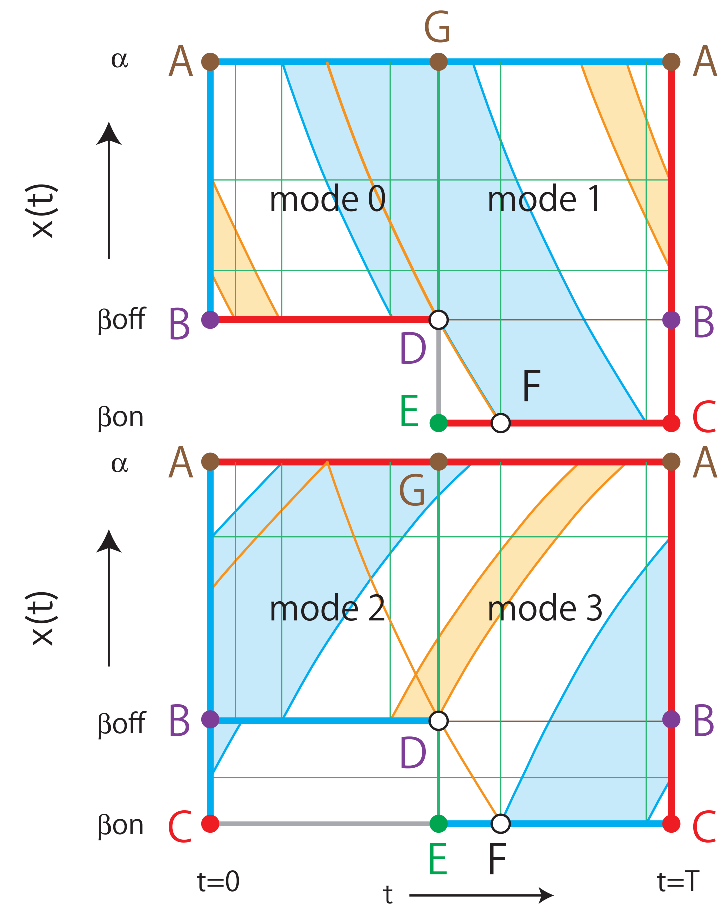


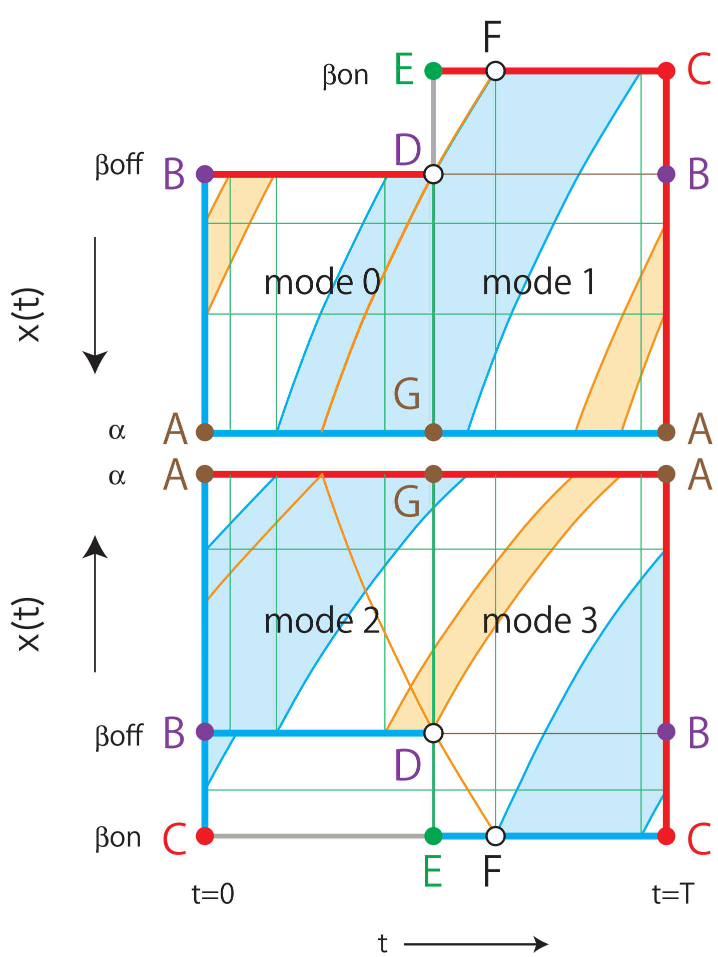
p(t)=0
p(t)=1
T
光入力 p(t)
dT
t=0
t=(1-d)T
t=T
from mode 2
G
from mode 3
α
A
A
comparator
q(t)=0
x(t)
mode 0
mode 1
βoff
B
D
B
βon
to mode 2
E
C
A
F
to mode 3
to mode 0
G
to mode 1
α
A
A
comparator
q(t)=1
x(t)
mode 2
mode 3
A
βoff
B
D
B
A
βon
C
from mode 0
E
C
F
from mode 1
t

