



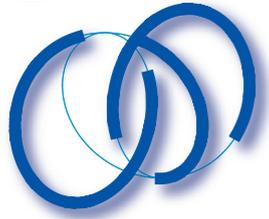
吉川研 ゼミ

リズムと同期現象 —LEDホタルの同期—



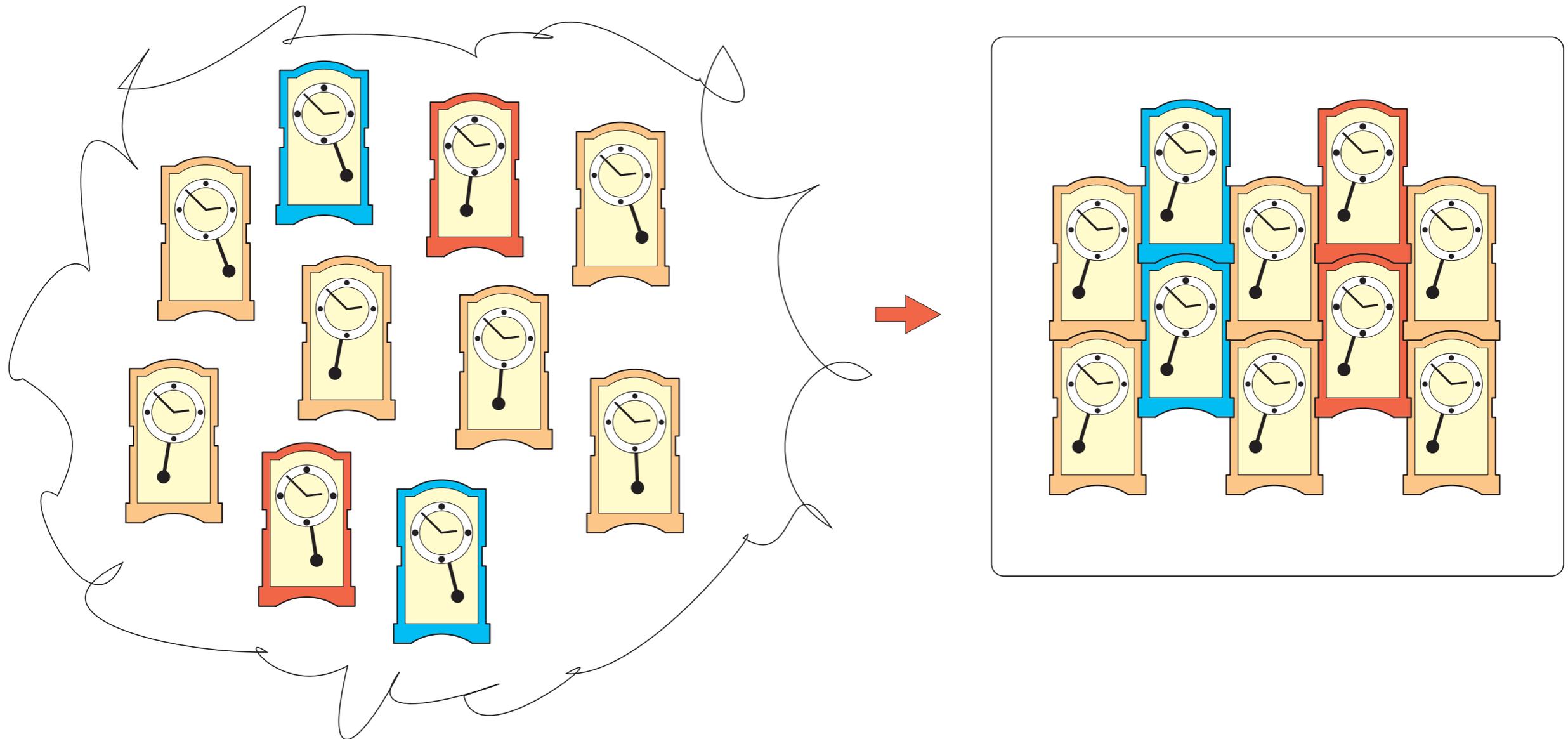
川上 博

2014(H26).02.20



話の概要

リズムは同期する



Christiaan Huygens (オランダ 1629-1695)

1656年 振り子時計を発明：複数の振り子時計が同期することを発見



話の発端：合原プロジェクトの具体例

1976年 Synchronous Fireflies, J. Buck and E. Buck,
Sci. Am. Vol. 234, No. 5

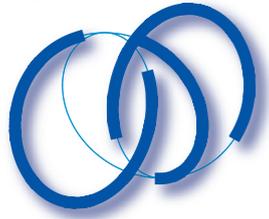
1980年代後半～2000年：吉川研一先生，塩水Osc., PetBottle Osc. etc

1995年 Electronic Fireflies, W. Garver and F. Moss

1996年(H.8) 年方形波発振器を用いた電子ホタルの同期現象
NLP96-5：高坂，川上，上田

1998年(H.10) 年 Trans. IEICE Vol. E-81-A, No. 4

2011年(H.23) 年春夏からLEDホタルをつくり実験する。共同研究者：
最先端研究開発支援プログラム(FIRST合原最先端数理モデルプロジェクト)
合原一幸，木本圭子，関川宗久，合原一究，
上田哲史，伊藤大輔，辻 明典，川上 博



