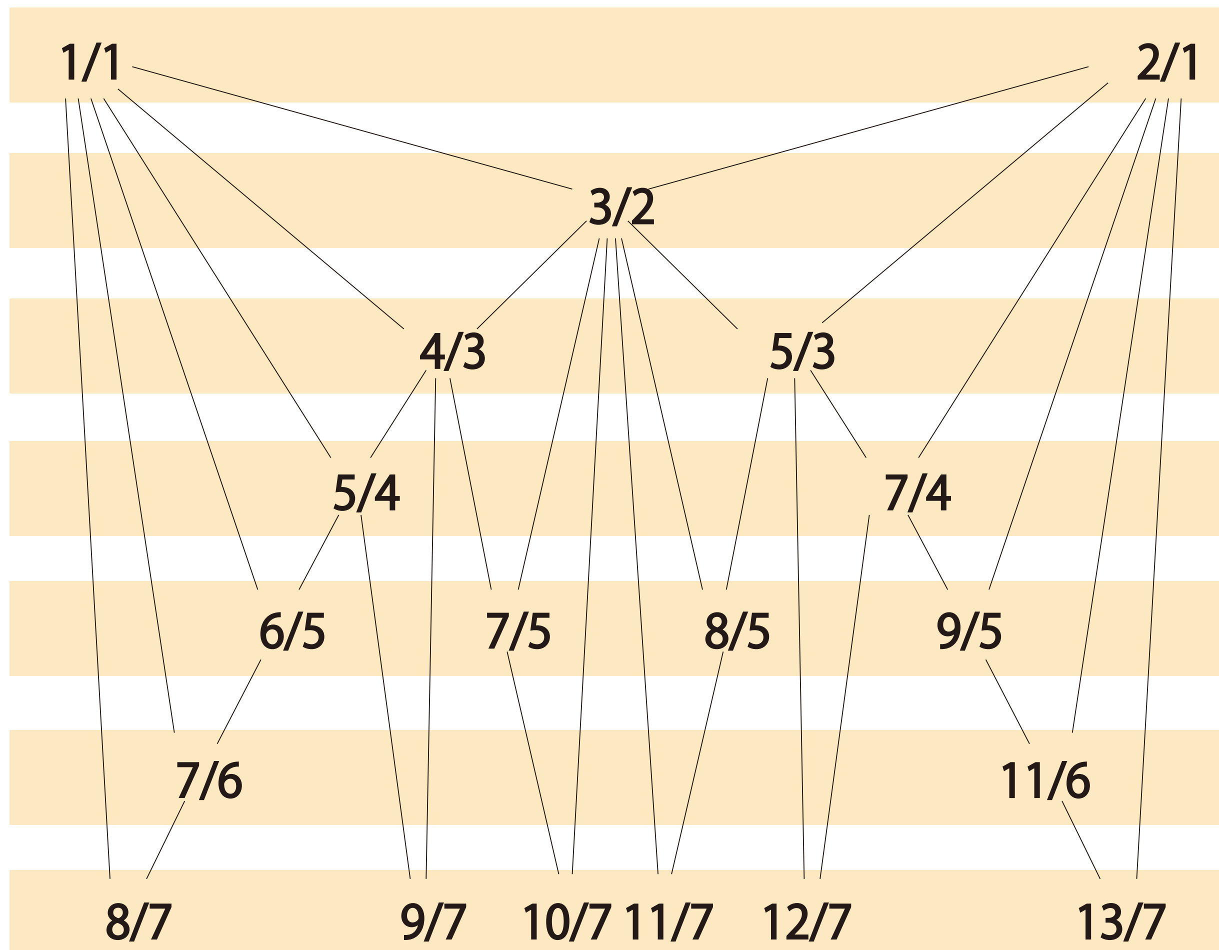
上田研　ゼミ

LEDホタルA1の強制振動 (II)



川上　博

2014(H26).10.27

話の流れ

1．力学系の導出

◎ ホタルの状態空間：貼り合わせ多様体

◎ イベントと状態の運動則

◎ 状態の時間発展

2．波形の持つ情報

◎ 波形の型：符号数付き波形

3．周期波形とその分岐

◎ 周期波形の分類と分岐

4．非周期波形の存在

◎ 準周期解かカオス解か

周期波形と周期解

周期波形の型

◎ m個のフェーズイベントをもつ周期波形の符号数：

$$\{1(01)^{m-1}0\}$$

◎ \beta eventの数：

$$\#\beta_{off} + \#\beta_{on} + \#\beta = \#\beta_{off} + \#\beta_{on} + 1 = m$$

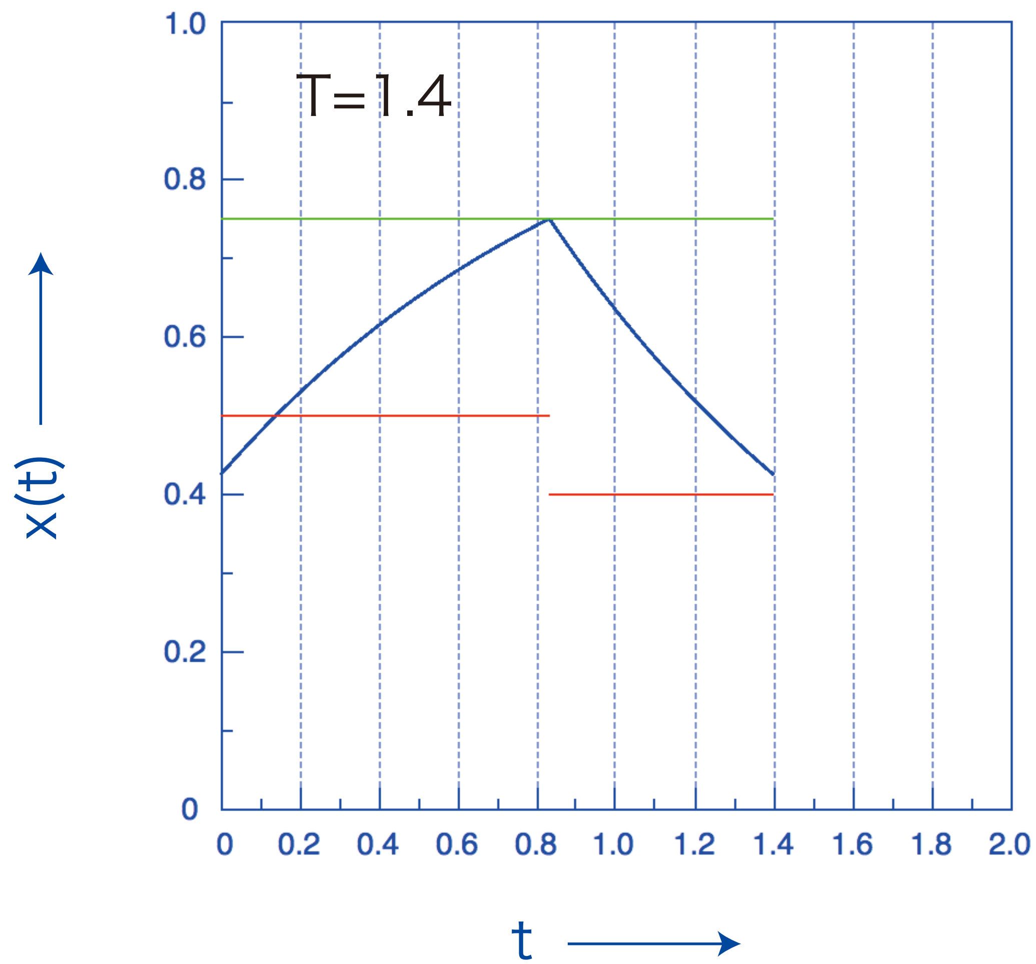
◎ n周期時間（タイマーイベントがn回）に埋め込むと

$$\{\{1(01)^{m-1}0\}, (\#\beta_{off}, \#\beta_{on}, \#\beta), n\}$$

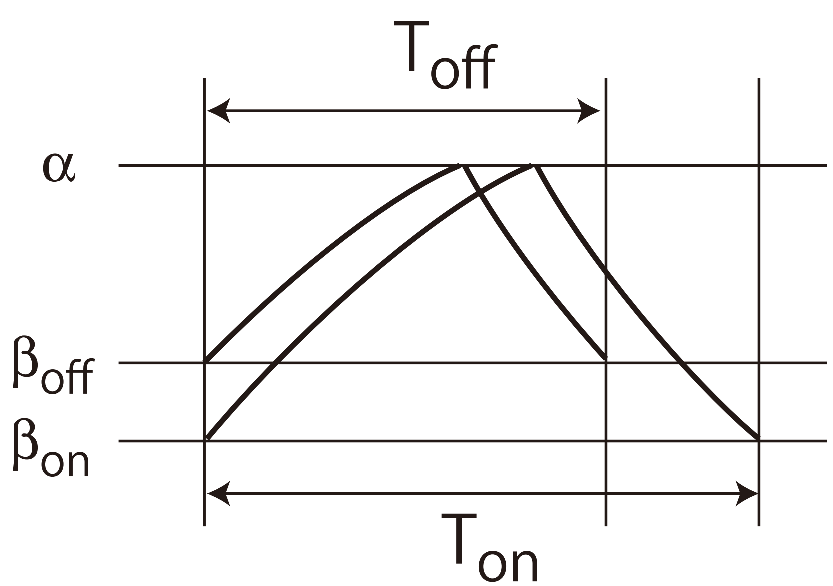
周期波形の型

m	{1(01)^(m-1)0}	(#boff,#bon,1)
1	10	(0,0,1)
2	1010	(1,0,1) (0,1,1)
3	(10)^3	(2,0,1) (1,1,1) (0,2,1)

{{10},(0,0,1),1}型周期解の例



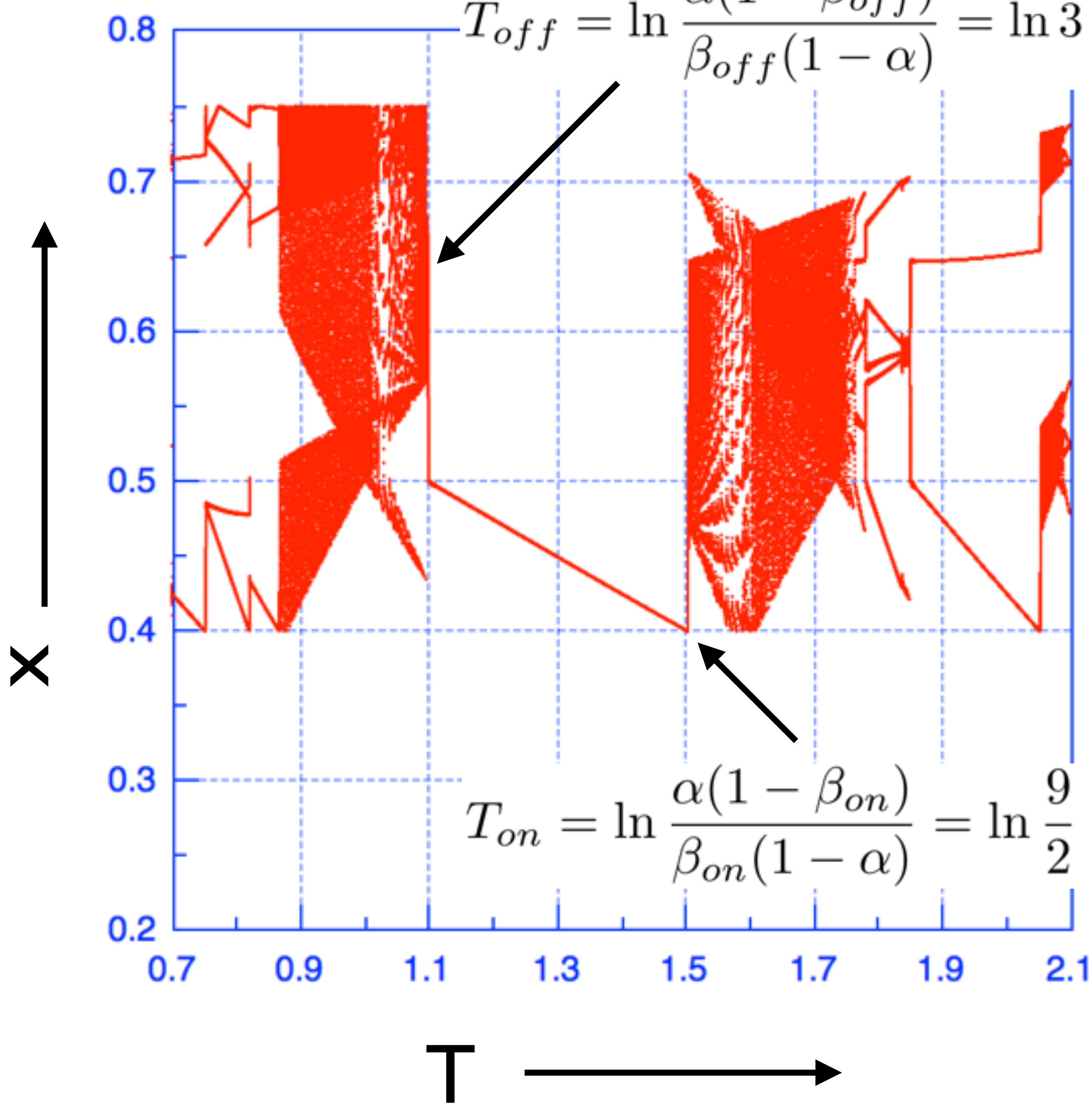
$$\alpha = 0.75, \beta_{off} = 0.5, \beta_{on} = 0.4$$



$$T_{off} = \ln \frac{\alpha(1-\beta_{off})}{\beta_{off}(1-\alpha)} = \ln 3 = 1.0986$$

$$T_{on} = \ln \frac{\alpha(1-\beta_{on})}{\beta_{on}(1-\alpha)} = \ln \frac{9}{2} = 1.5041$$