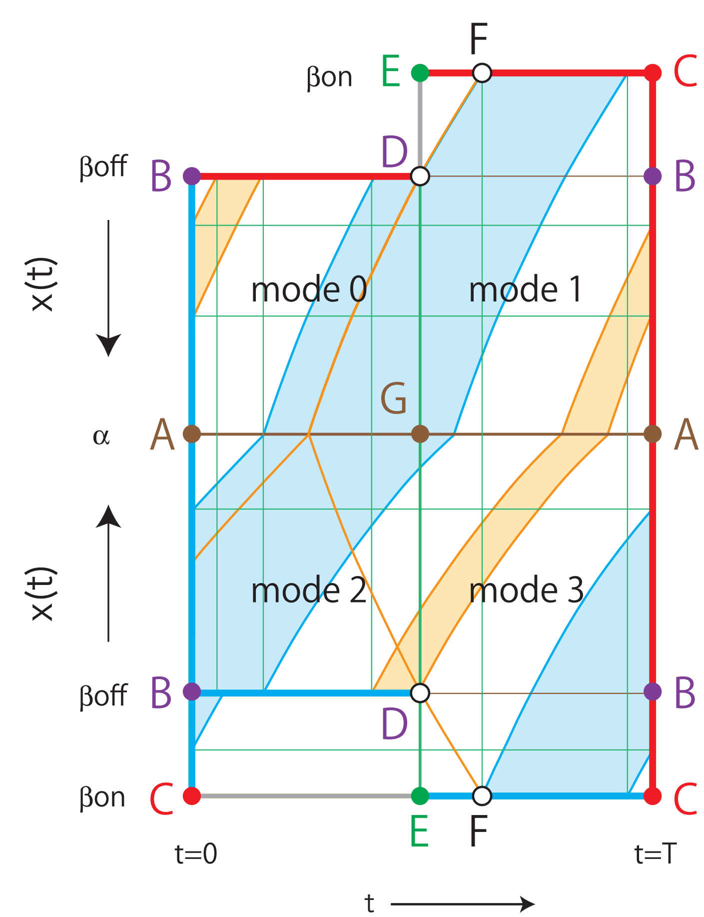


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
0.5
1.0
1.5
2.0
2.5
3.0

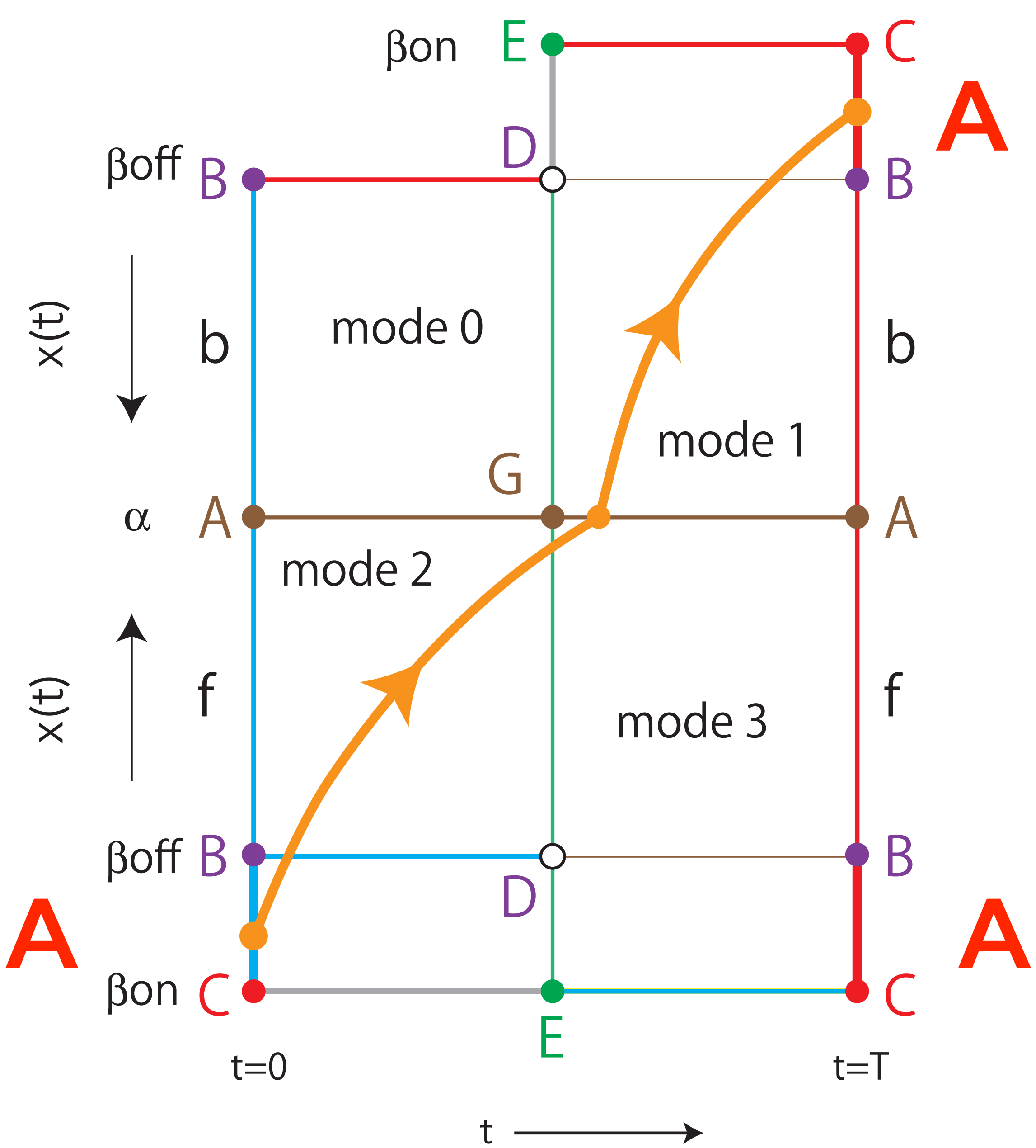

G
α A
A
x(t)
βoff B
B
D
βon C
C
E F
t=0
t=T
t


G
α A
A
x(t)
mode 0
mode 1
βoff B
B
D
F
βon
E
C
α A
A
G
x(t)
mode 2
mode 3
βoff B
B
D
βon C
C
E F
t=0
t
t=T