話の流れ

1. リズムをつくる：発振器の仕組み

◎ 方形波発振器：最も簡単なdigital-analog-hybrid系

2. 引き込み現象：強制同期

◎ LEDホタルを光方形波信号で強制振動させると？

3. 同期現象：相互同期

◎ LEDホタルを相互に結合する

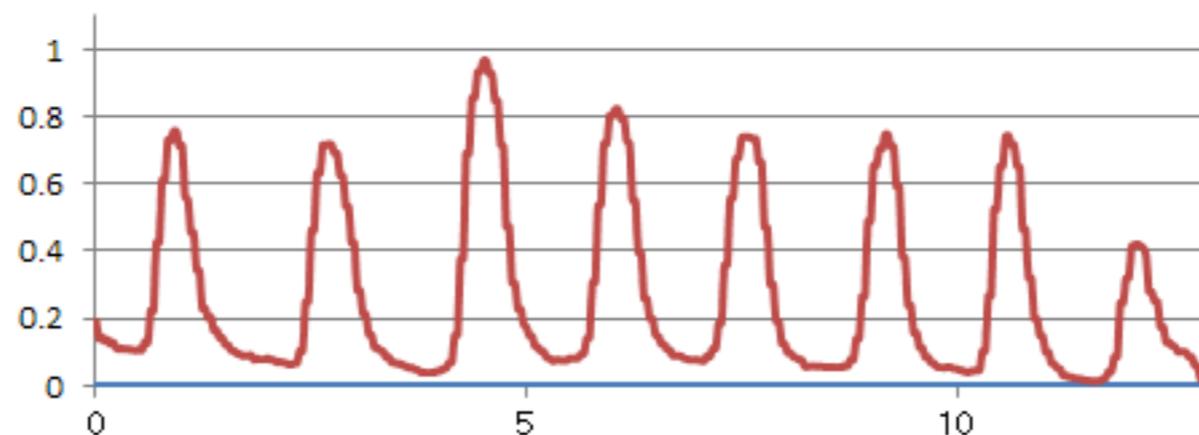
4. 解析：幾何学的アプローチ

◎ hybrid系：FSM+力学系を解析する

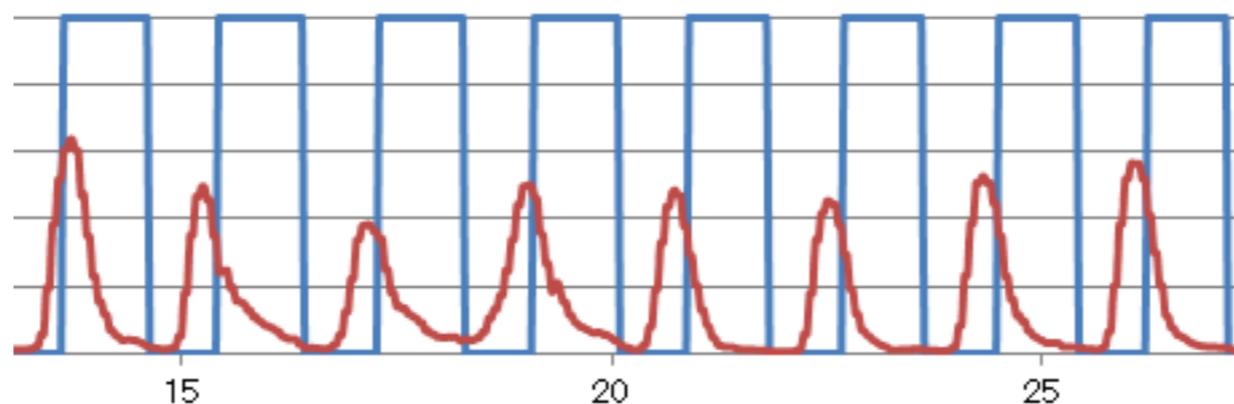


ゲンジホタルの発光：同期パターン

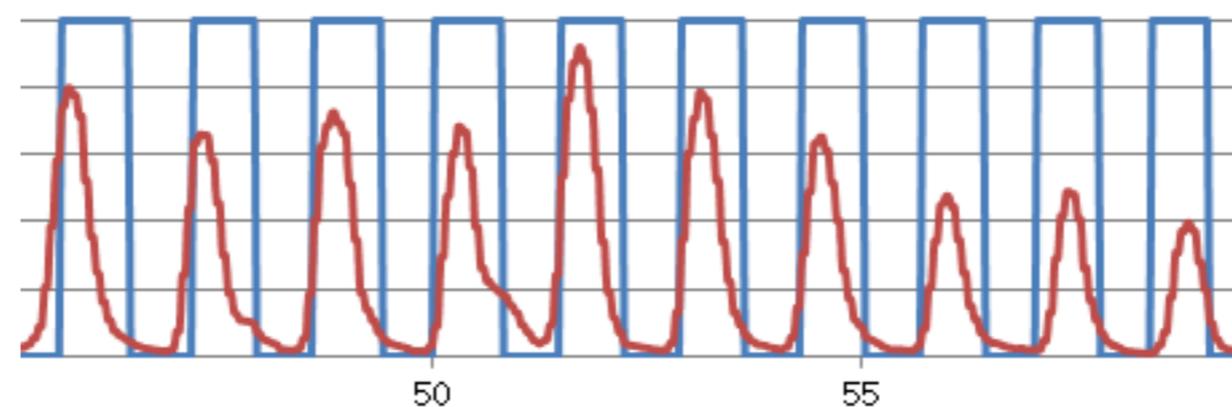
ホタルの発光



逆相同期

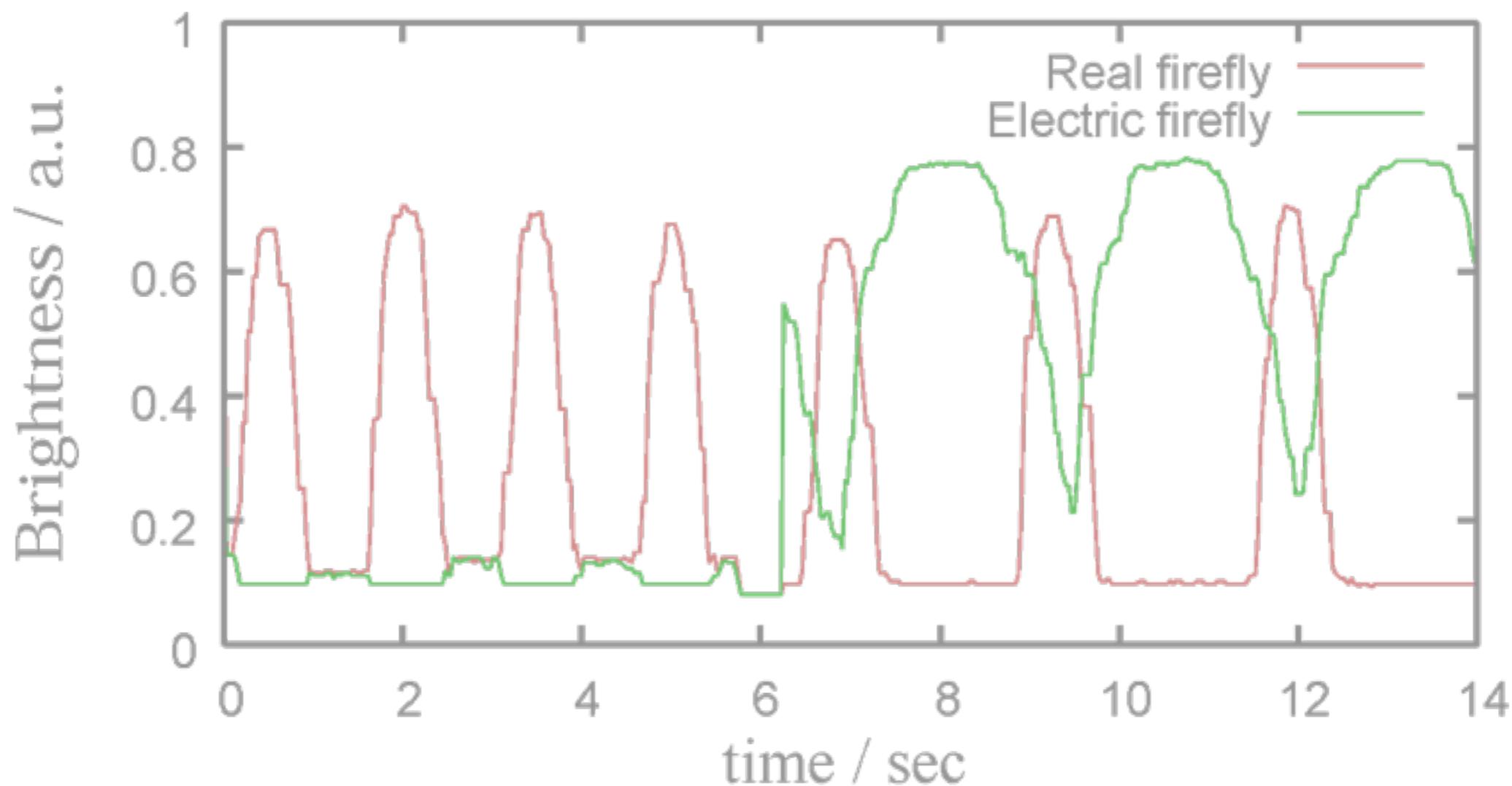


同相同期





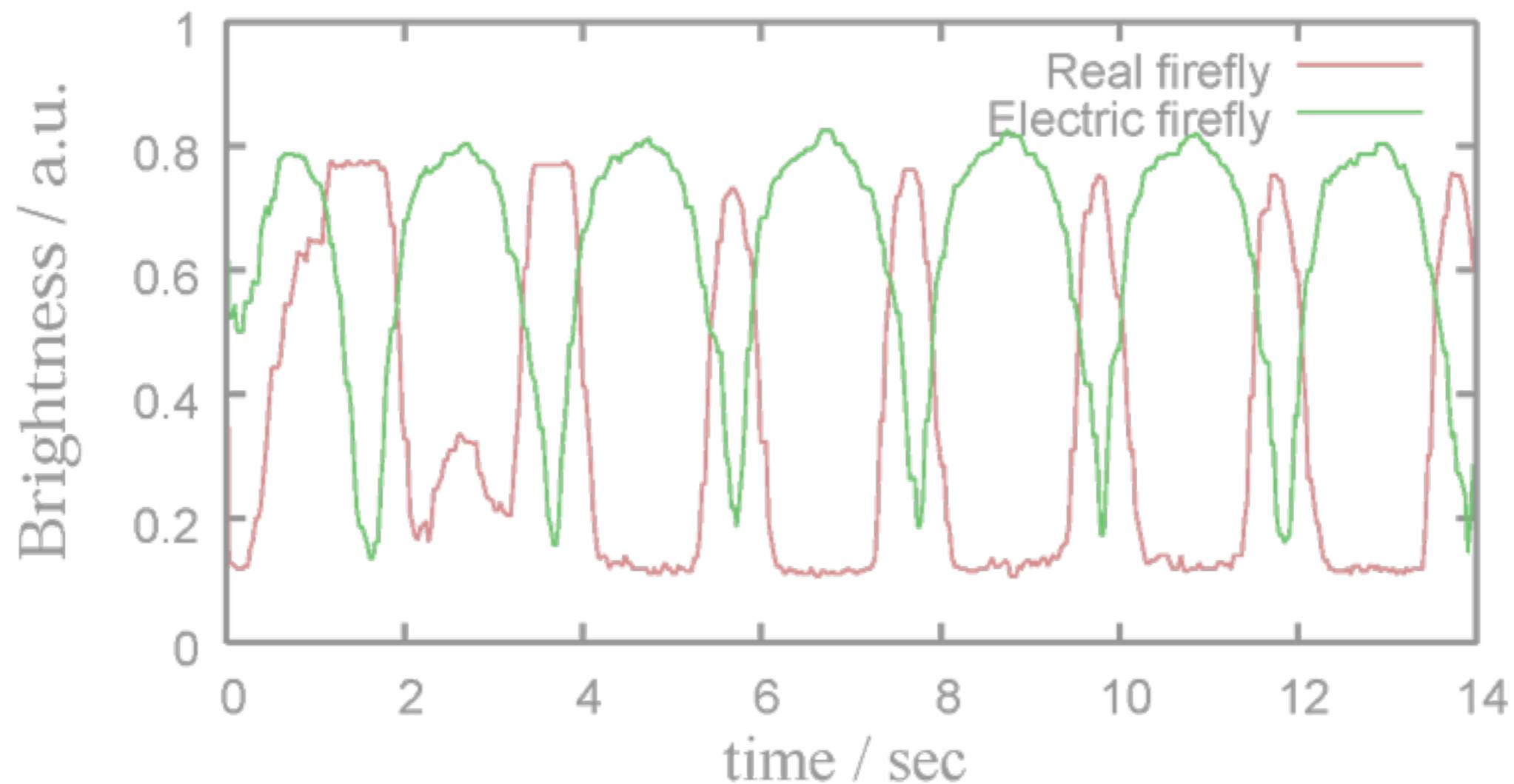
ゲンジホタルの発光：同期パターン



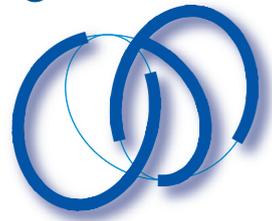
2012/06/12 徳島県吉野川市美郷川田川流域, 木本, 関川, 合原, 川上