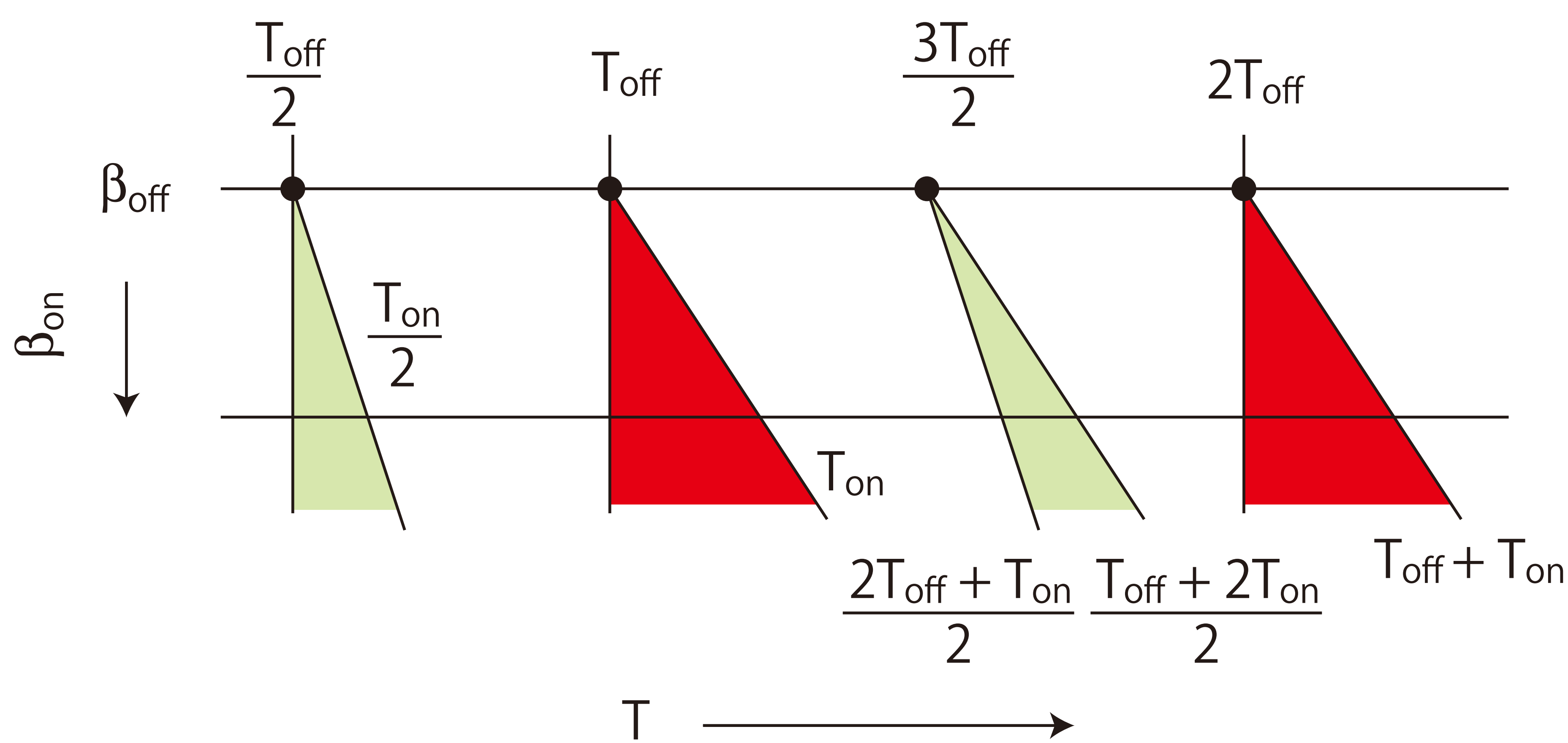
外力の周期を変化させた場合の分岐図



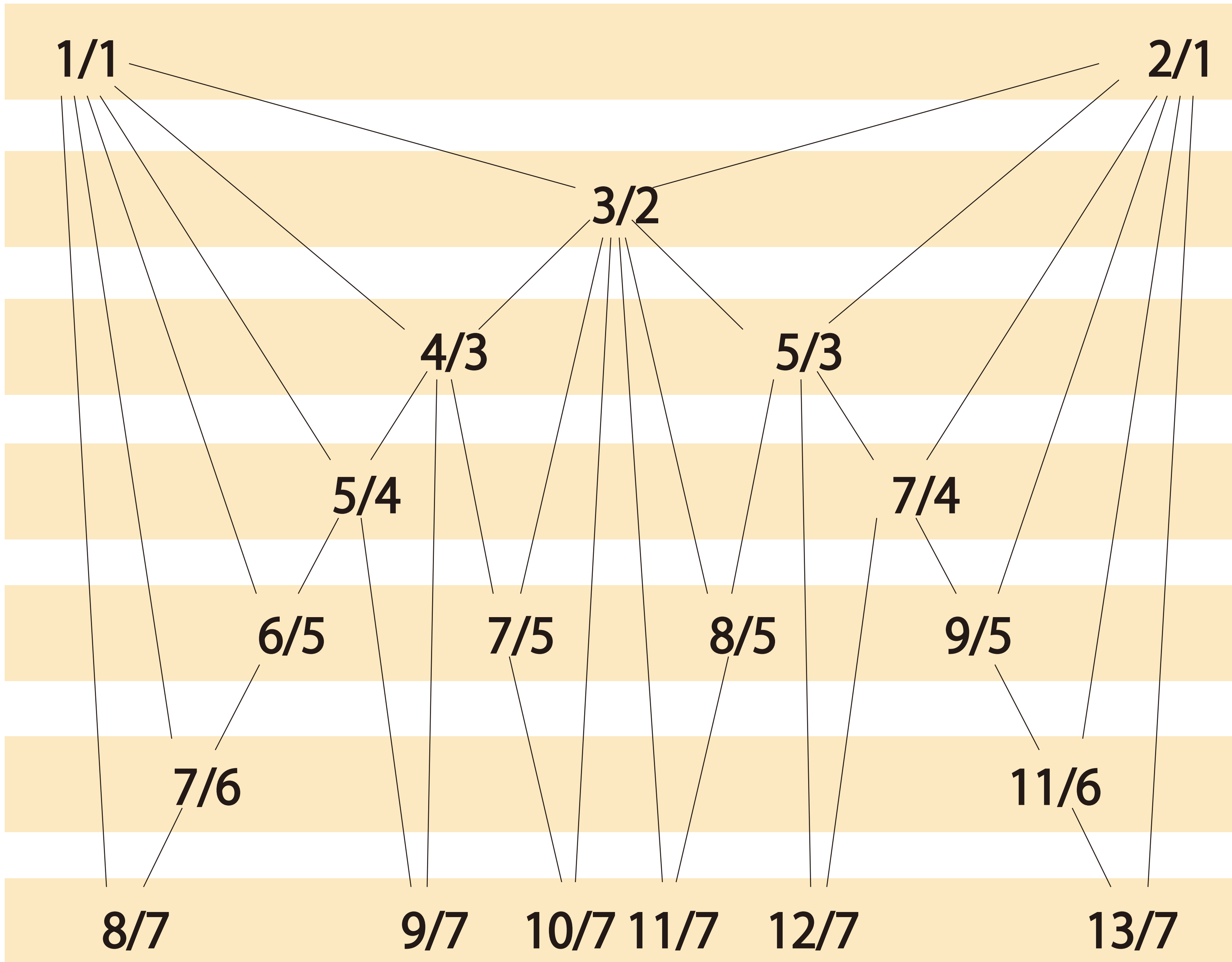
$$T_{off} = \ln \frac{\alpha(1-\beta_{off})}{\beta_{off}(1-\alpha)} = \ln 3 = 1.0986$$

$$T_{on} = \ln \frac{\alpha(1-\beta_{on})}{\beta_{on}(1-\alpha)} = \ln \frac{9}{2} = 1.5041$$

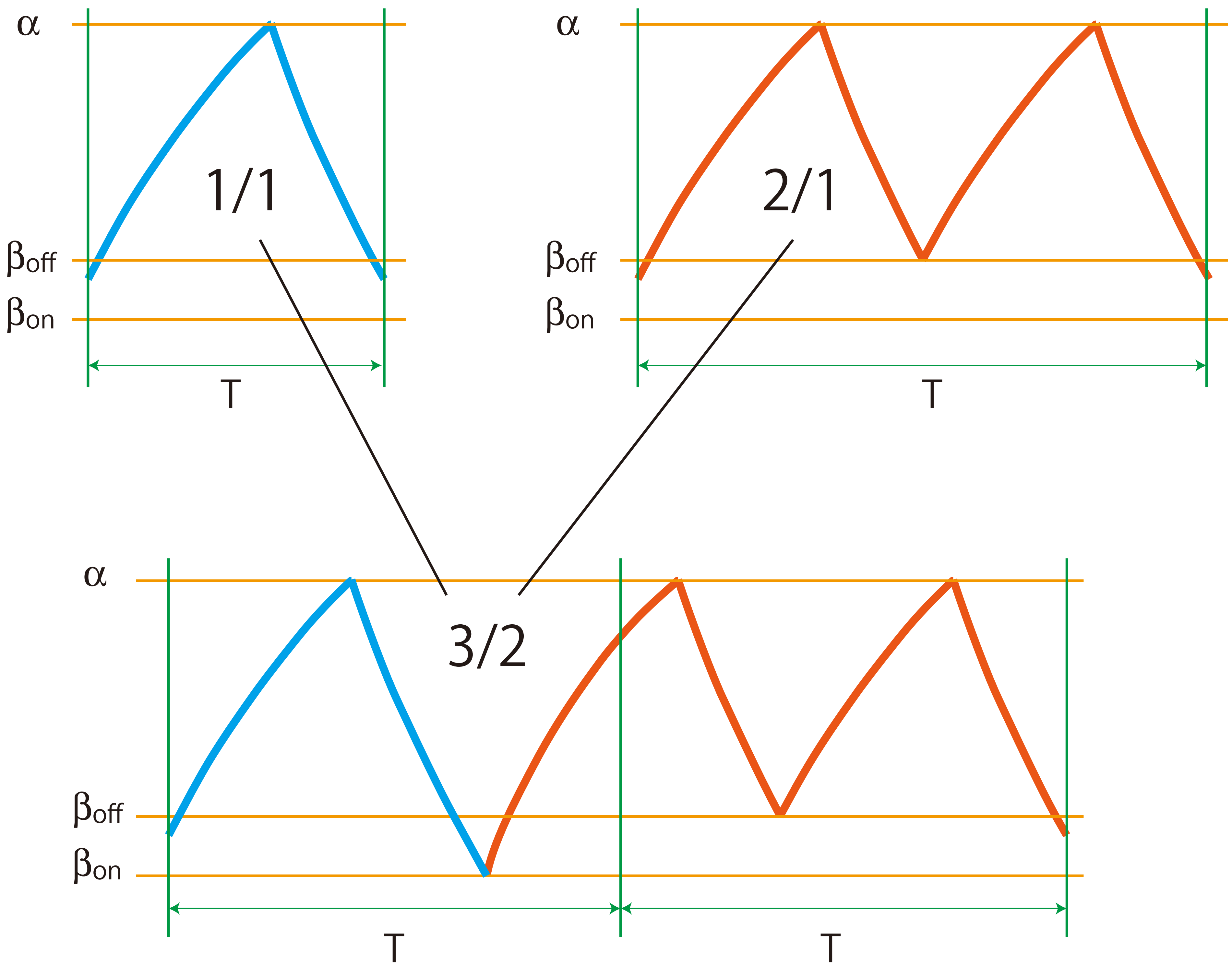
外力の周期とbeta_on変化させた場合の分岐図



Farey 数列

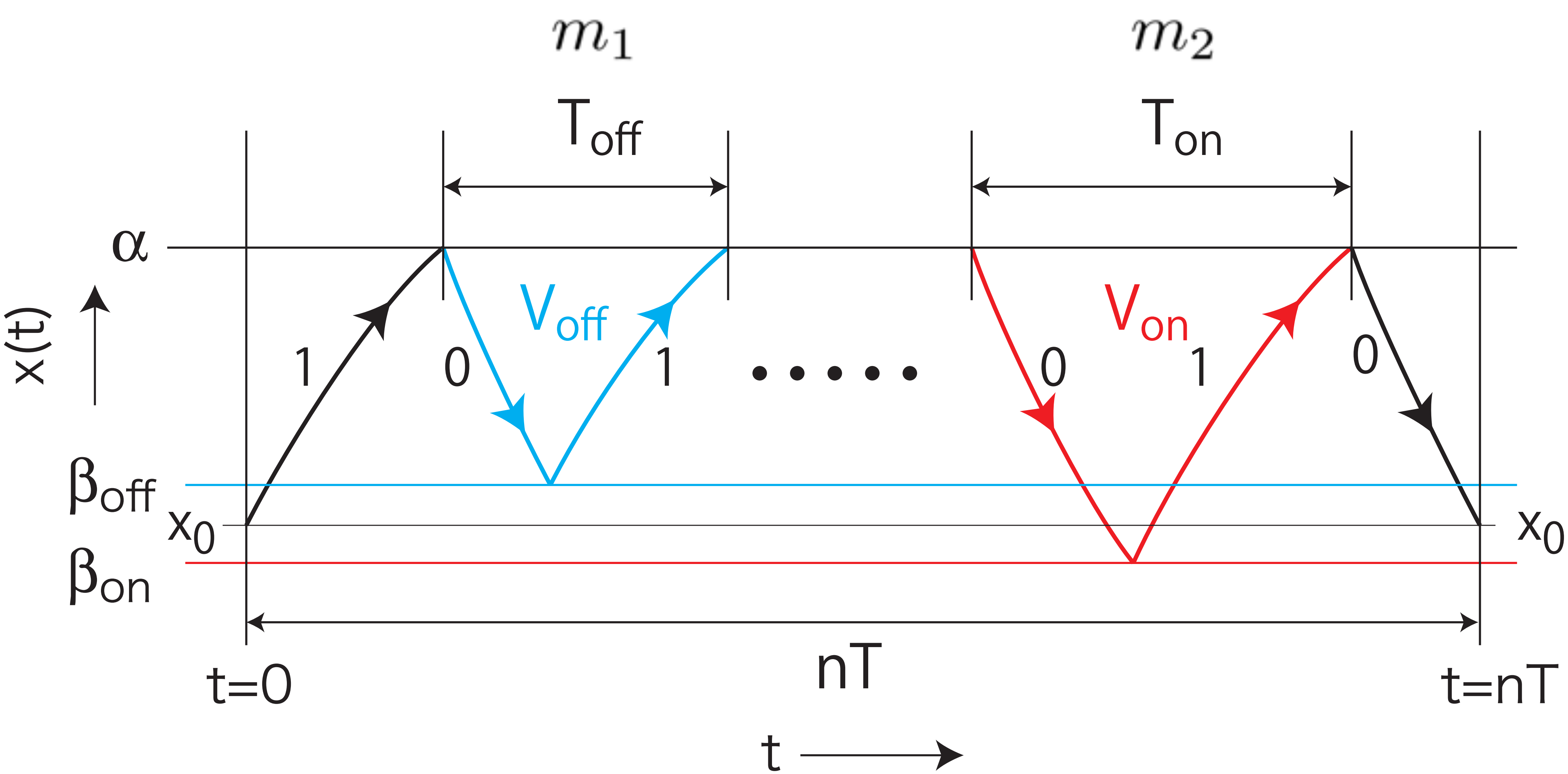


Farey 数列の例： 3/2 == 1/1 + 2/1



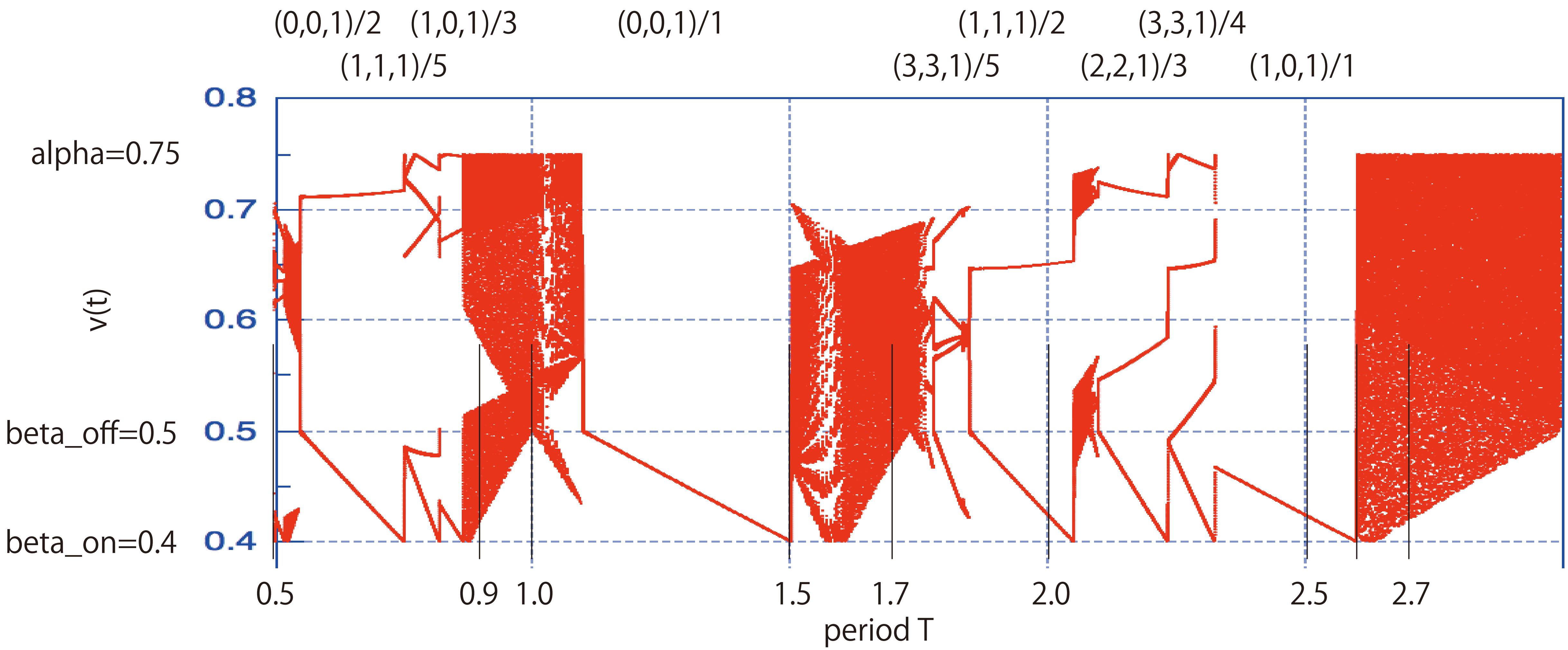
周期解の一般形

$$\{\{1(01)^{m-1}0\},\ (m_1, m_2, 1), nT\}$$



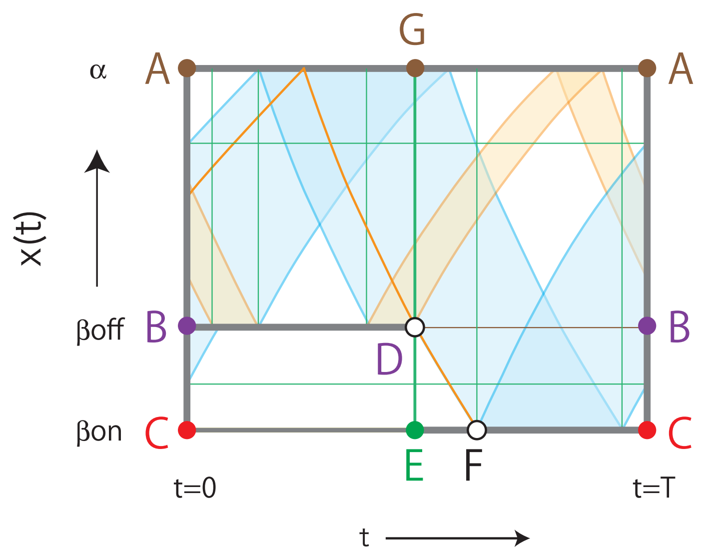
$$T_{off}^{m_1+1} + T_{on}^{m_2} < nT < T_{off}^{m_1} + T_{on}^{m_2+1}$$