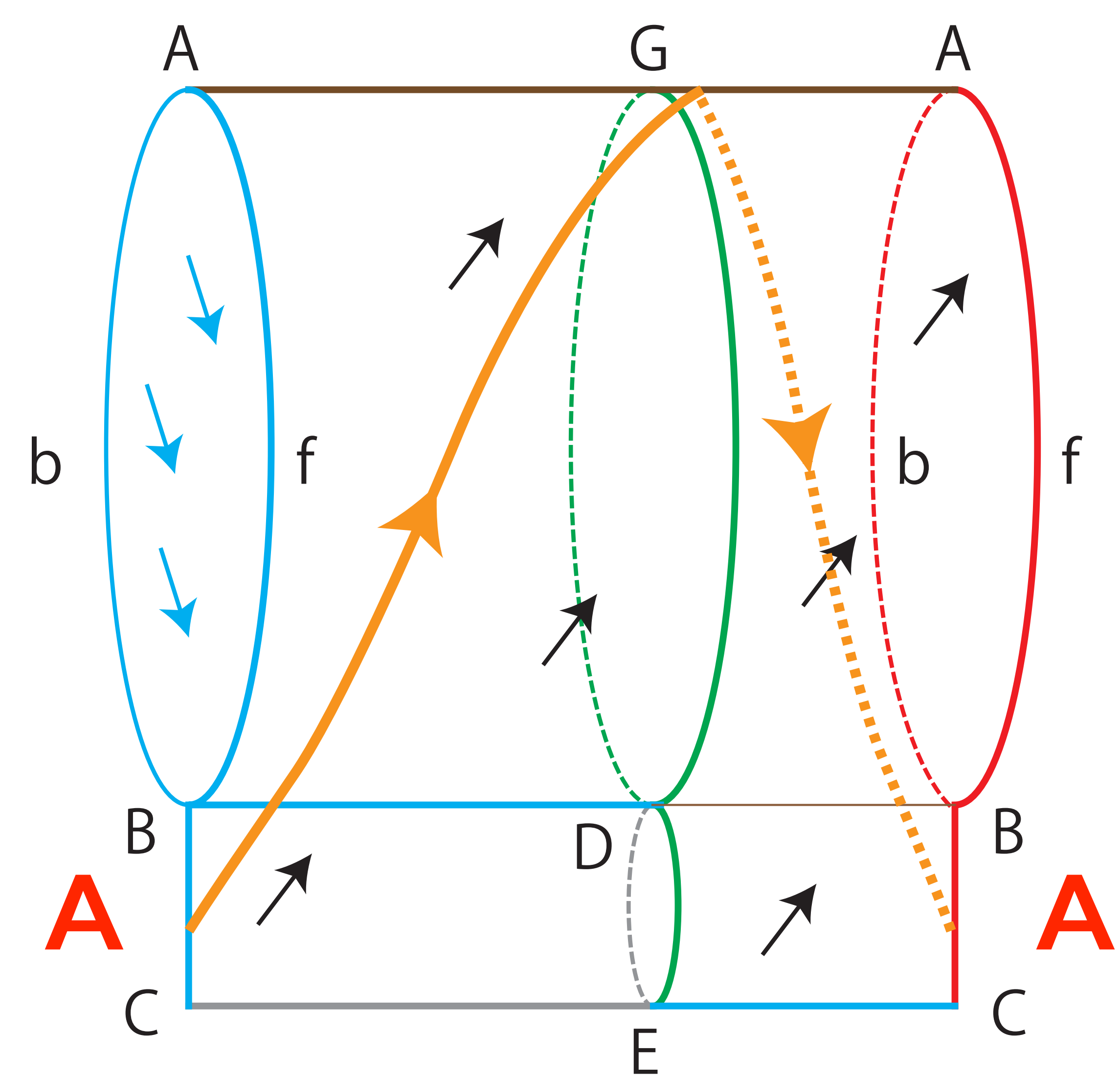

F
βon E
C
D
βoff B
B
x(t)
mode 0
mode 1
G
α A
A
α A
A
G
x(t)
mode 2
mode 3
βoff B
B
D
βon C
C
E F
t=0
t=T
t


F
βon E
C
D
βoff B
B
x(t)
mode 0
mode 1
G
α A
A
x(t)
mode 2
mode 3
βoff B
B
D
βon C
C
E F
t=0
t=T
t