ゲンジホタルの発光：同期パターン



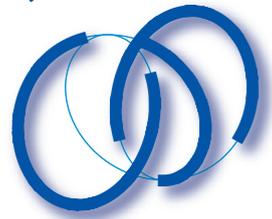
2012/06/12 徳島県吉野川市美郷川田川流域, 木本, 関川, 合原, 川上



1. リズムをつくる：発振器の仕組み

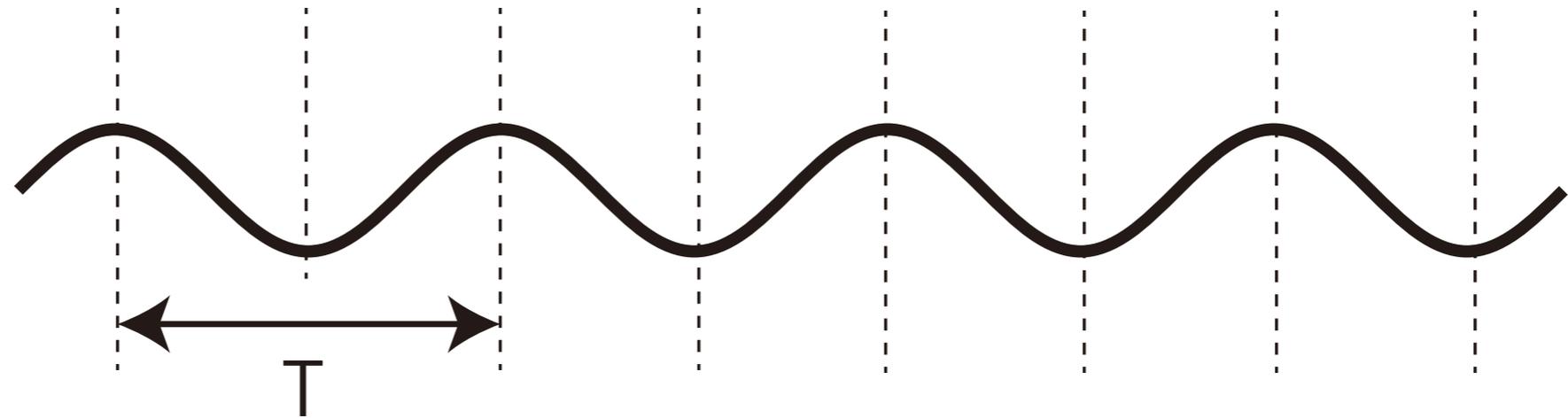


場所：徳島県吉野川市美郷川田川流域
ゲンジボタル発祥の地「国の天然記念物」により保護
日時：2013年6月29日20時～22時
温度，湿度：22度，85%
対象：ゲンジボタル（オス）の個体

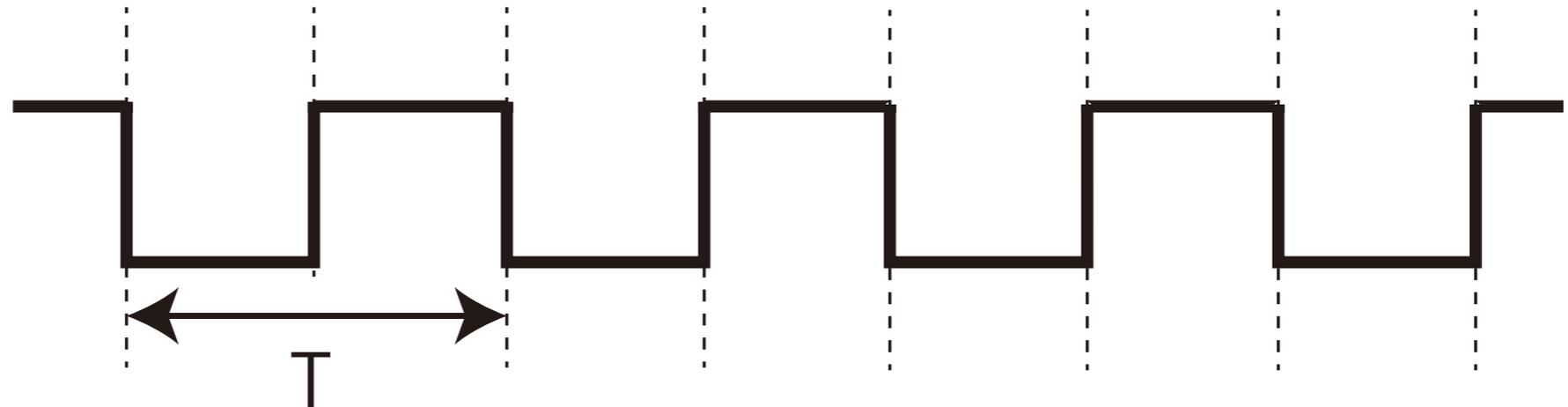


2つの典型的なリズム波形

正弦波
analog



方形波
digital

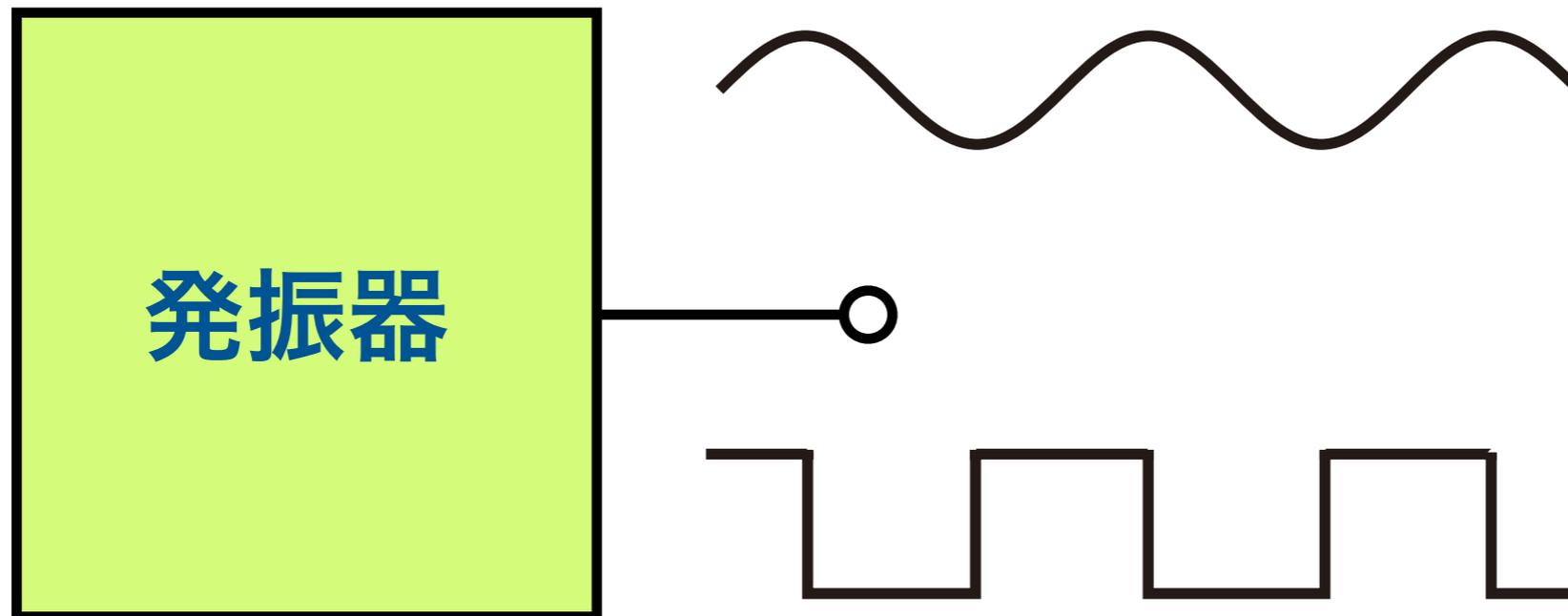


t



リズム波形をつくる：発振器

正弦波発振器(harmonic oscillator)