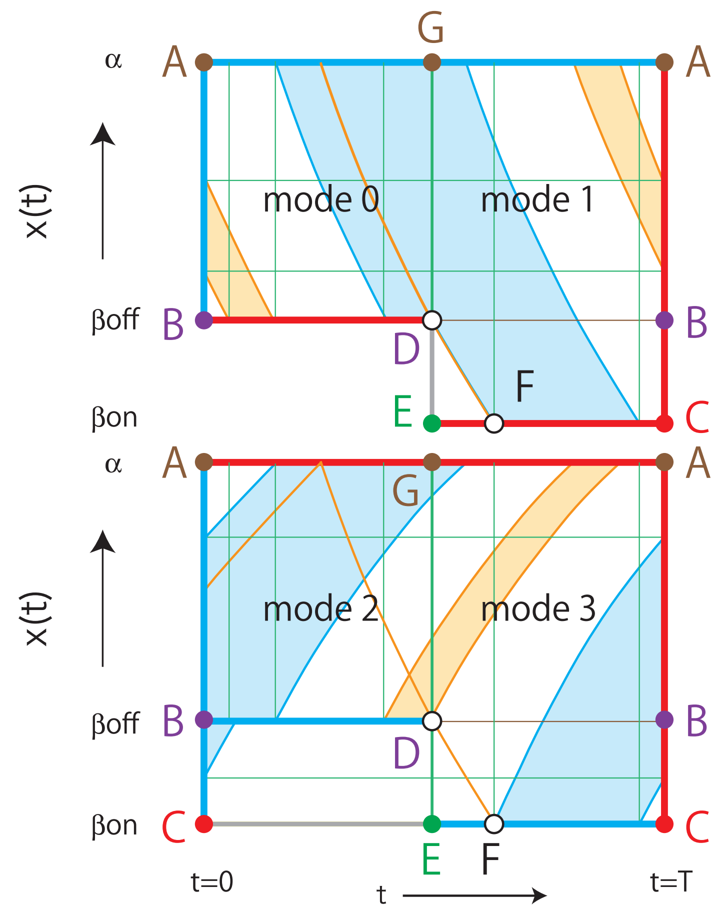
周期解の例

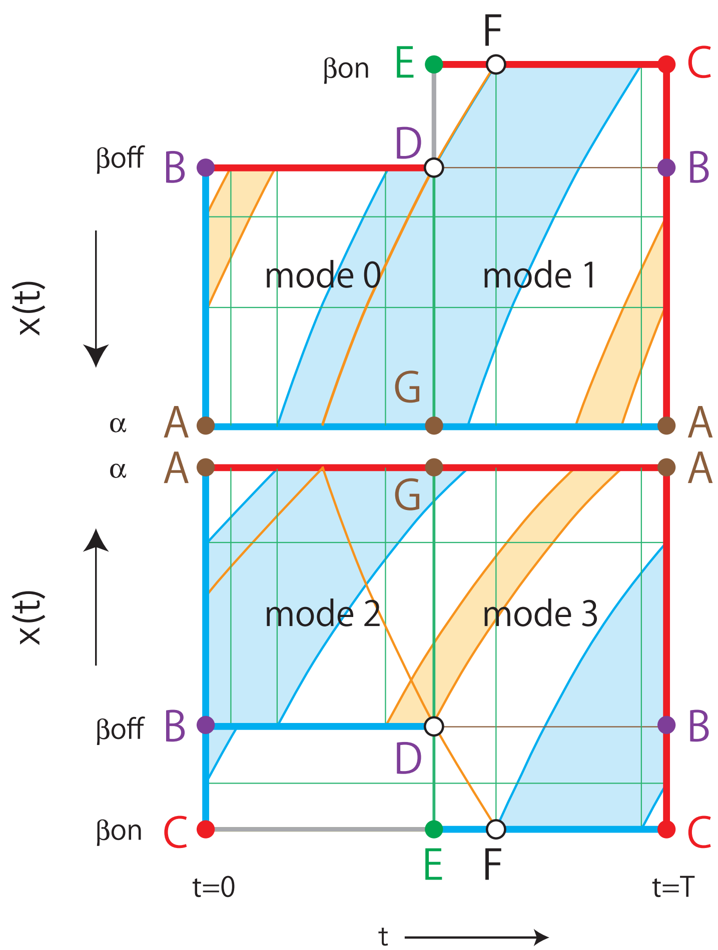


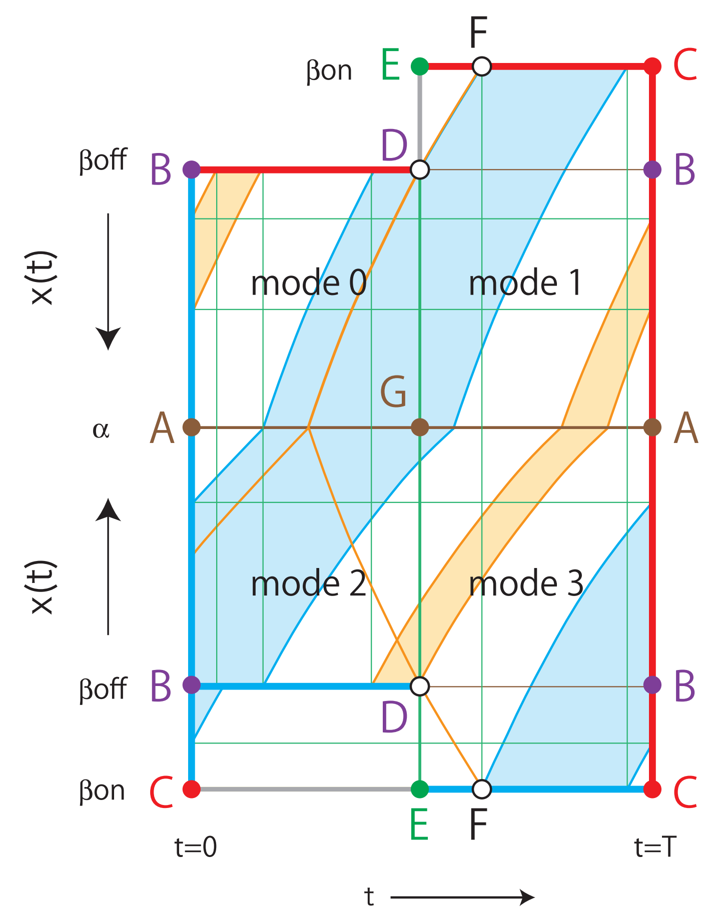
(0,0,1)/2
(1,0,1)/3
(0,0,1)/1
(1,1,1)/2
(3,3,1)/4
(1,1,1)/5
(3,3,1)/5
(2,2,1)/3
(1,0,1)/1
0.8
0.7
0.6
0.5
0.4
alpha=0.75
v(t)
beta_off=0.5
beta_on=0.4
0.5
0.9
1.0
1.5
1.7
2.0
2.5
2.7
period T

貼り合わせトーラスと
その上のPoincare 写像

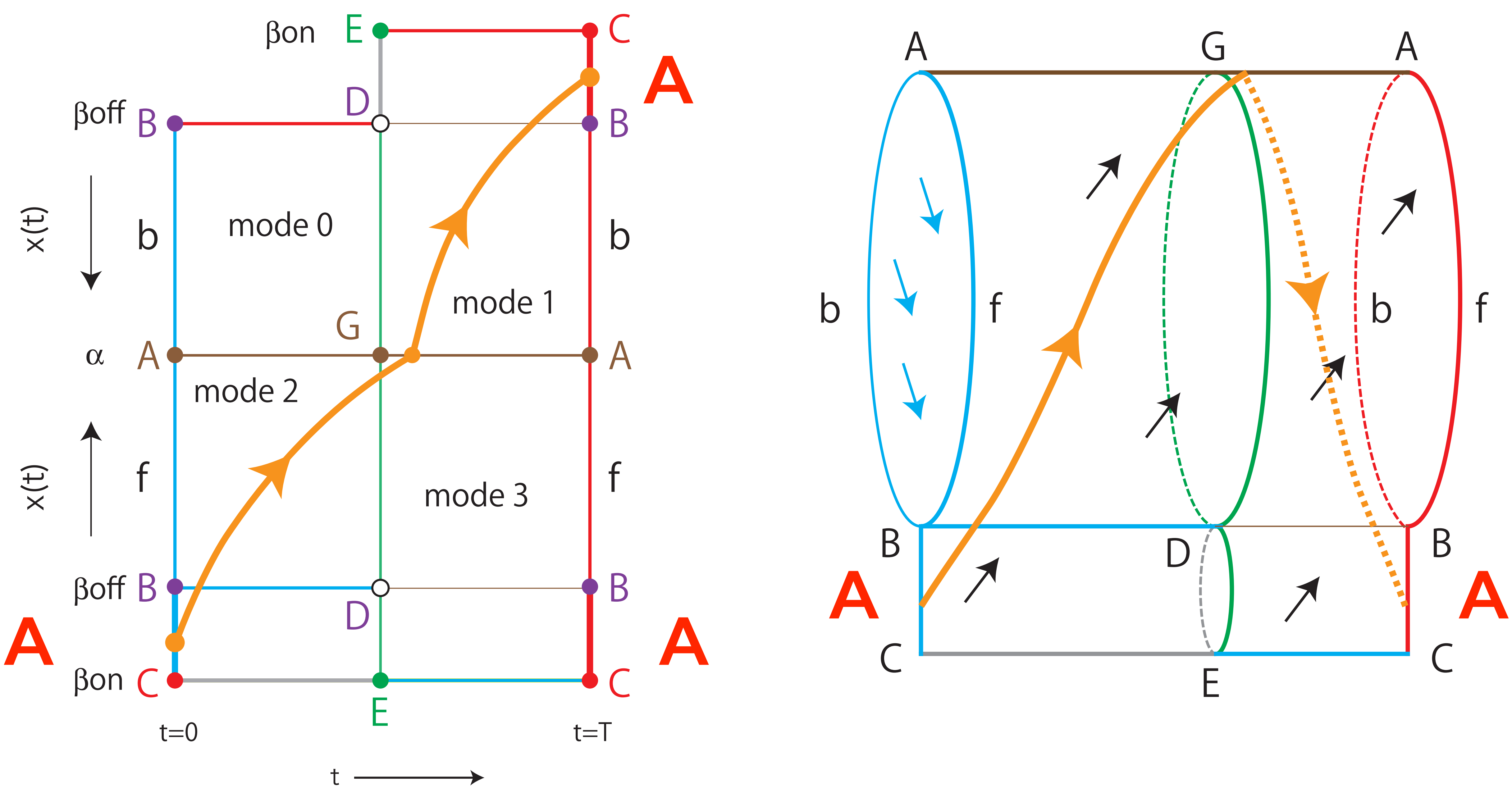

G
α
A
A
x(t)
βoff
B
B
D
βon
C
C
E
F
t=0
t=T
t


G
α
A
A
x(t)
mode 0
mode 1
βoff
B
B
D
F
βon
E
C
α
A
A
G
x(t)
mode 2
mode 3
βoff
B
B
D
βon
C
C
E
F
t=0
t=T
t


F
βon
E
C
βoff
B
B
D
x(t)
mode 0
mode 1
G
α
A
A
α
A
A
G
x(t)
mode 2
mode 3
βoff
B
B
D
βon
C
C
E
F
t=0
t=T
t


F
βon
E
C
βoff
B
B
D
x(t)
mode 0
mode 1
G
α
A
A
x(t)
mode 2
mode 3
βoff
B
B
D
βon
C
C
E
F
t=0
t=T
t

貼り合わせトーラス



(a)　　(b)

波形型と経過時間

{1(01)^n}型

$$x_1 - 1 = e^{-T}\left(\frac{\alpha(\beta-1)}{\beta(\alpha-1)}\right)^n (x_0 - 1)$$

{1(01)^n0}型

$$x_1 = e^{-T}\left(\frac{\alpha(\beta-1)}{\beta(\alpha-1)}\right)^n \frac{\alpha}{\alpha-1}(x_0 - 1)$$