貼り合わせトーラス

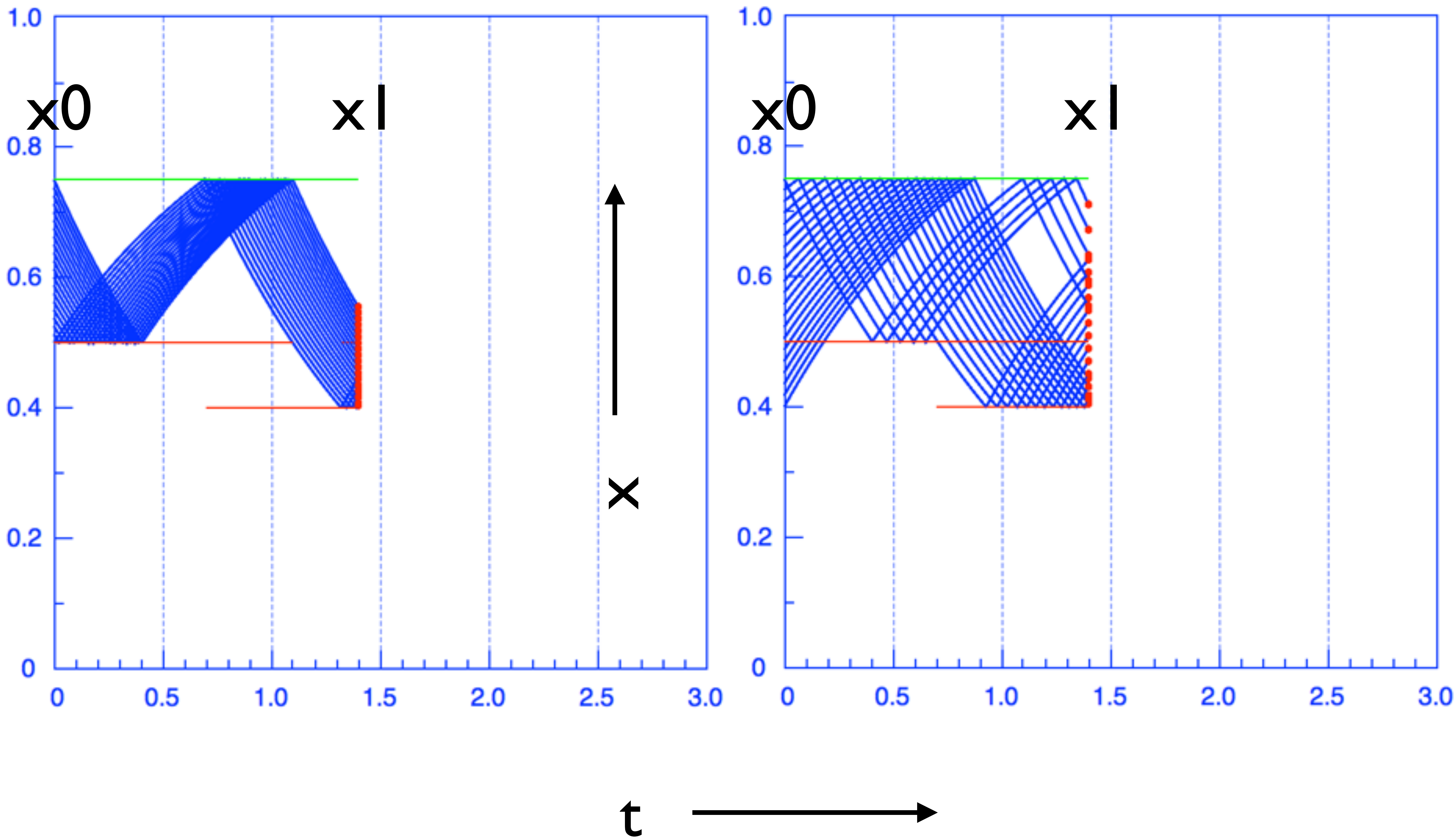


(a)