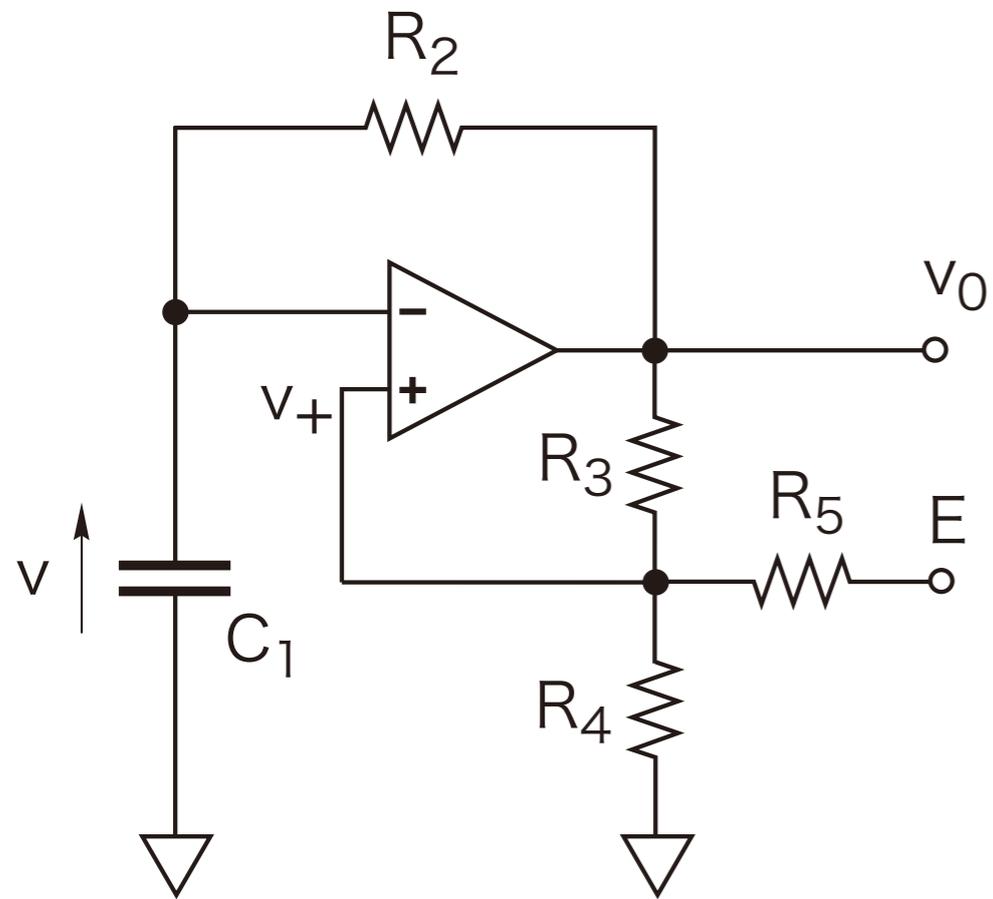


方形波発振器(square wave oscillator)

弛張振動(relaxation oscillation)



RC方形波発振器



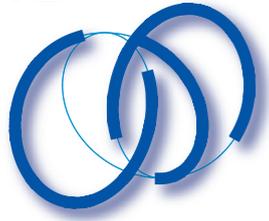
RC square wave oscillator



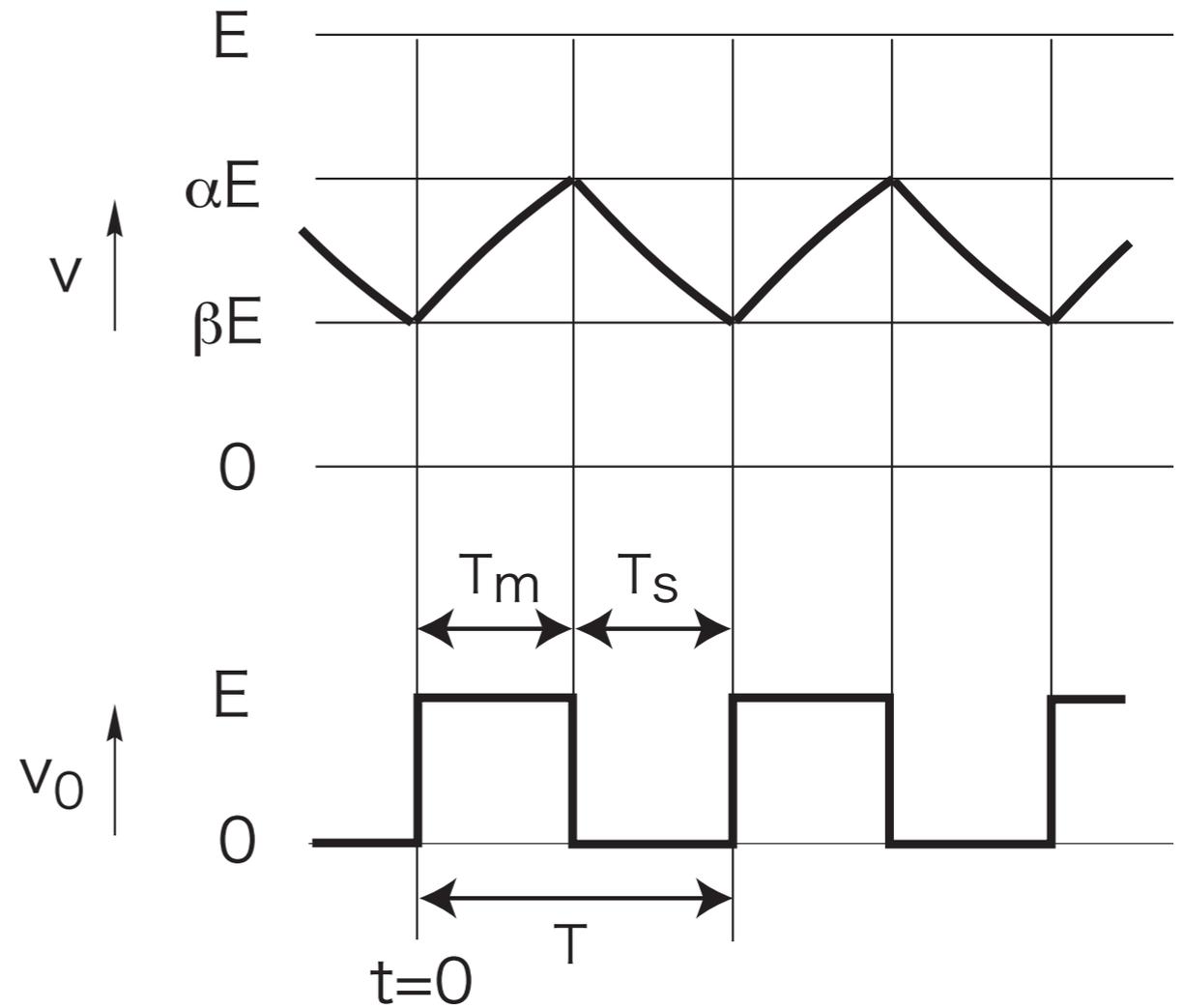
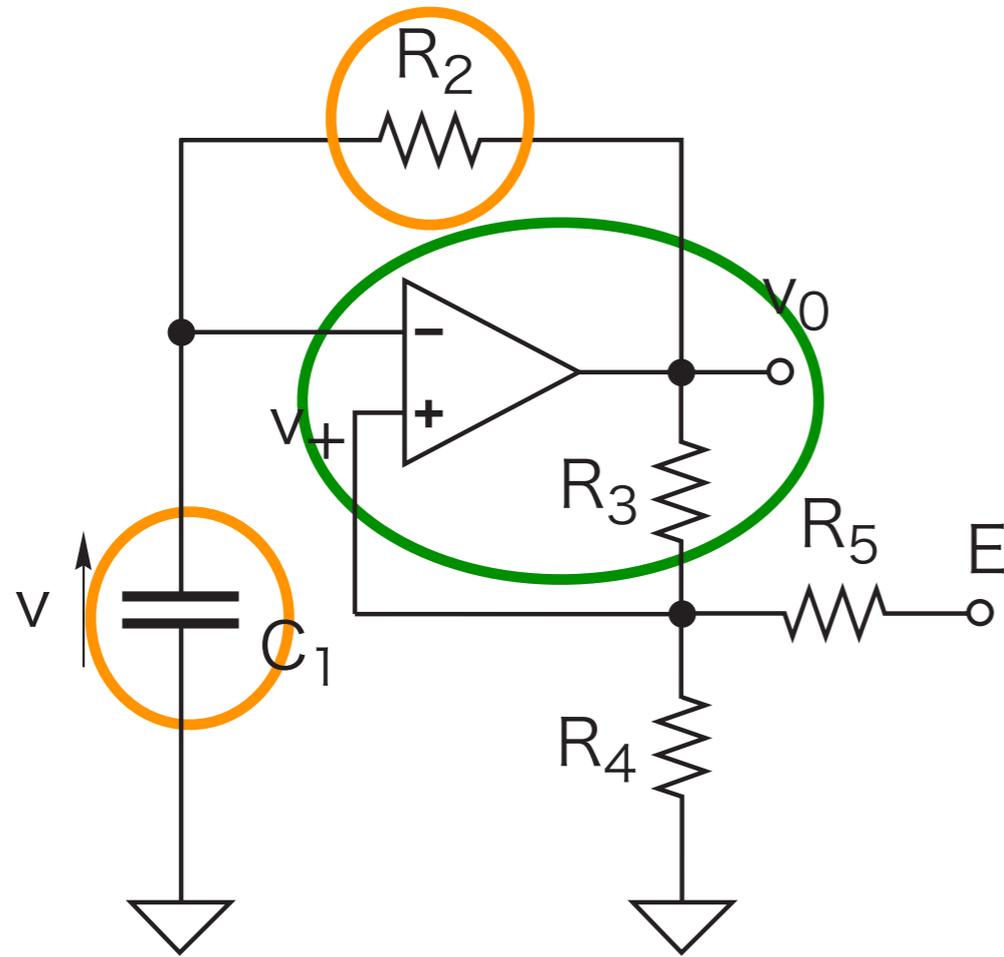
鹿おどし