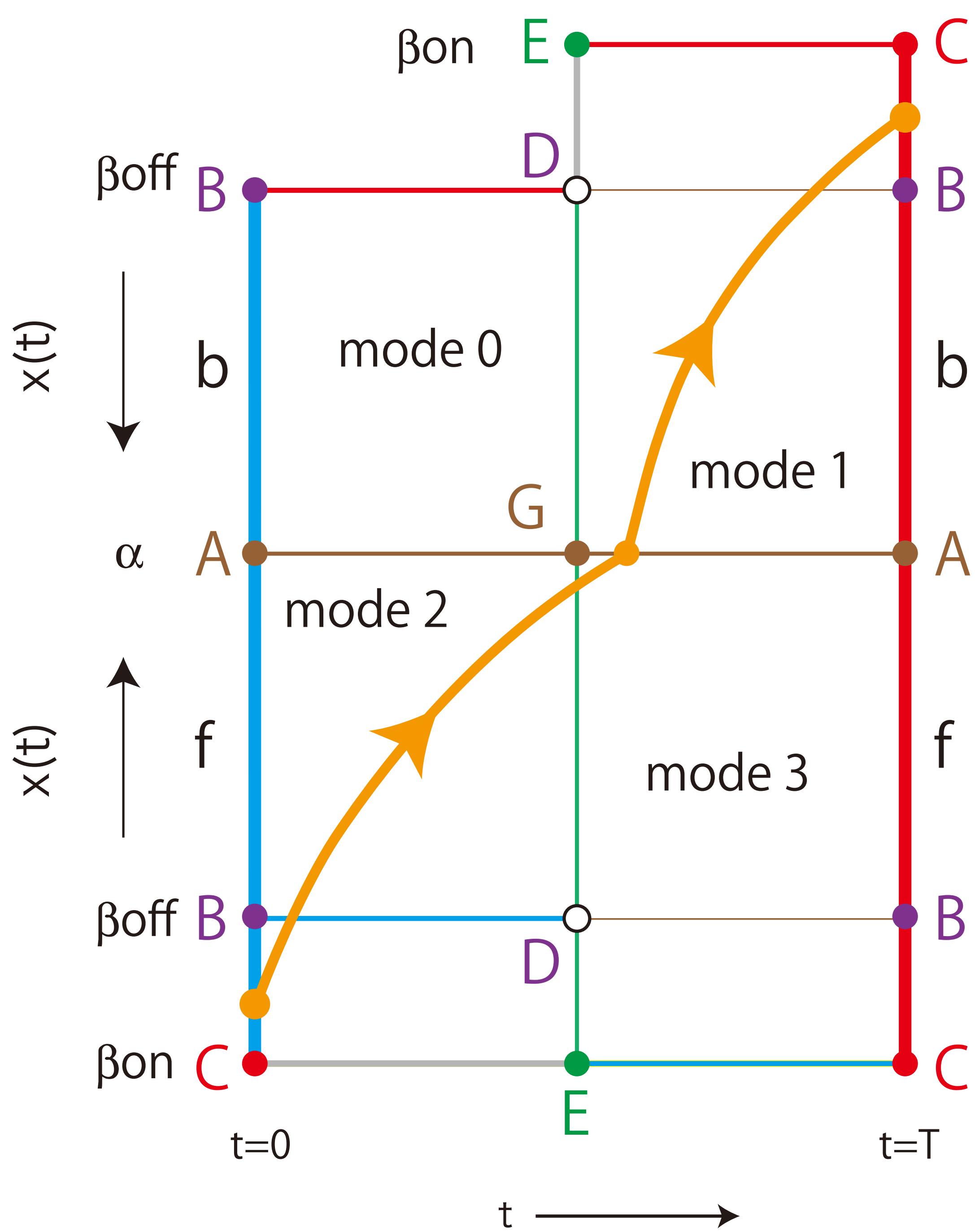
{0(10)^n}型

$$x_1 = e^{-T}\left(\frac{\alpha(\beta-1)}{\beta(\alpha-1)}\right)^n x_0$$

{0(10)^n1}型

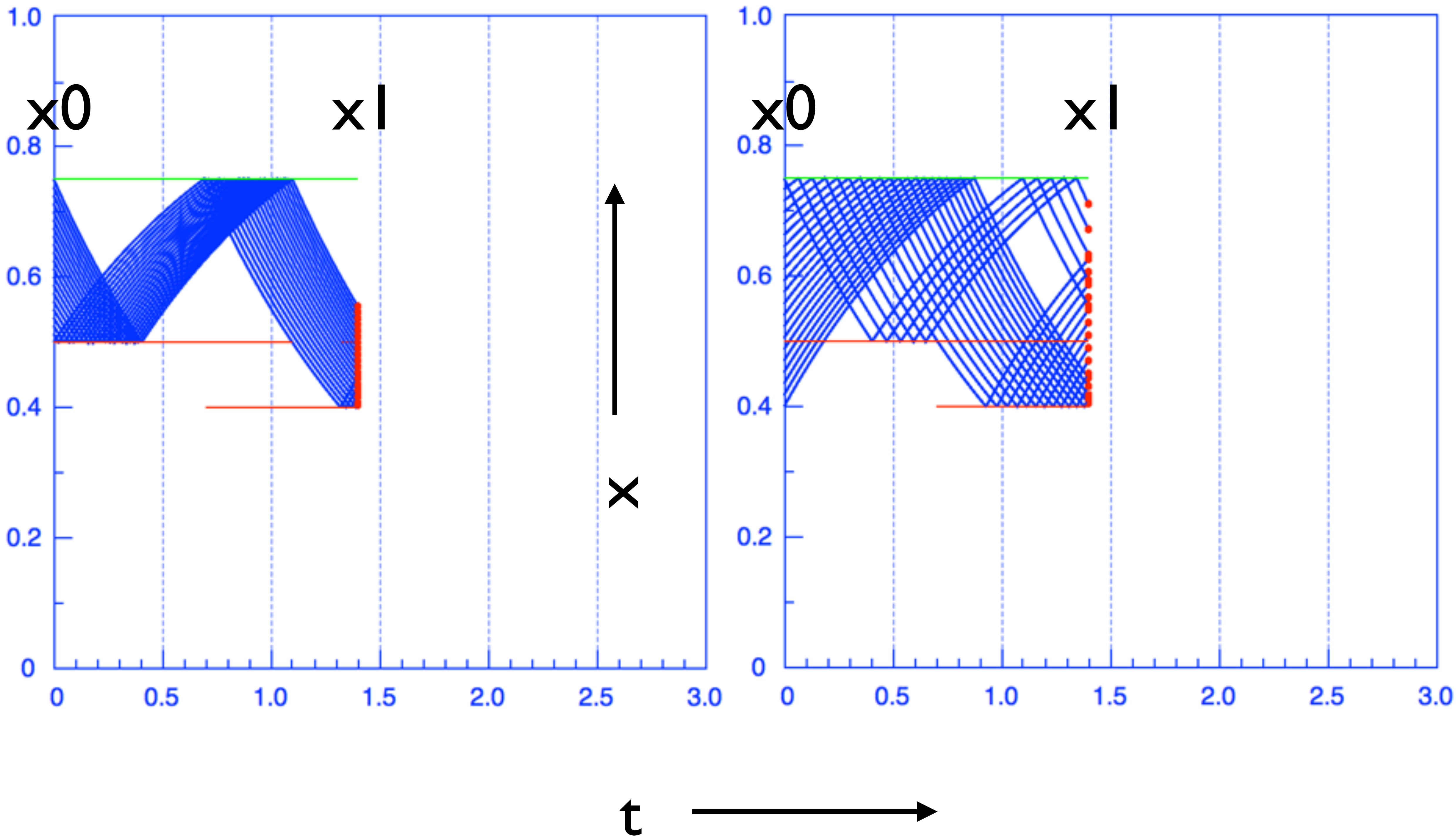
$$x_1 - 1 = e^{-T}\left(\frac{\alpha(\beta-1)}{\beta(\alpha-1)}\right)^n \frac{\beta-1}{\beta}x_0$$