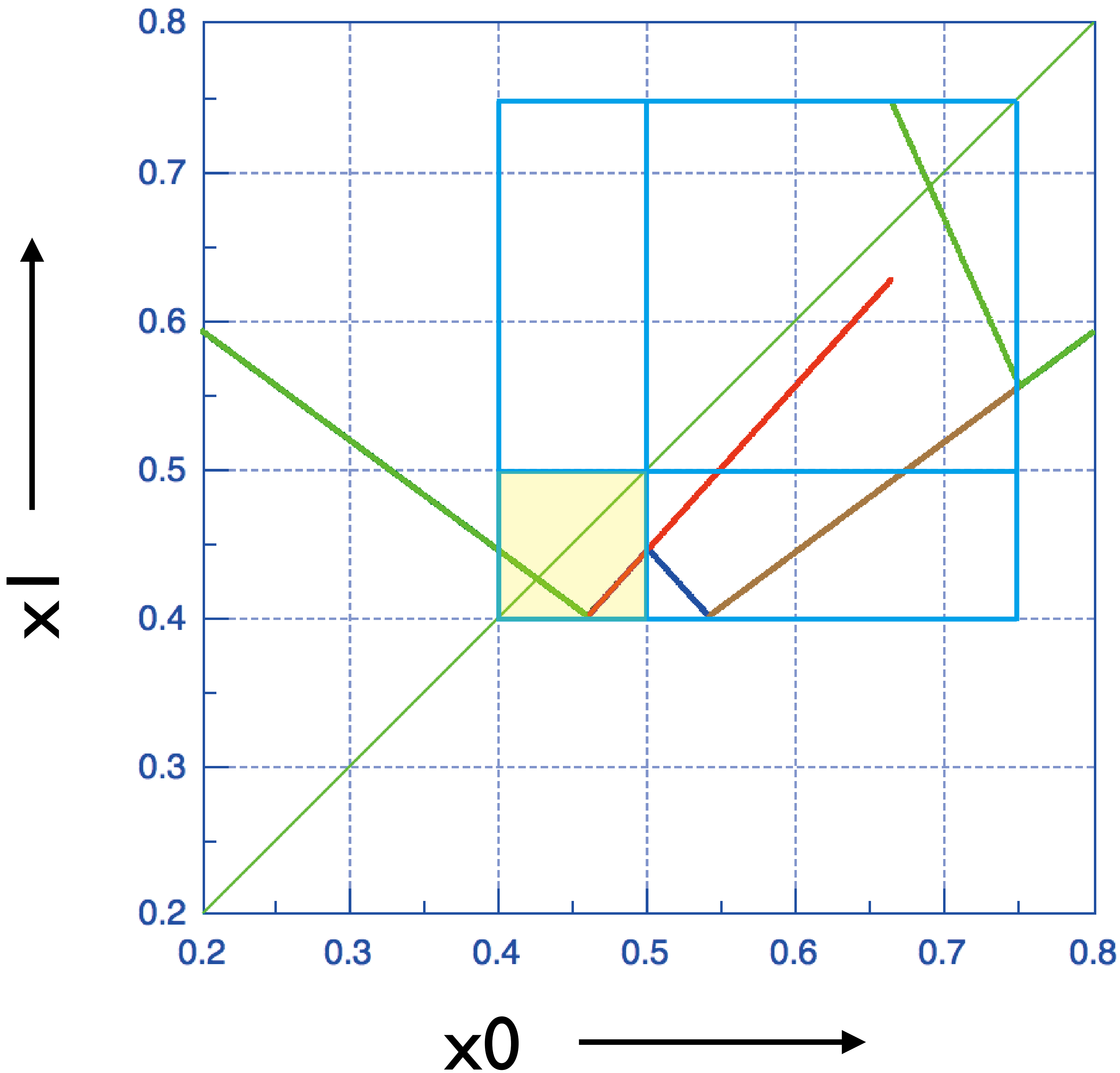


(b)