pet bottle oscillator



RC方形波発振器と発振波形

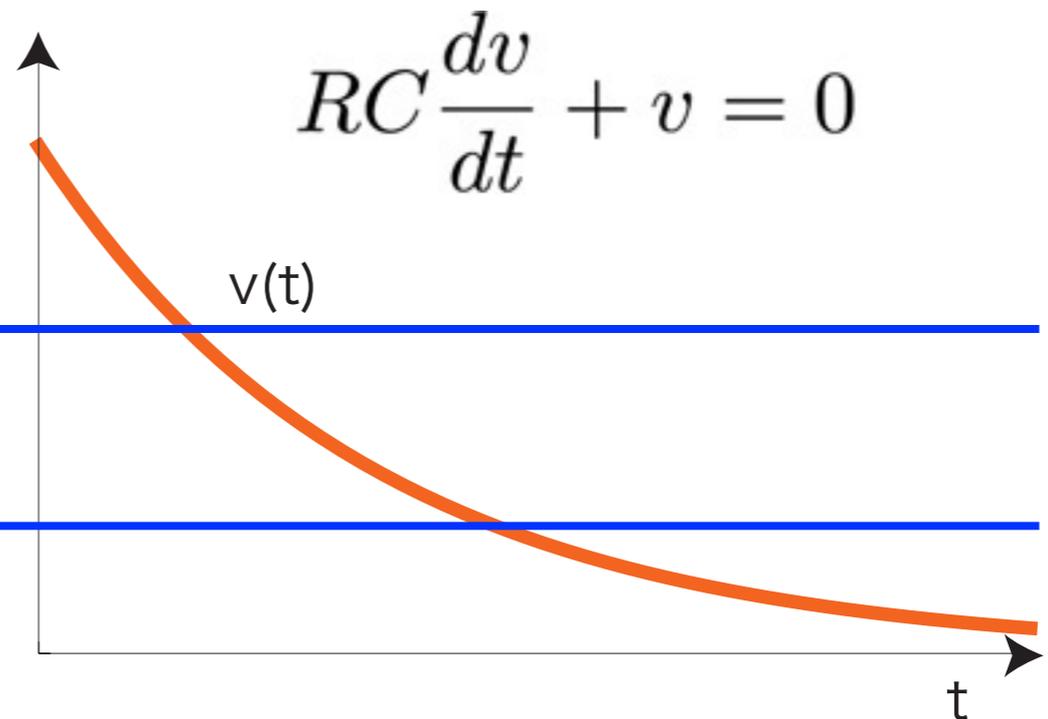
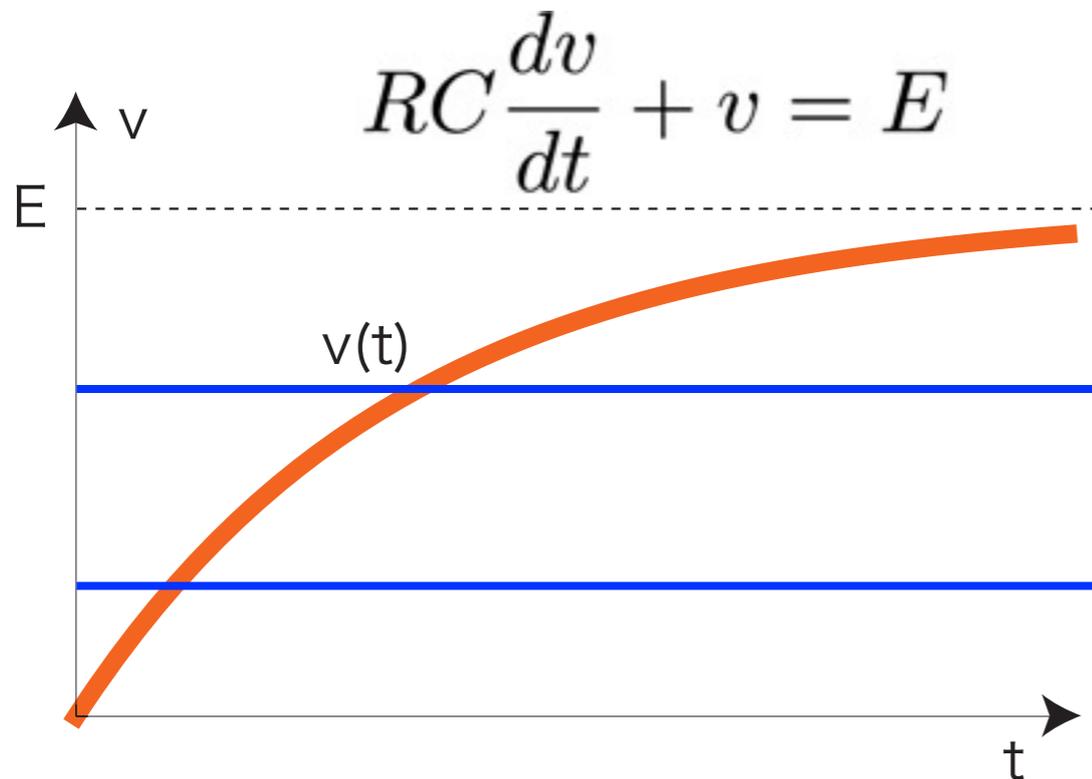
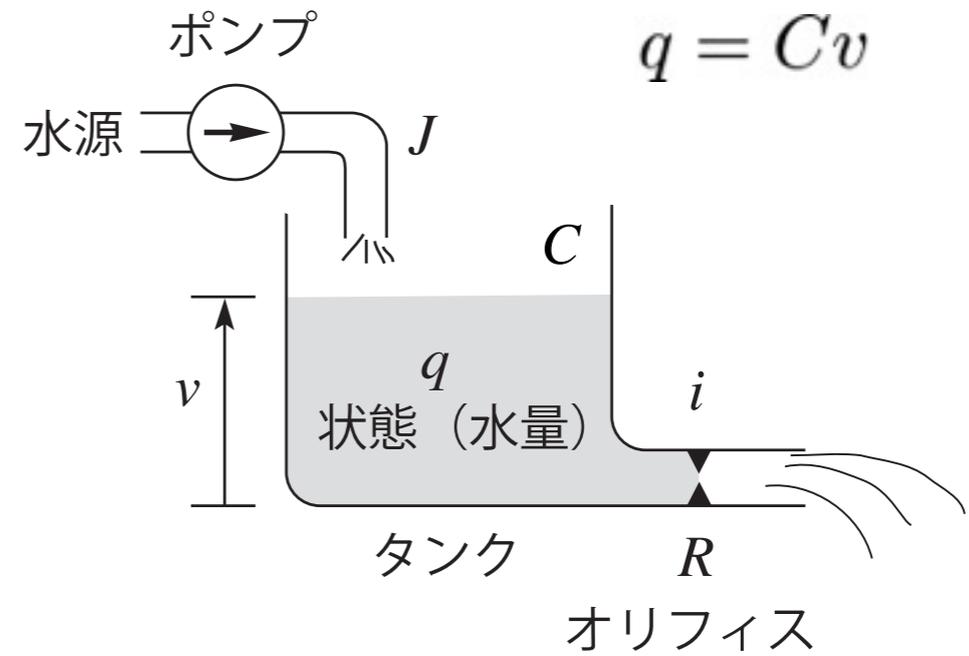
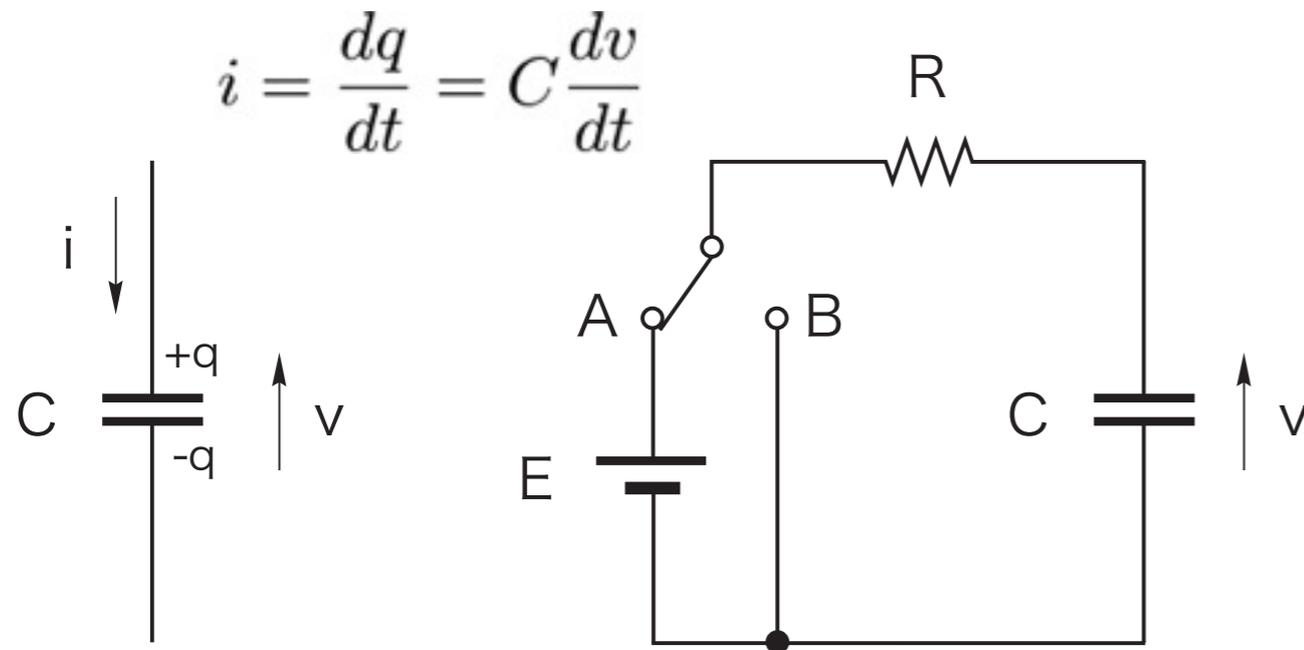