Time One Map: Poincare 写像

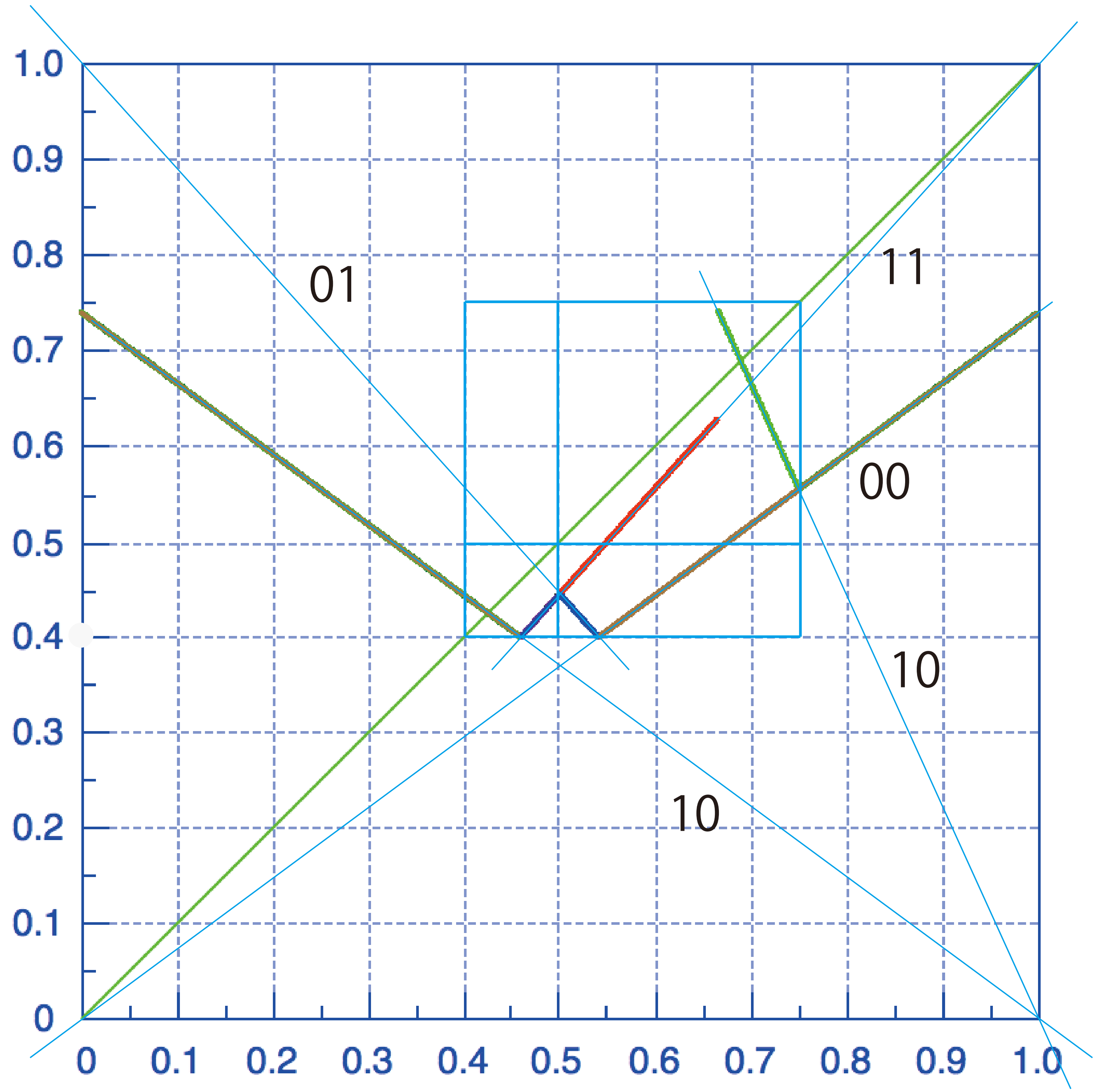


BbAfBC => CBbAfBC
=> BbAfBC => BbAfBC

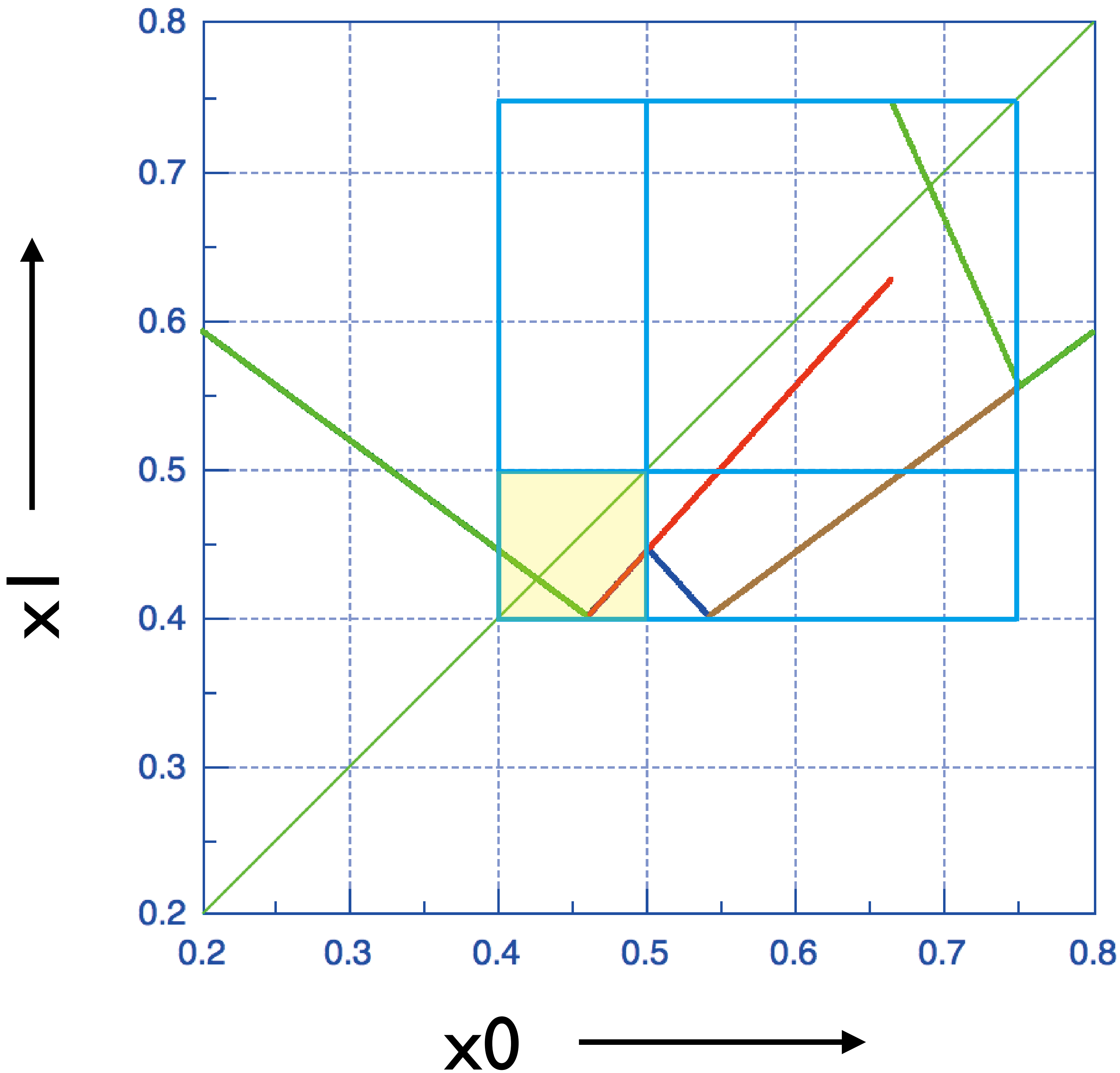
Time One Map: T=1.4



Time T 写像 = Poincare 写像



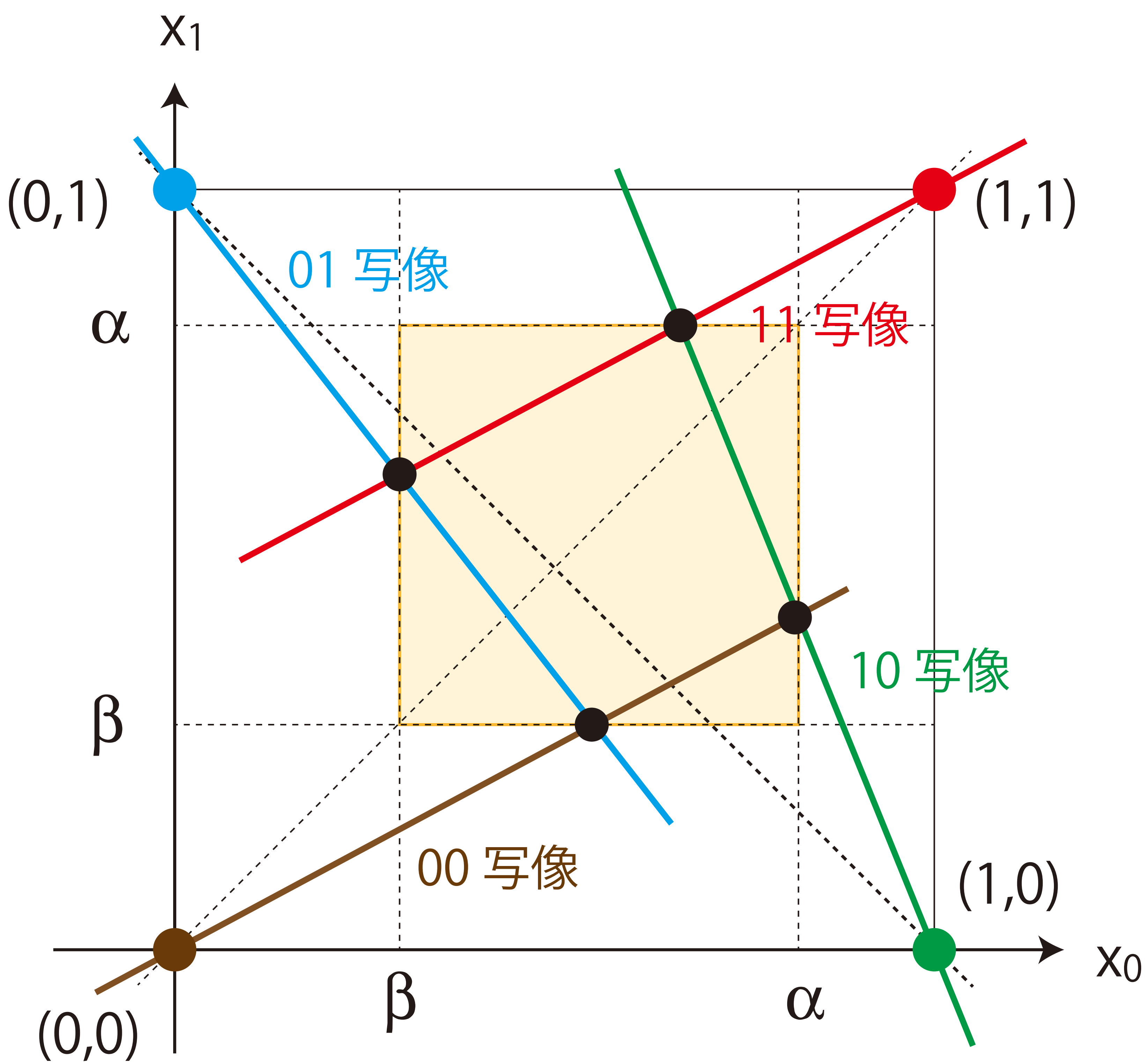
Time T 写像: T=1.4



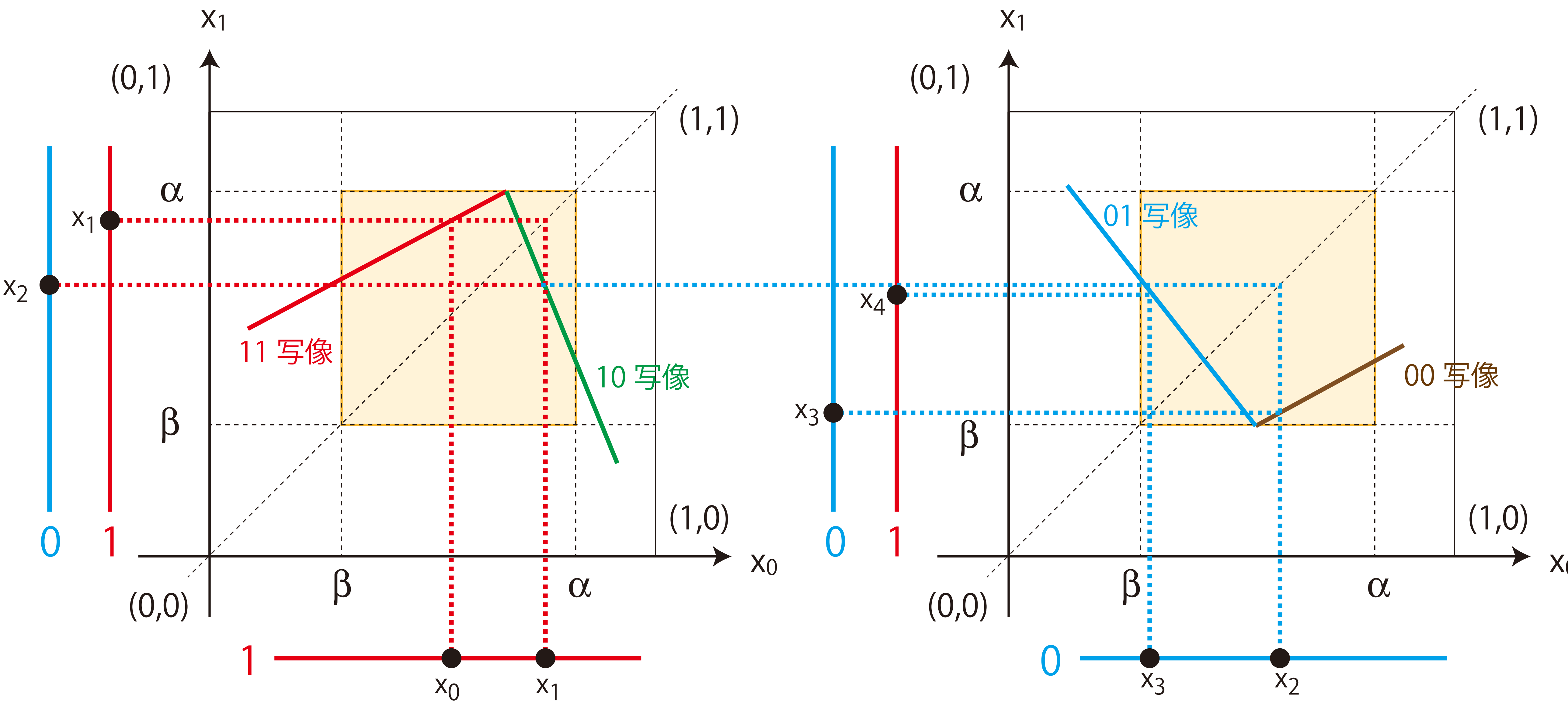
道草：準周期振動をみる

LED FF: betaOn=betaOff

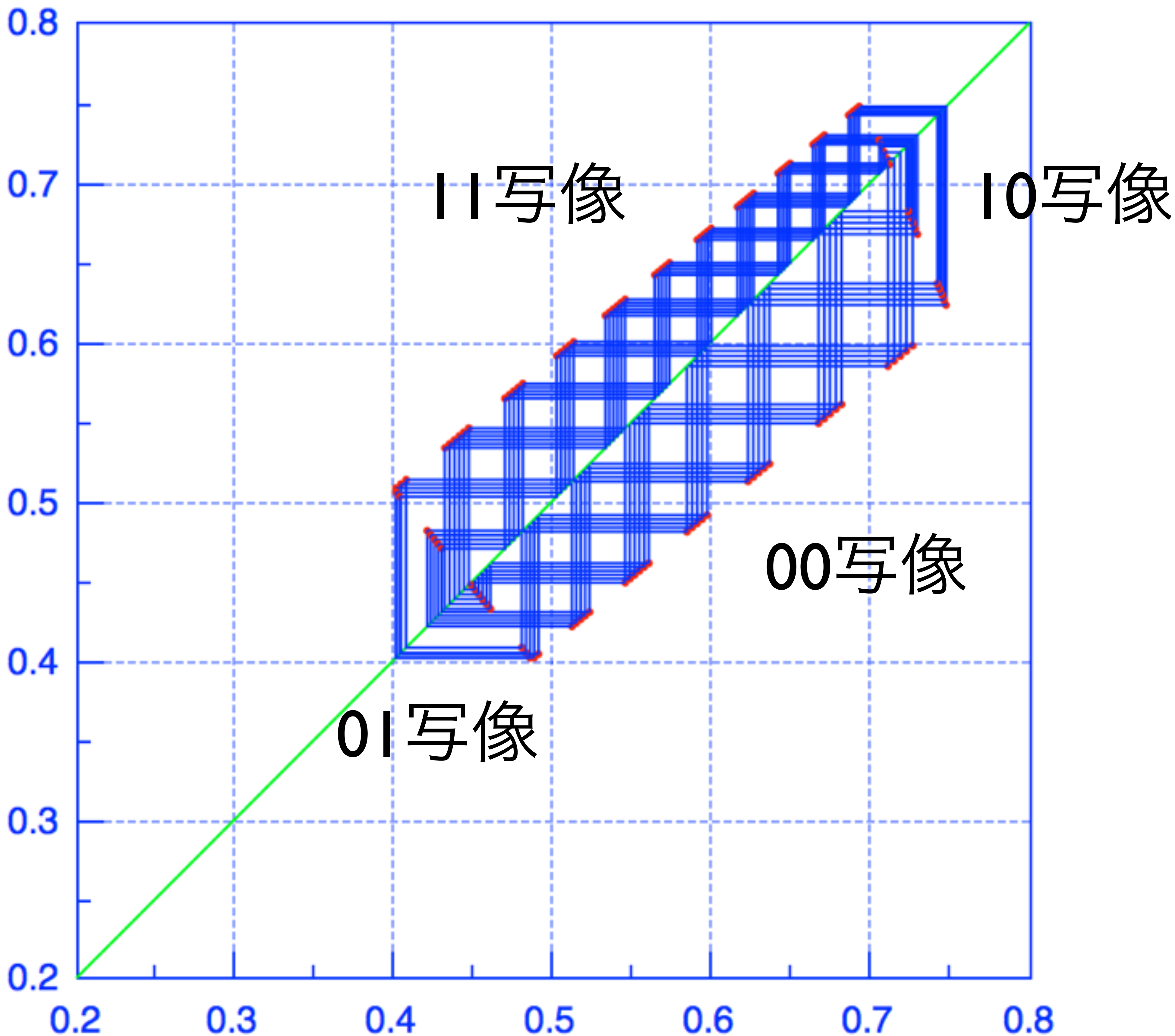
Poincare写像：betaOn=betaOff



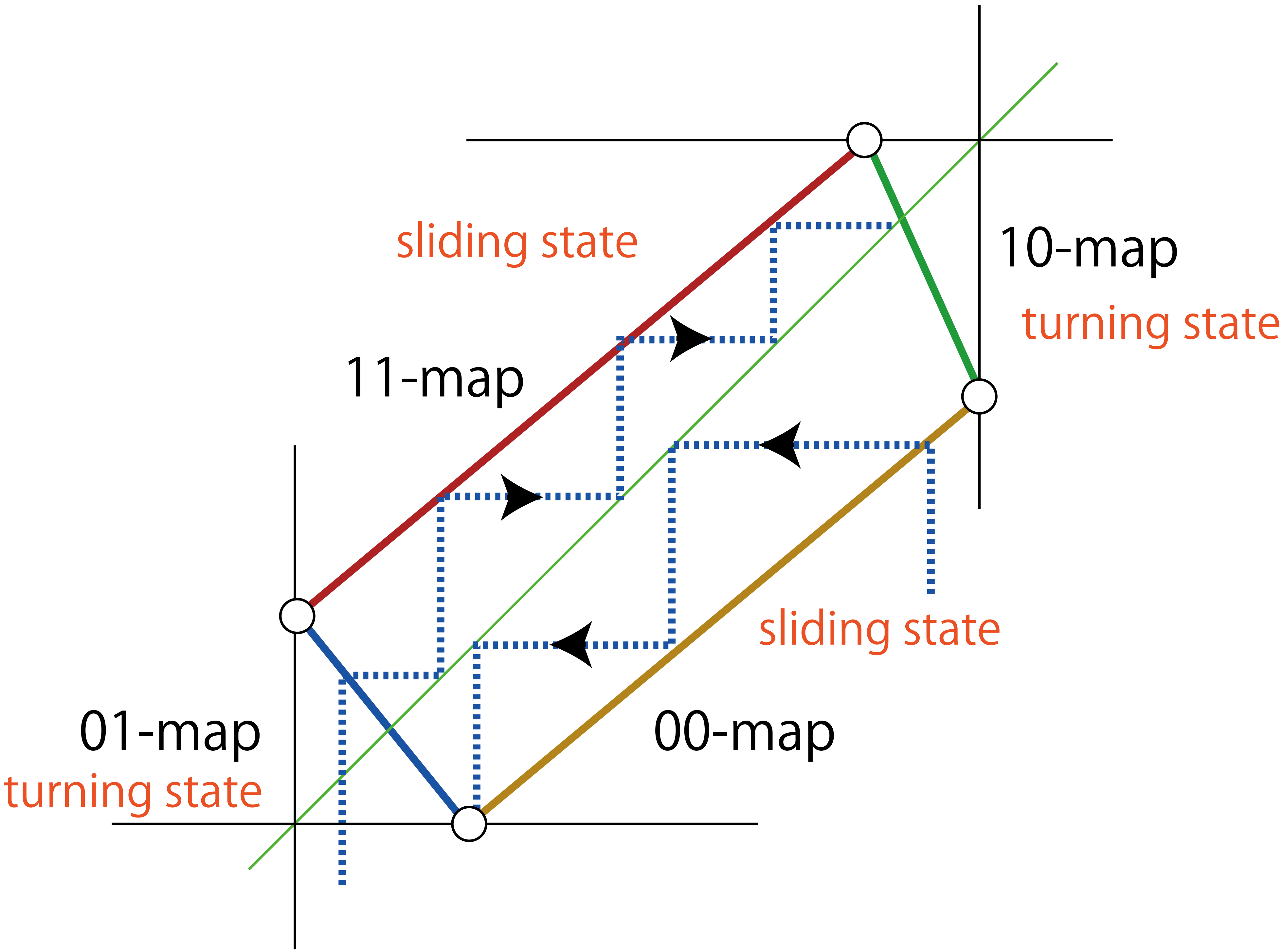
Poincare写像：写像の意味