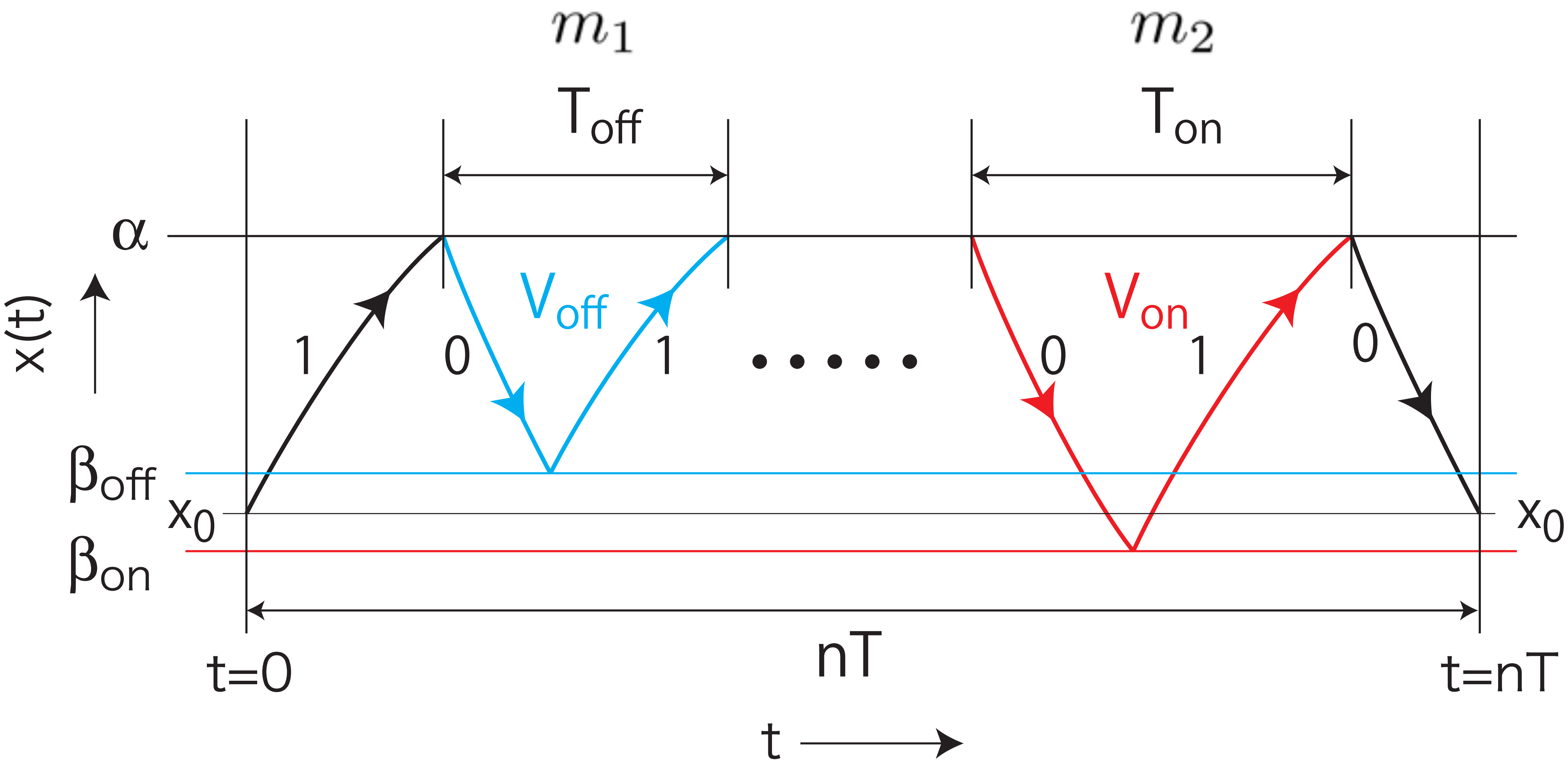
Time One Map: T=1.4



Time One Map: T=1.4



周期解の一般形



$$T_{off}^{m_1+1} + T_{on}^{m_2} < nT < T_{off}^{m_1} + T_{on}^{m_2+1}$$

$$T_{off}^{m_1+1} + T_{on}^{m_2} < \frac{T}{n} < T_{off}^{m_1} + T_{on}^{m_2+1}$$

ハイブリッド回路の定性的解析法

１．数学モデルの定式化

a) digital part: mode数の決定，mode遷移の導出

b) analog part: vector field, 回路方程式の導出

２．貼合せ多様体をつくる

Poincaré断面を定義し，Poincaré写像をつくる

３．運動の解析

各種不変集合を求め，安定性や分岐を考察する