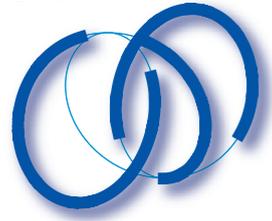
$$R_2 C_1 \frac{dv}{dt} + v = E$$

$$R_2 C_1 \frac{dv}{dt} + v = 0$$



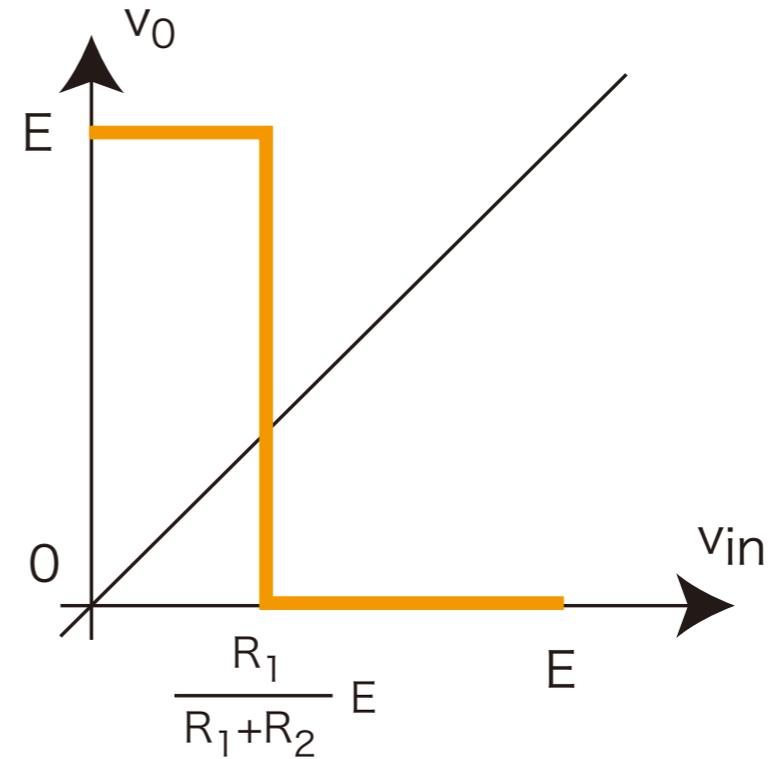
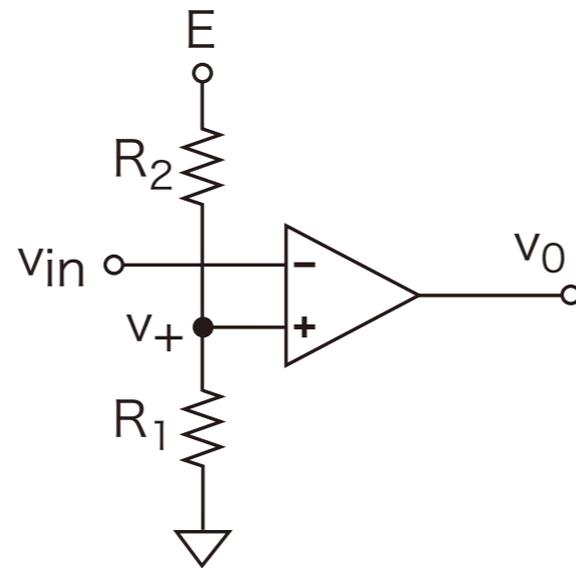
キャパシタとRC回路





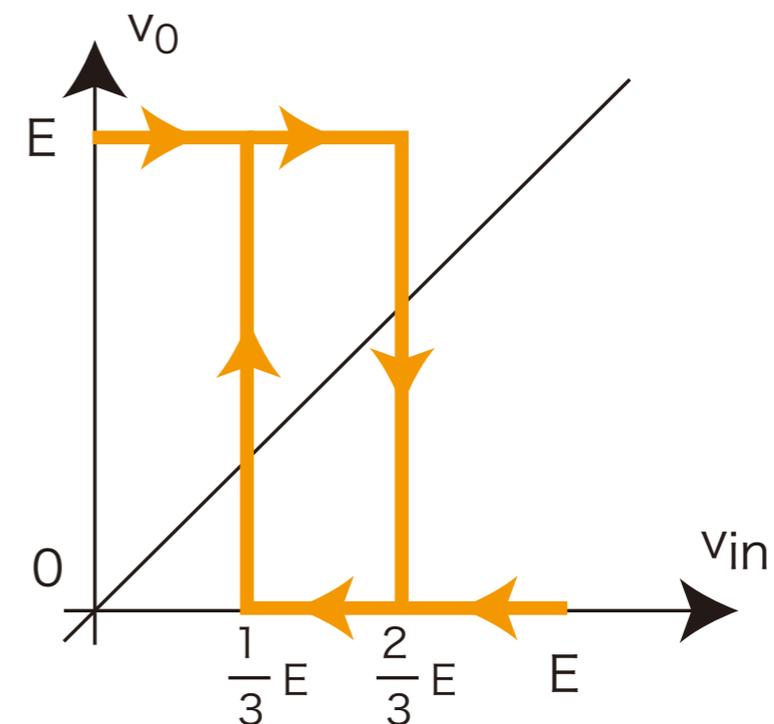
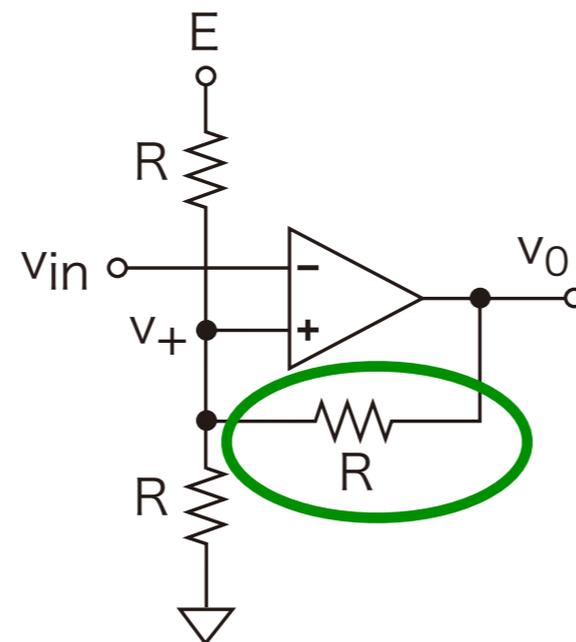
Comparator