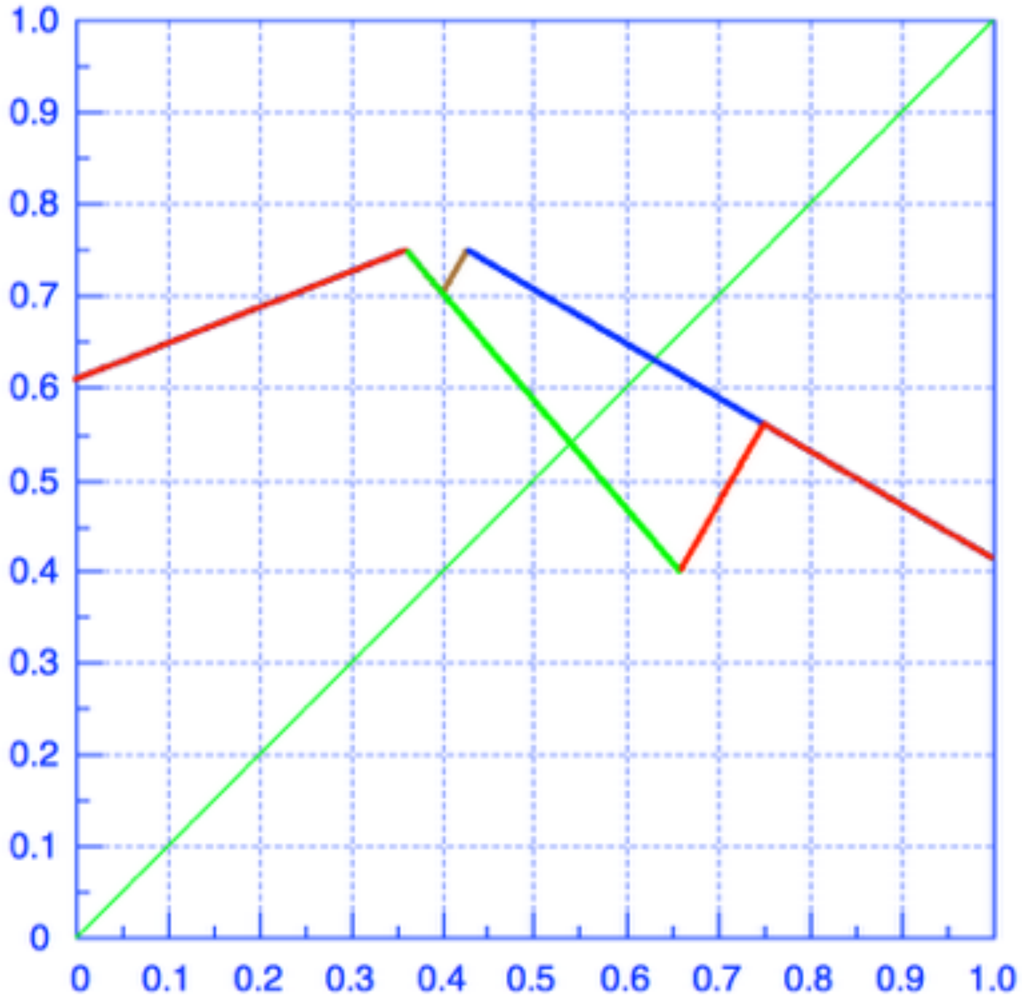
T=1.7

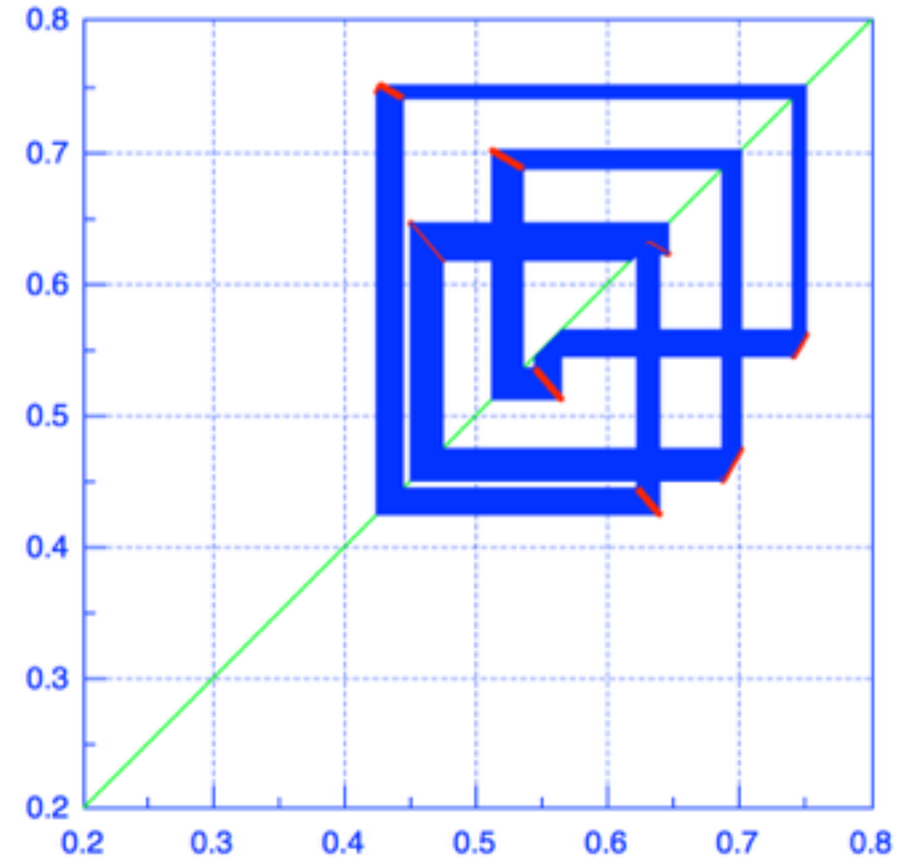


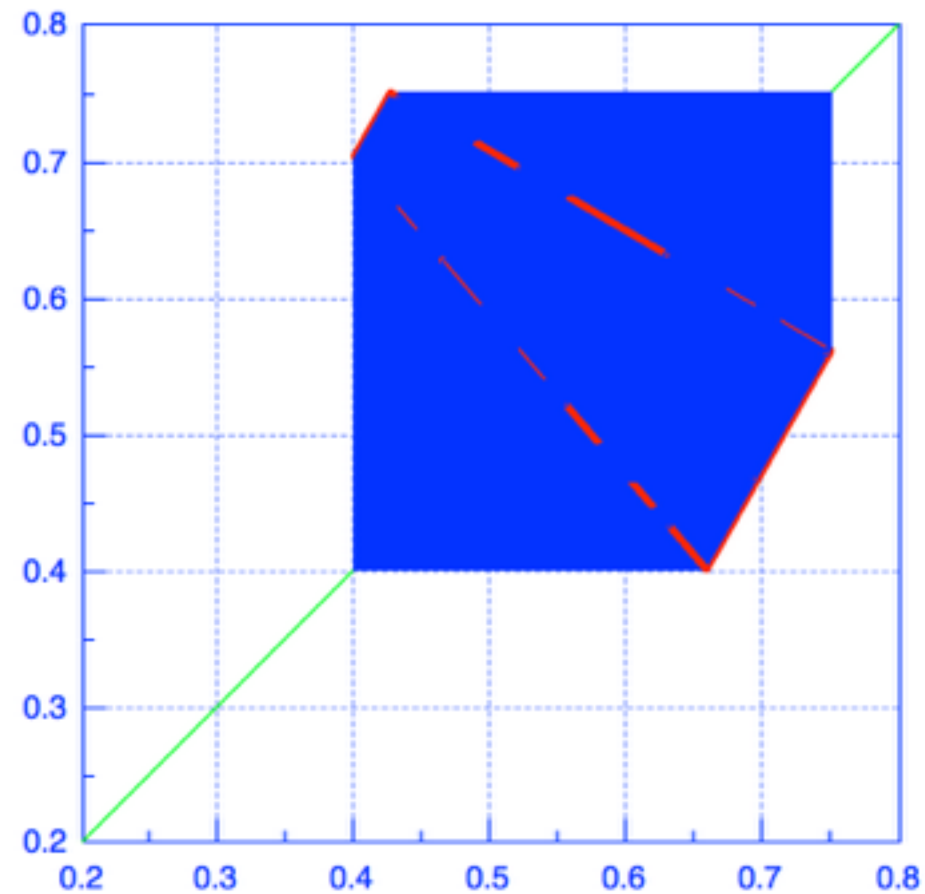
Quasi-Periodic States



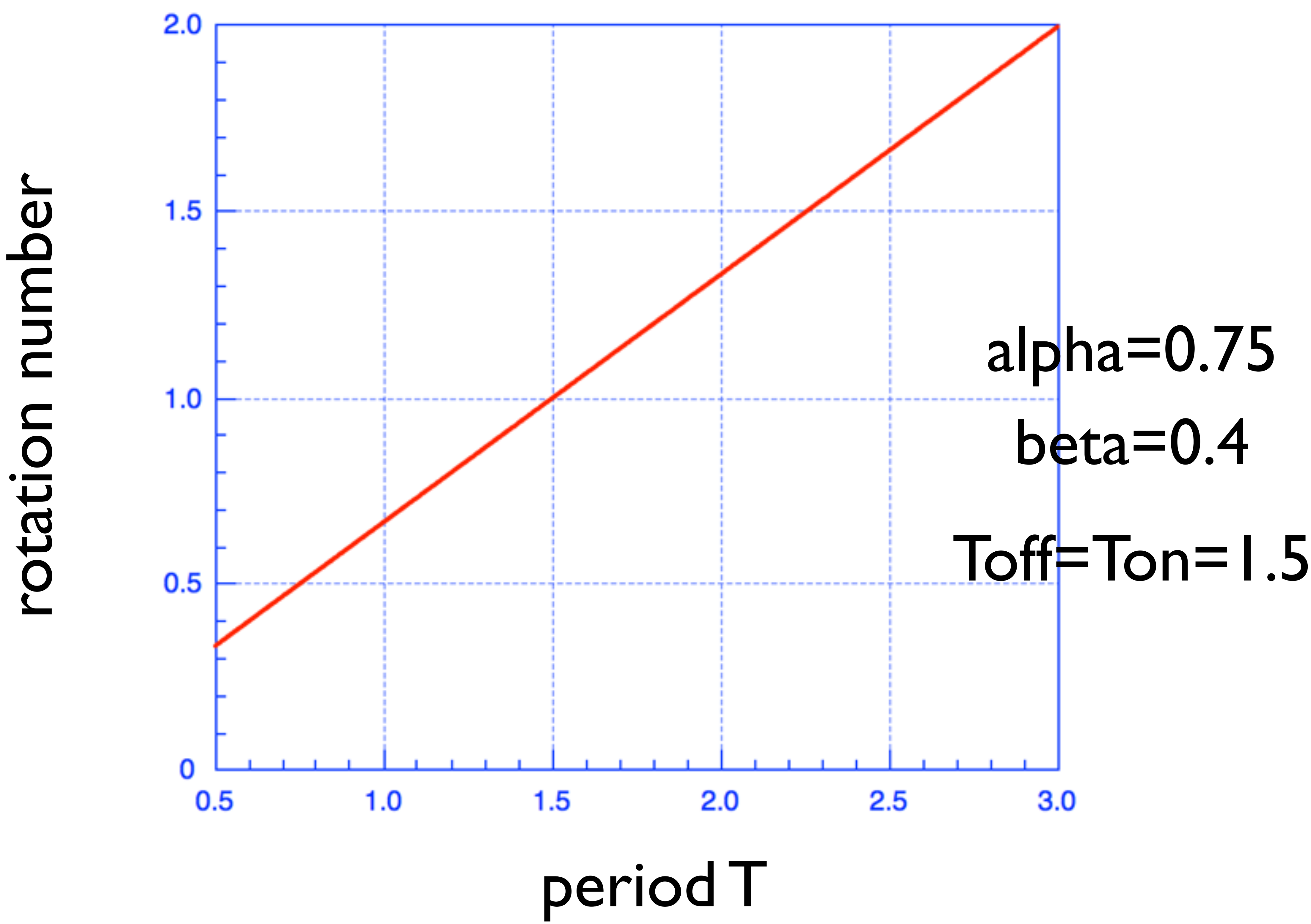
T=0.94


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0


0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.2 0.3 0.4 0.5 0.6 0.7 0.8

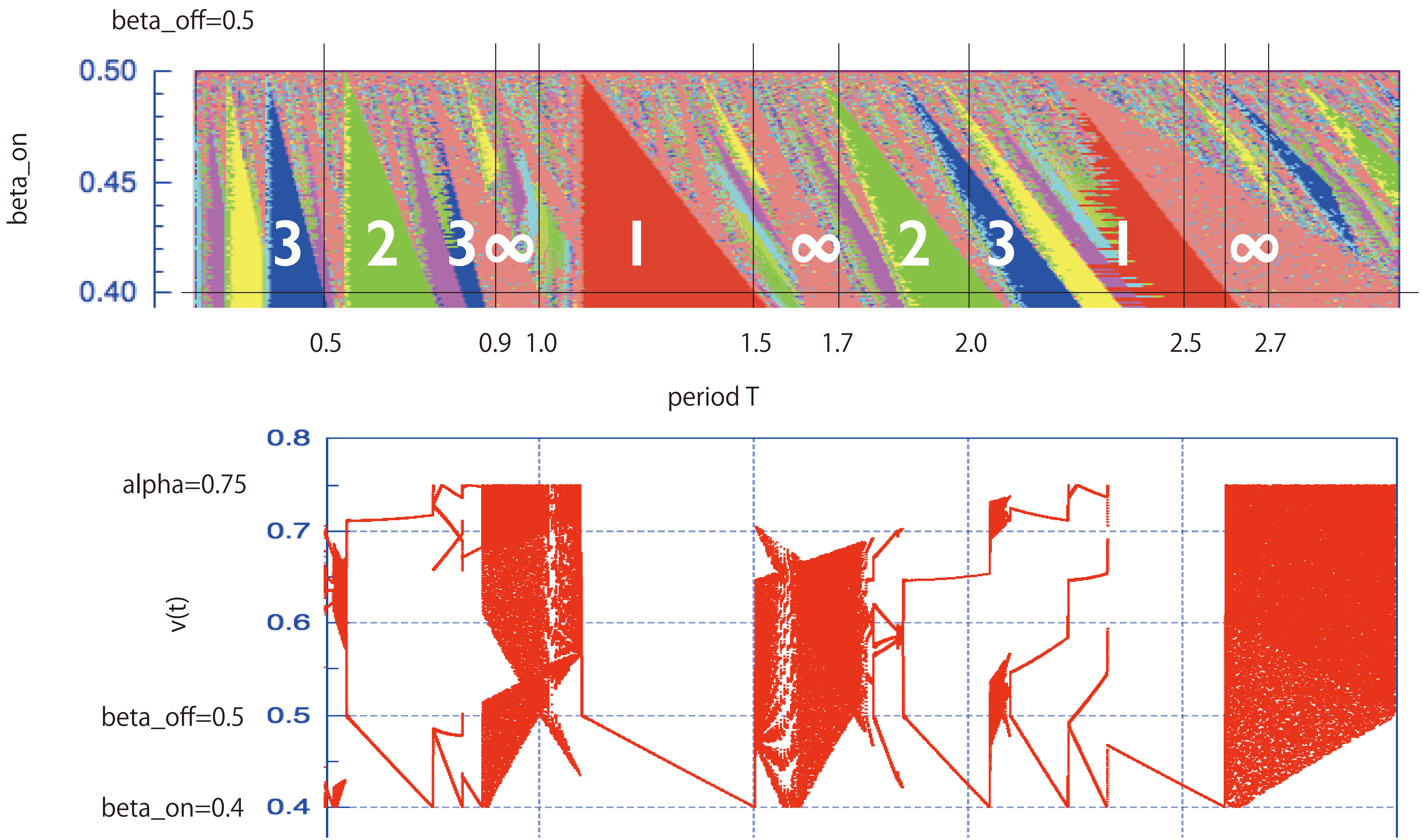

0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.2 0.3 0.4 0.5 0.6 0.7 0.8

rotation number=alpha event/timer event