comparator



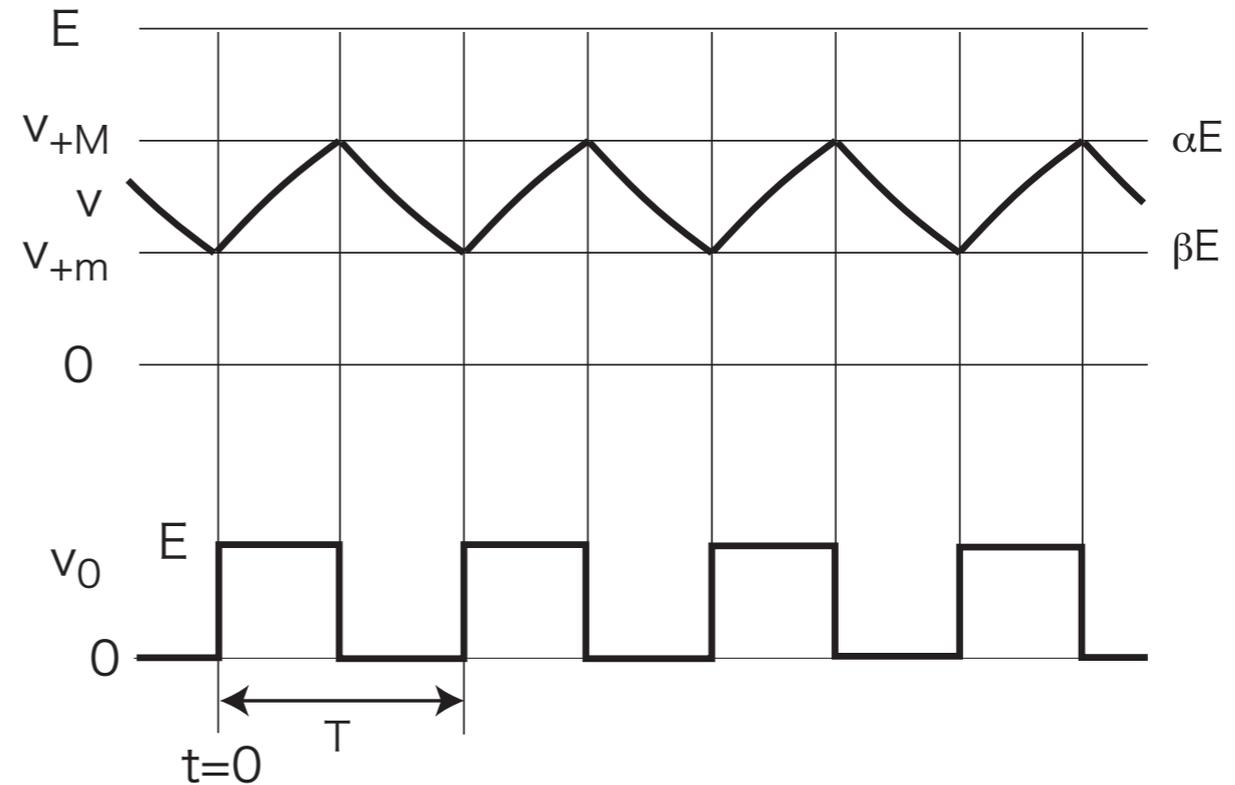
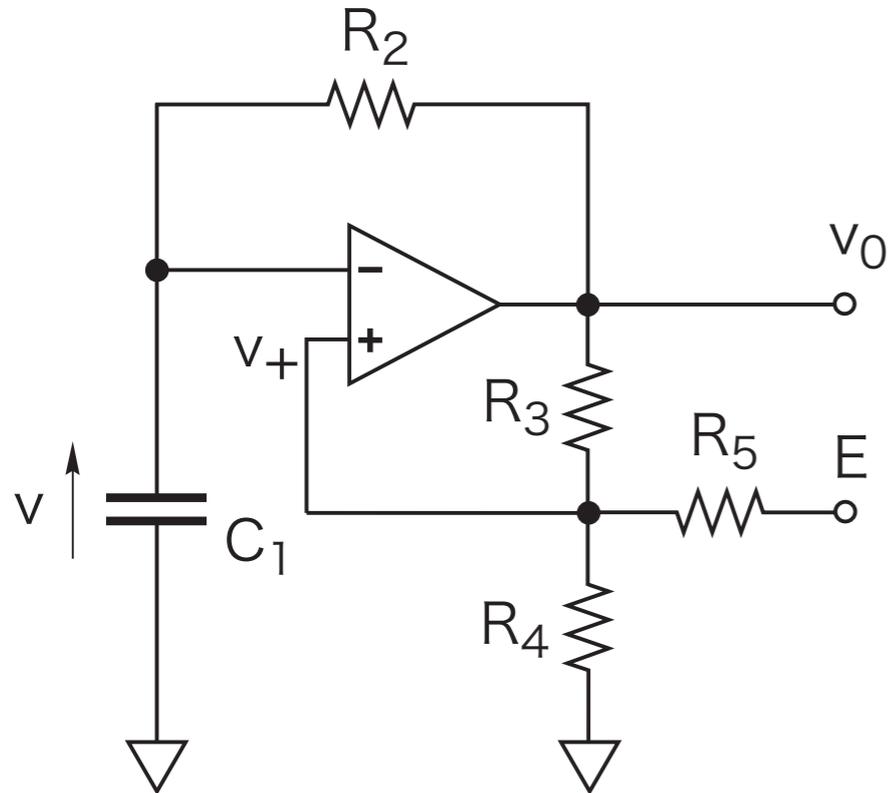
$$v_0 = A(V_+ - V_{in})$$