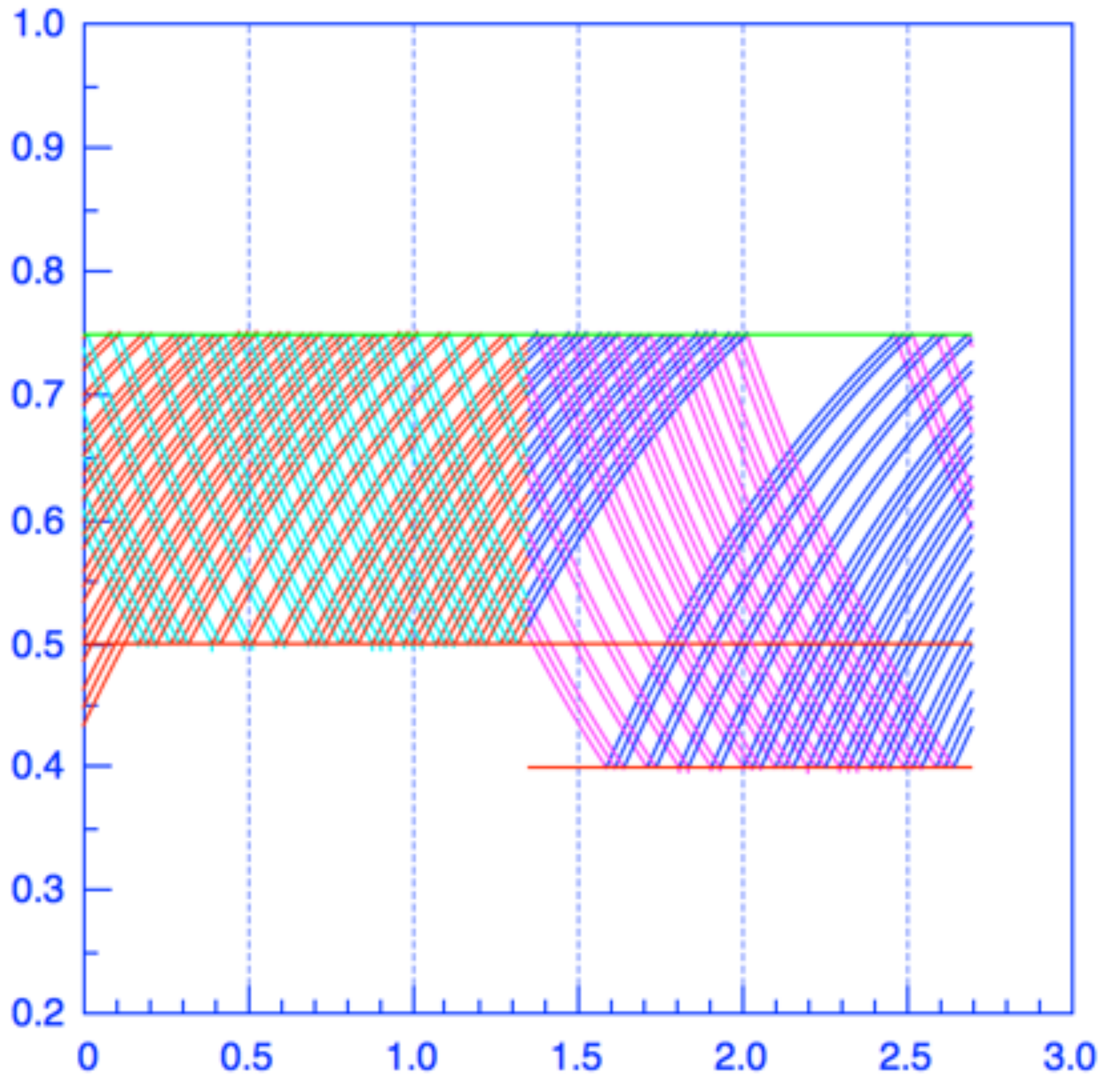


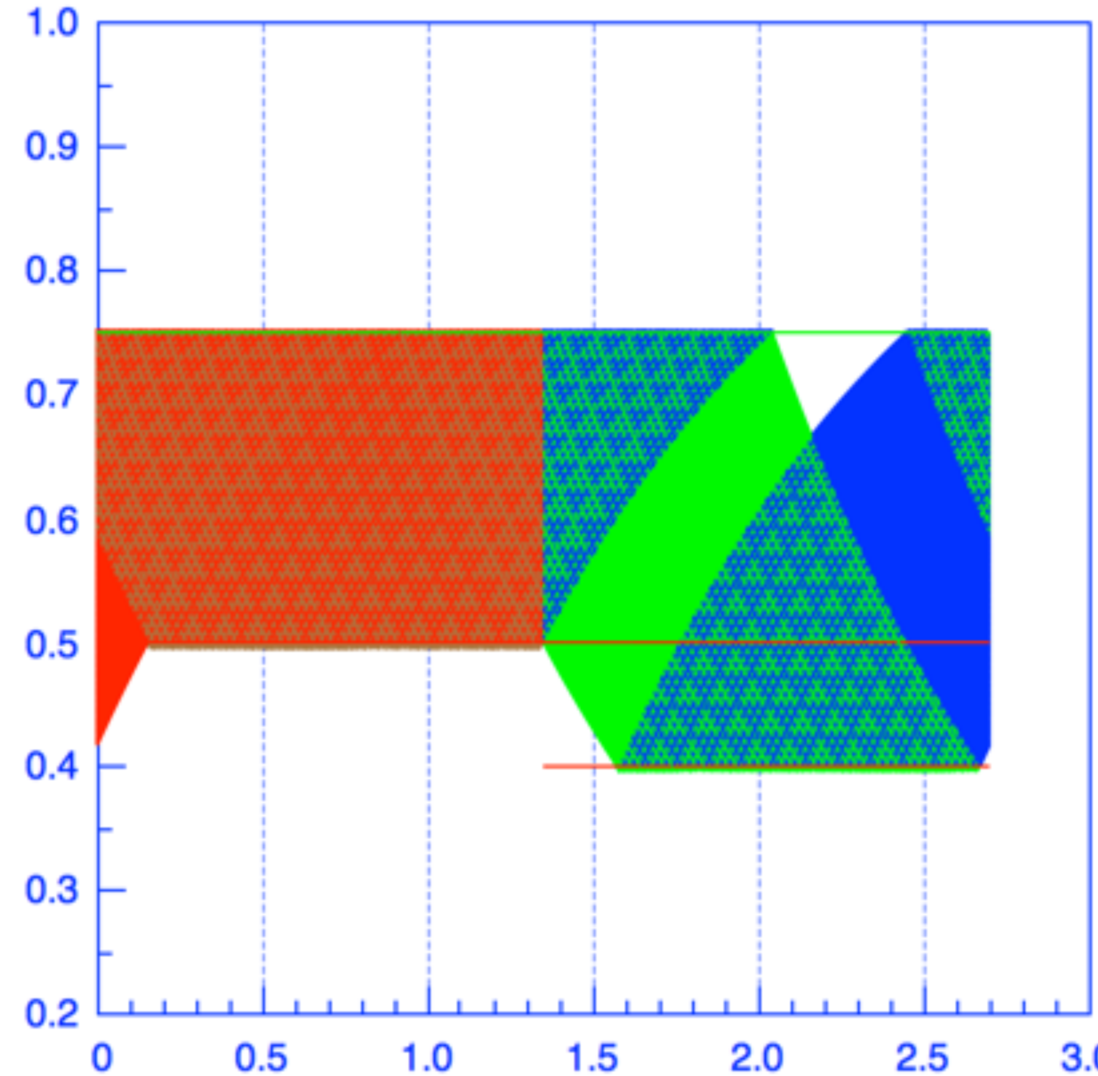
LEDホタルの非周期解

分岐図：duty cycle=0.5

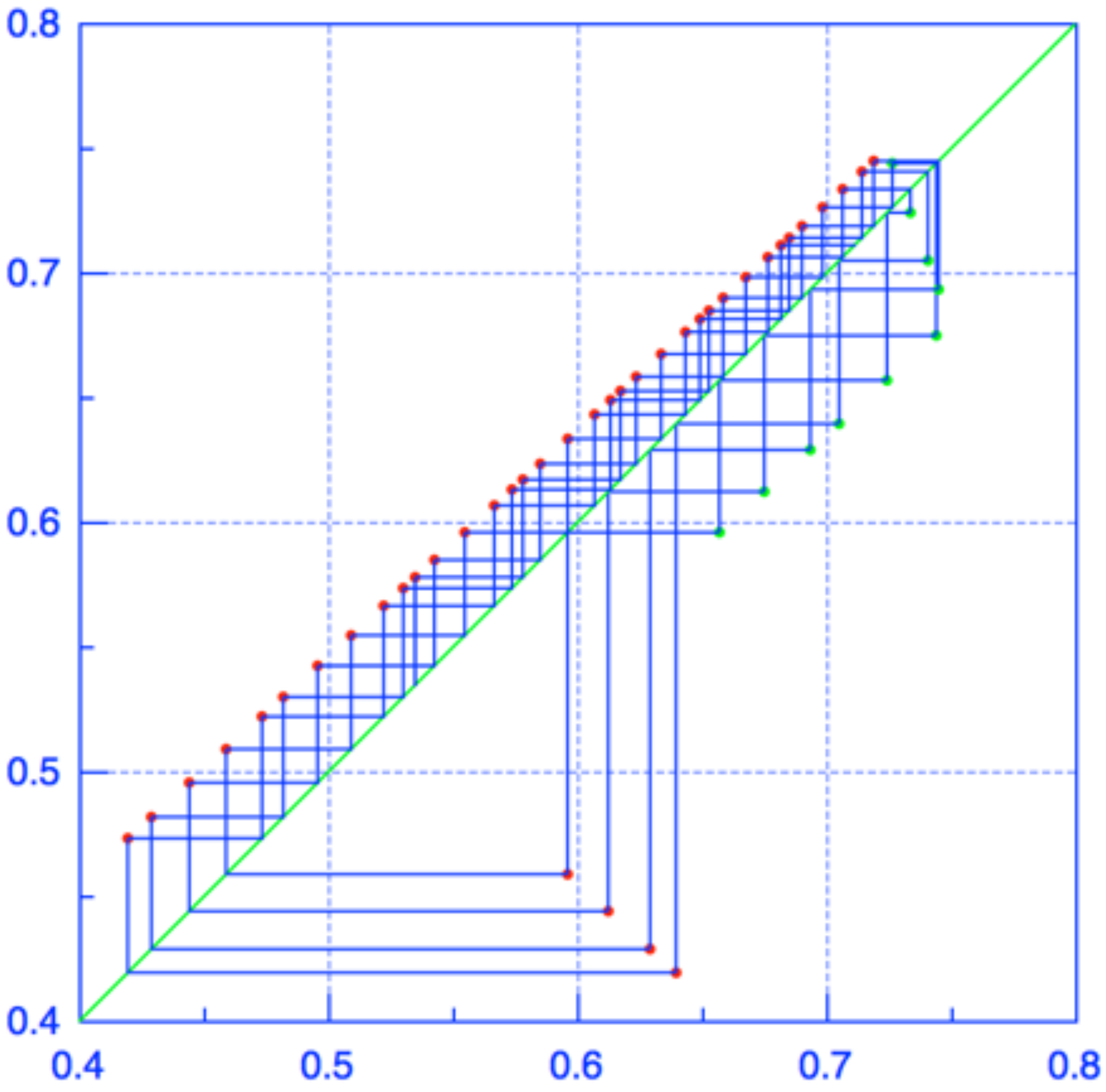


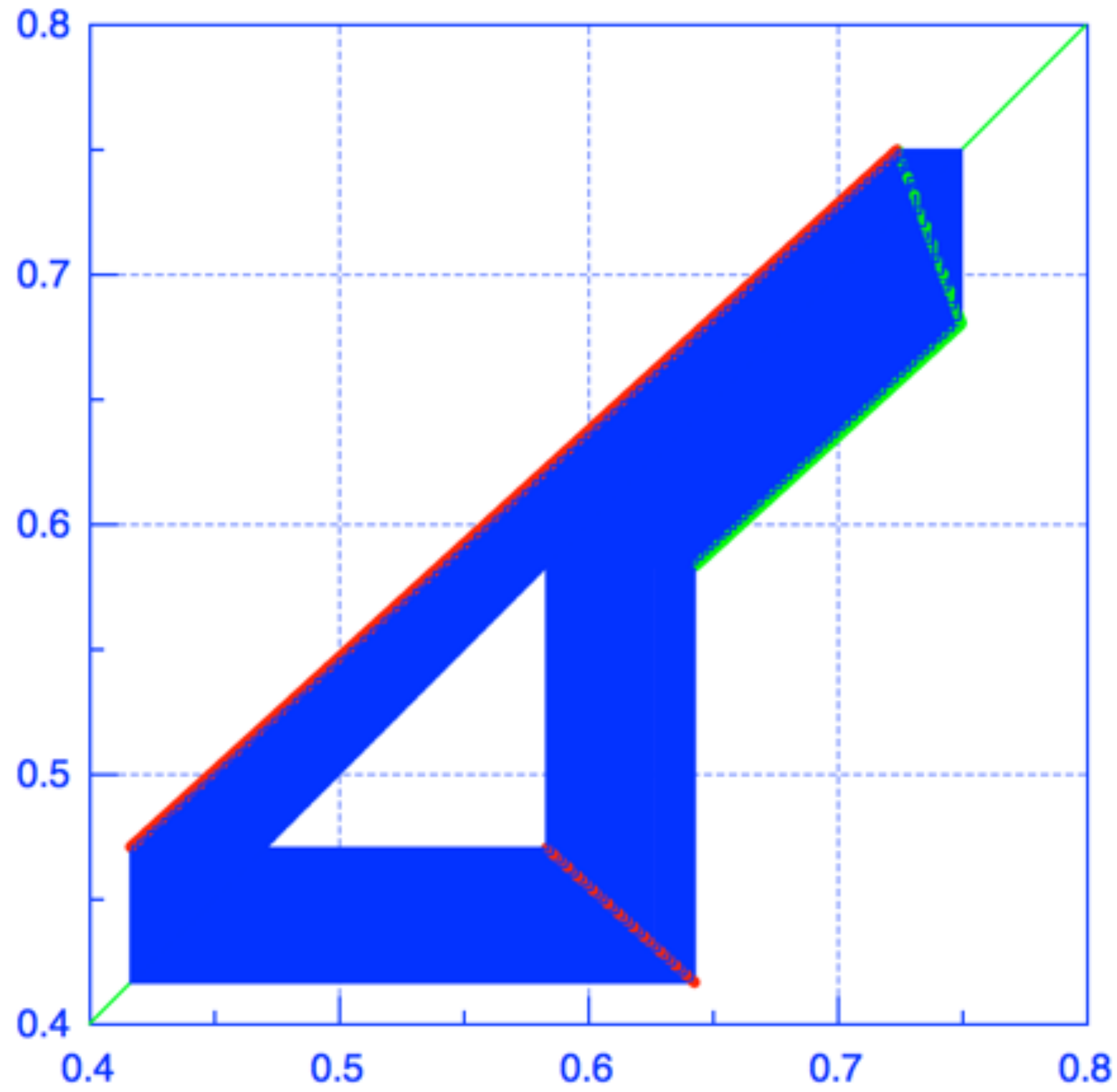
waveform for T=2.7



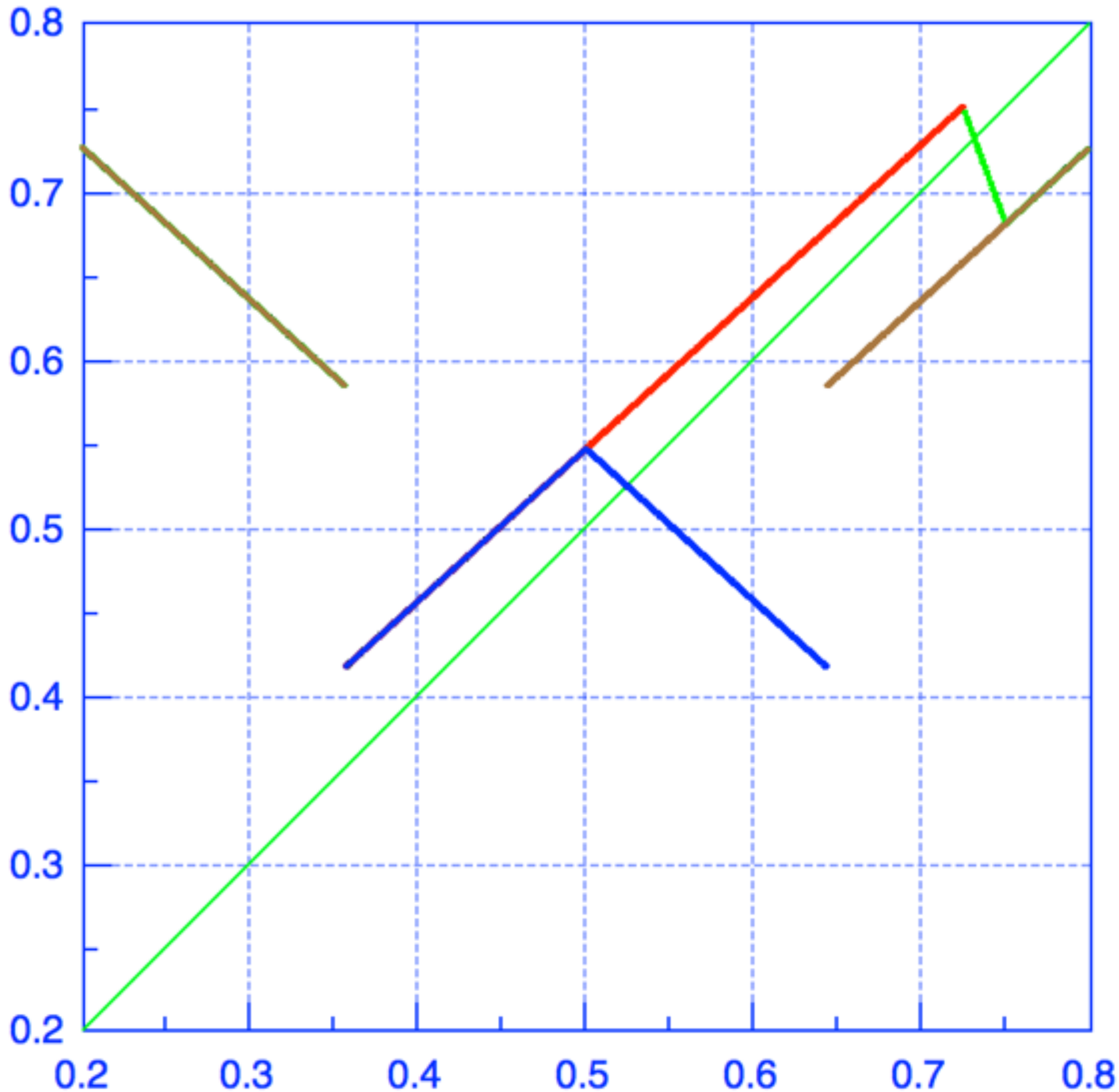


time T map trajectories for T=2.7



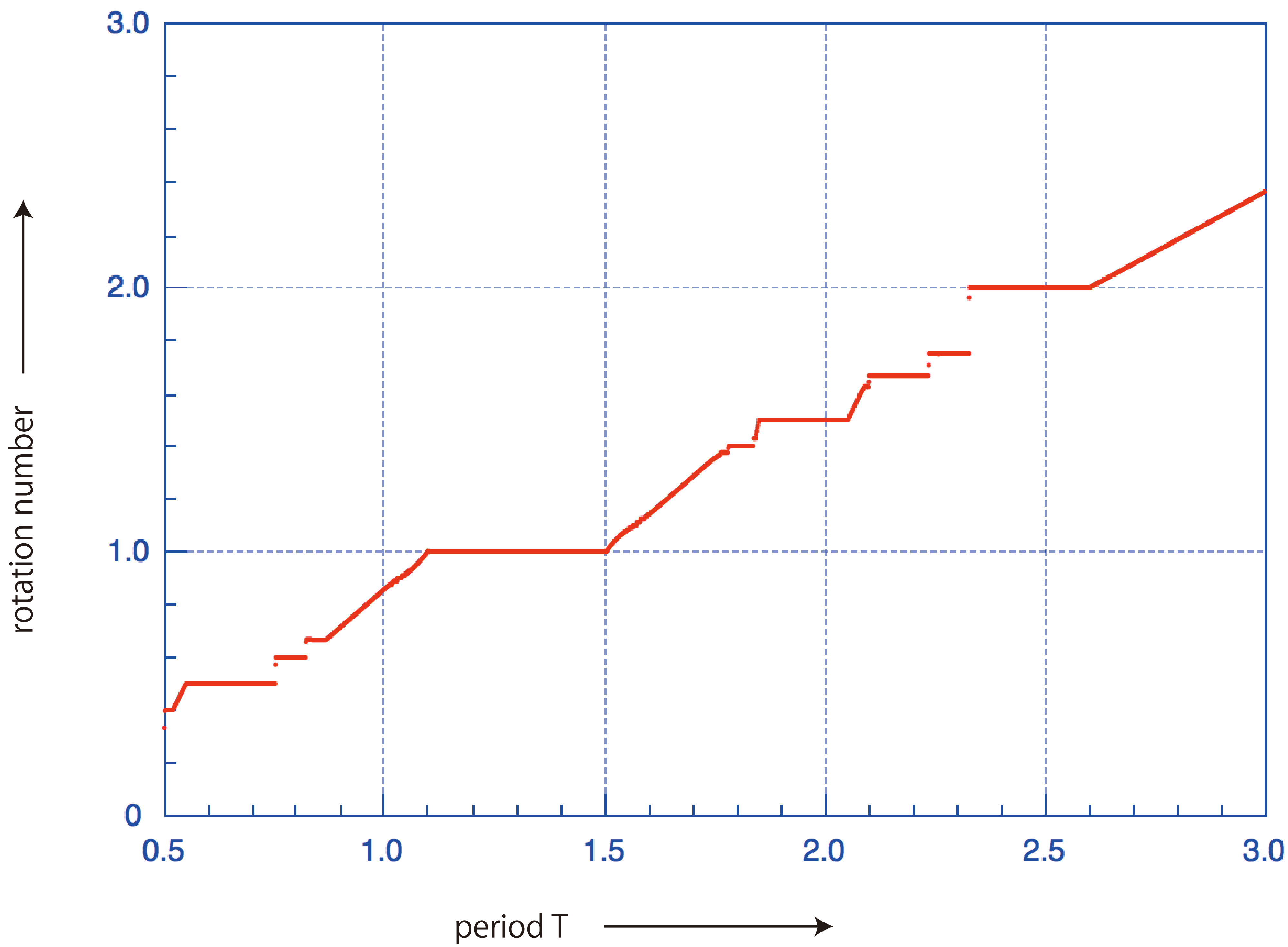


time T map : T=2.7

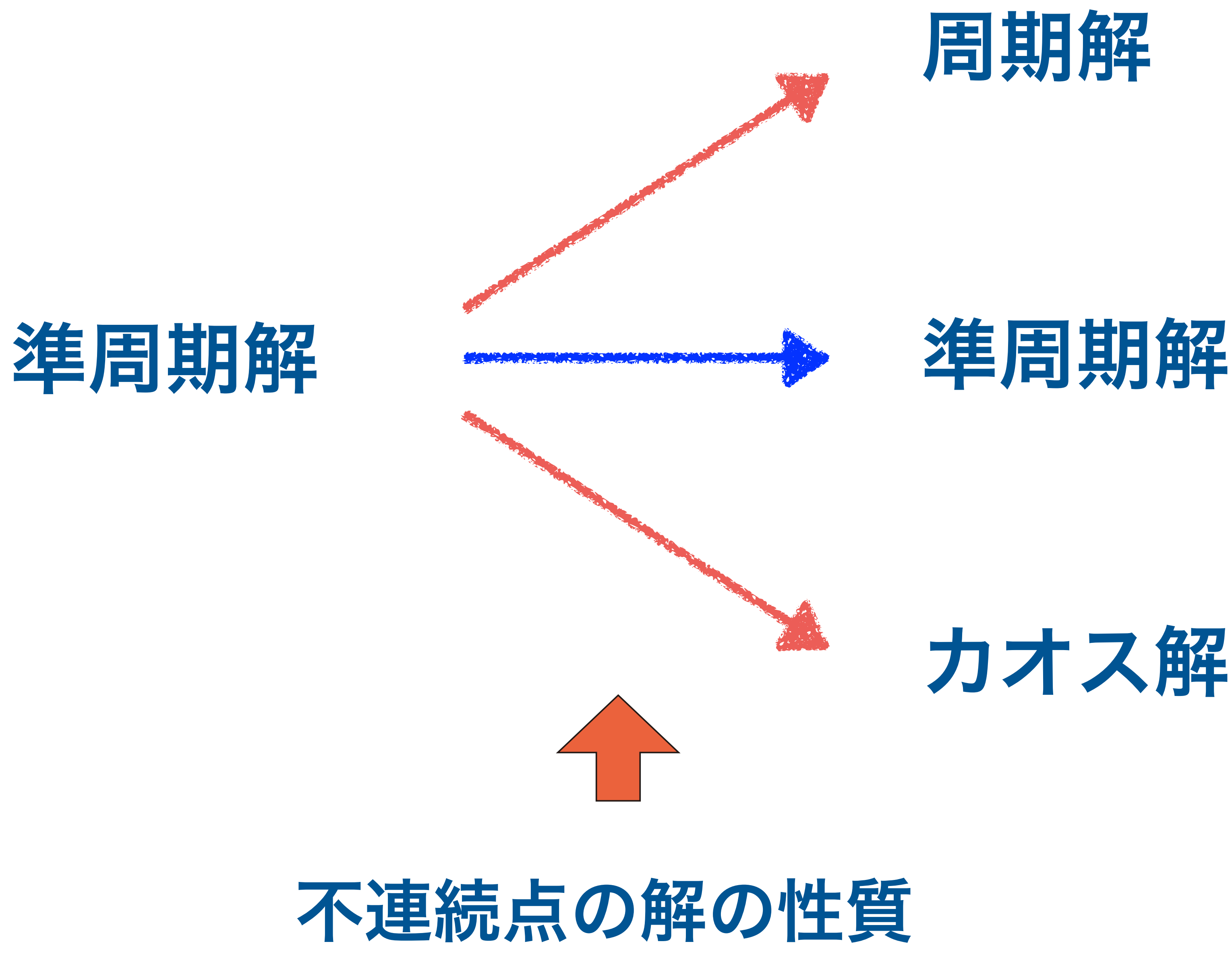


rotation number=alpha event/timer event

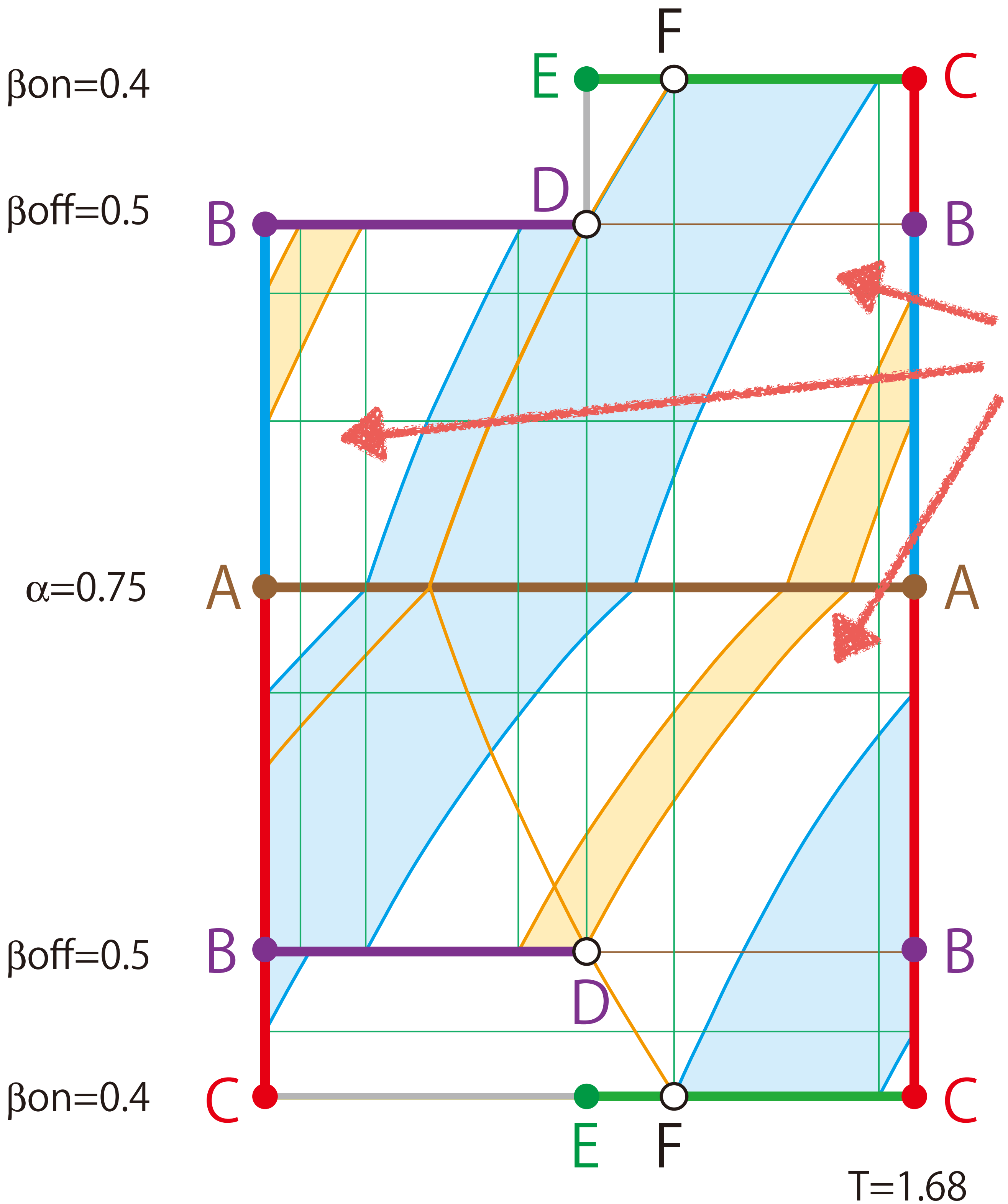
alpha=0.75, beta_off=0.5, beta_on=0.4



LEDホタルの定性論

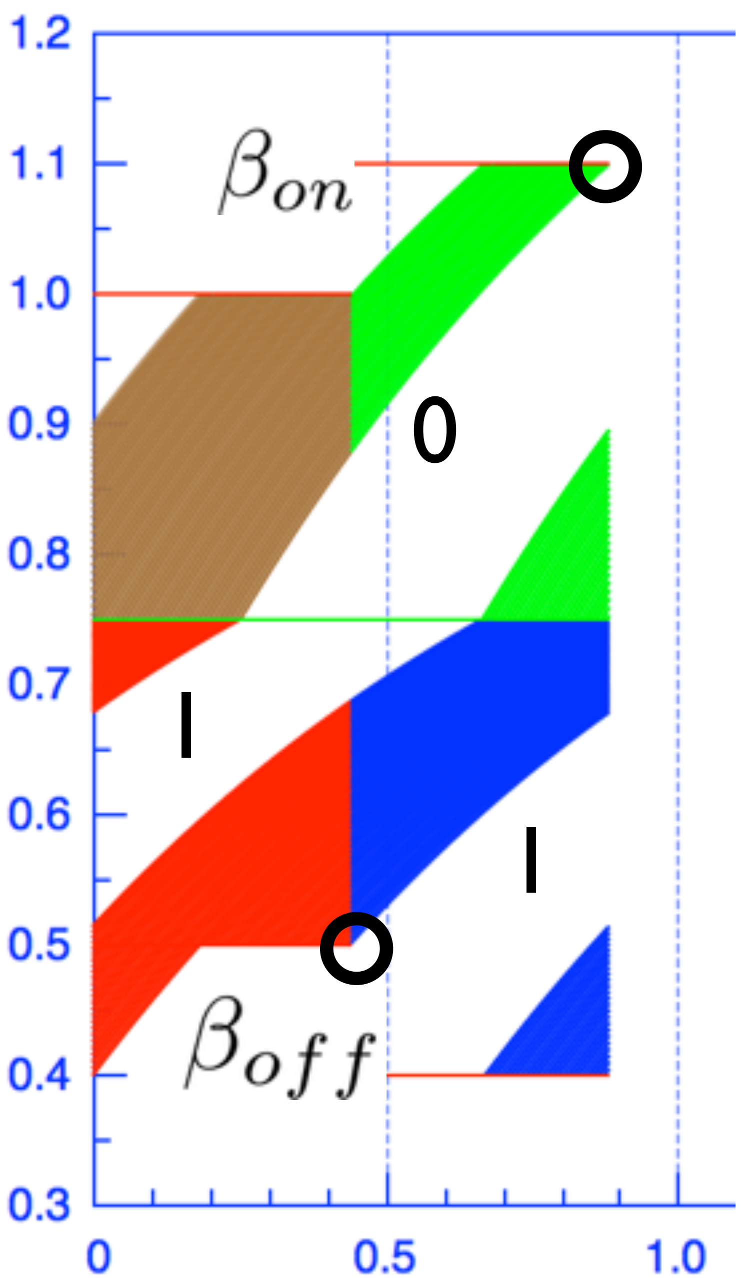


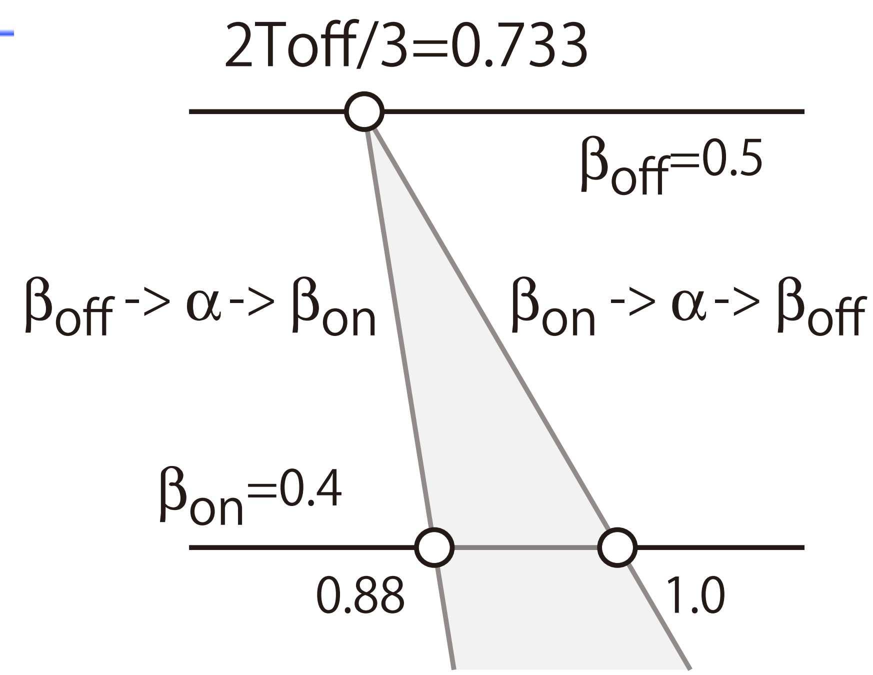
非周期解の存在条件

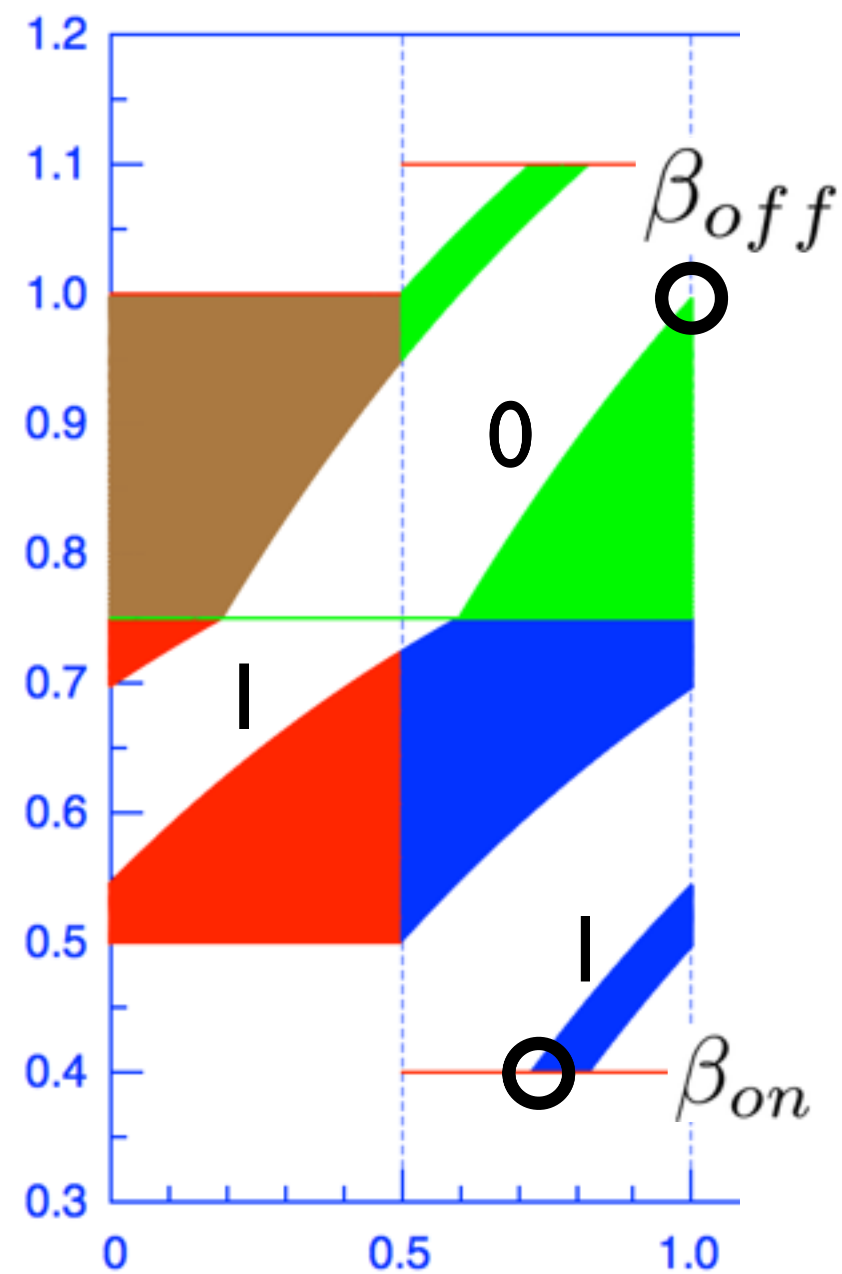


この領域に区間BCが
入れば周期解なし

非周期解の存在条件


1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
0.5
1.0
β_{on}
β_{off}
0
I
I


2Toff/3=0.733
β_{off}=0.5
β_{off} -> α -> β_{on}
β_{on} -> α -> β_{off}
β_{on}=0.4
0.88
1.0


1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
0.5
1.0
β_{off}
β_{on}
0
I
I