hysteresis comparator





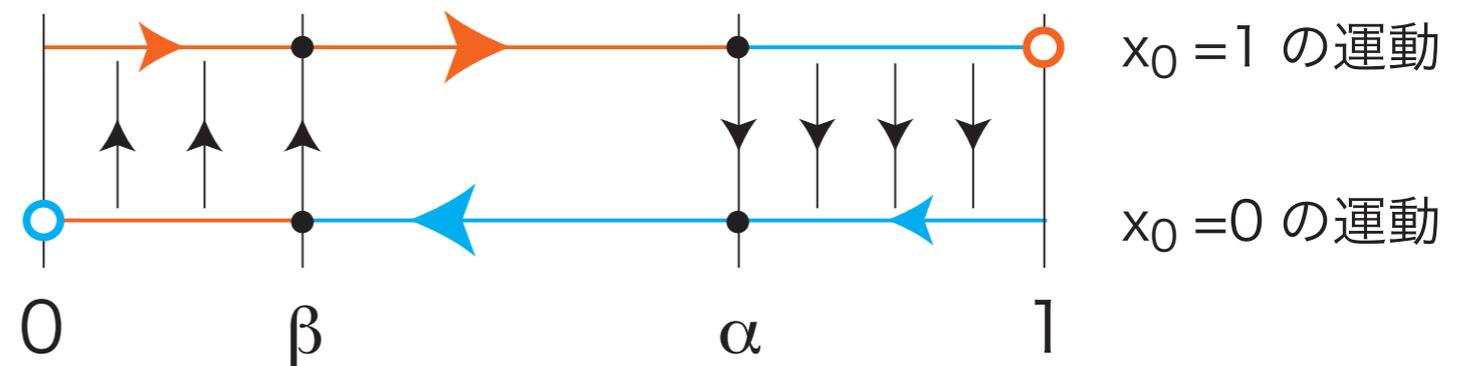
RC方形波発振器

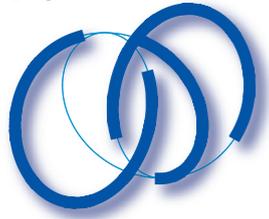


$$R_2 C_1 \frac{dv}{dt} + v = v_0 : E \text{ or } 0$$

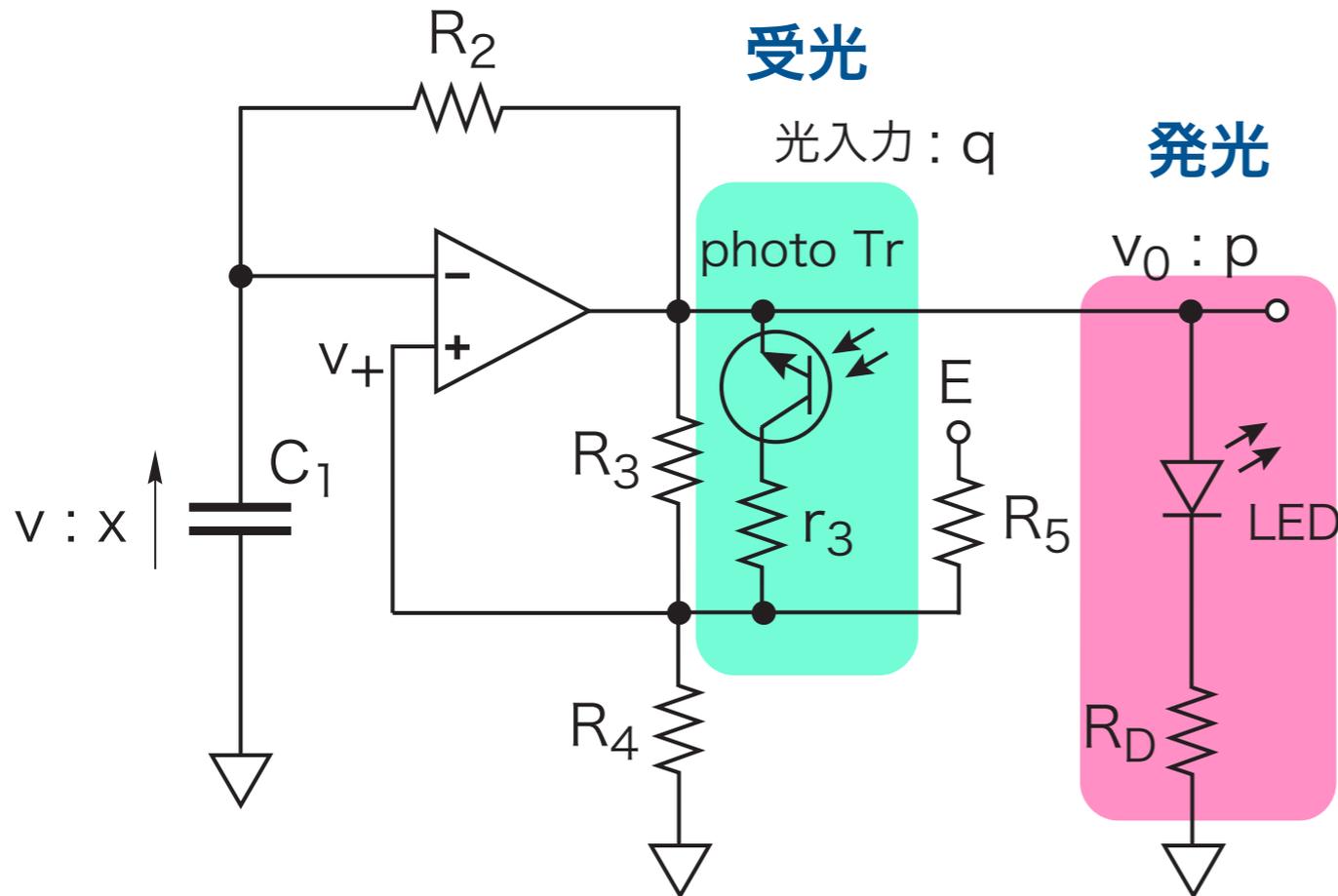
$$t = R_2 C_1 \tau, v = E x$$

$$\frac{dx}{dt} + x = x_0 : 1 \text{ or } 0$$



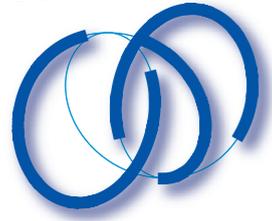


LEDホタルの回路例：type A1

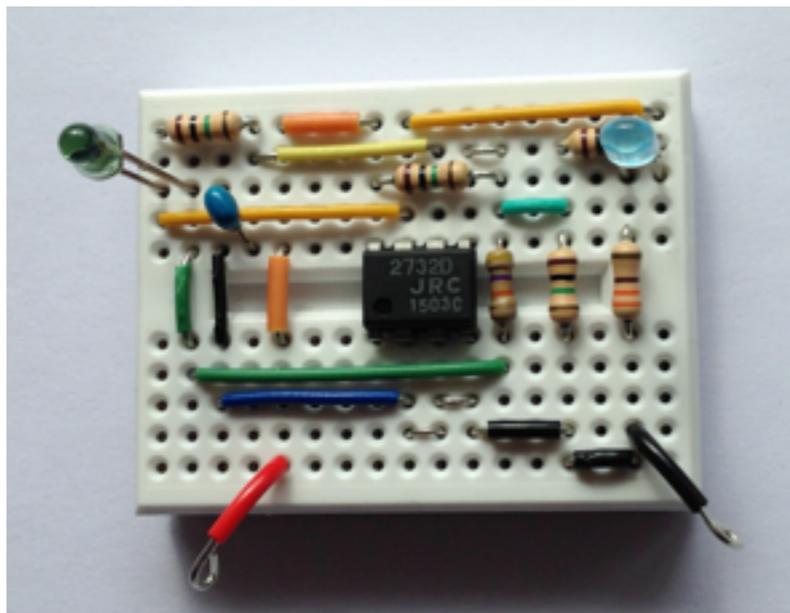
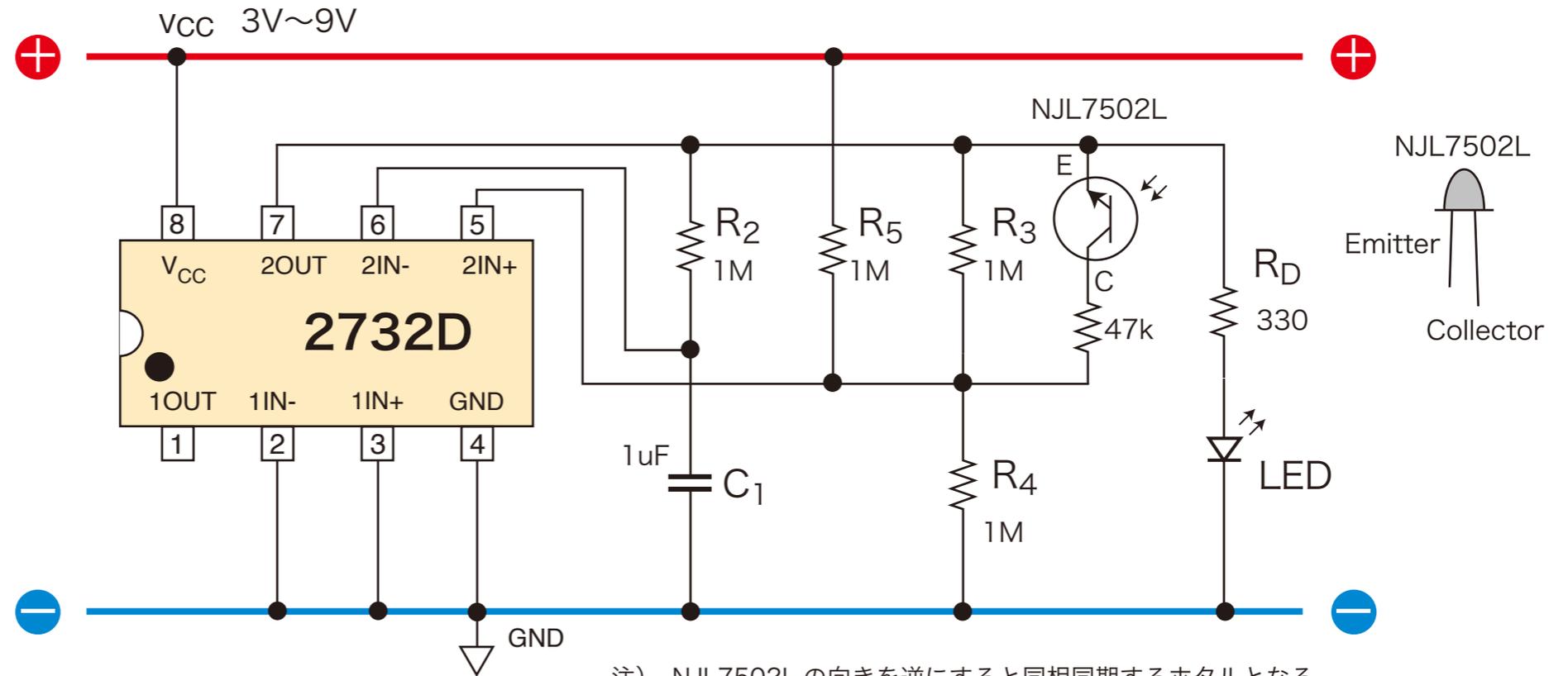


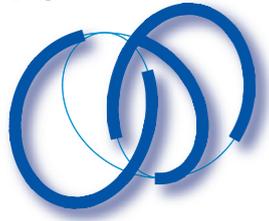
| | | comparator : p | |
|---------|-------|----------------|-------|
| | | on:1 | off:0 |
| 光入力 : q | on:1 | off:0 | on:1 |
| | off:0 | off:0 | off:0 |

- ◎ photo Trは、光入力qとコンパレータ出力pの条件で開閉するスイッチである
- ◎ この回路のdigital状態（モードという）はp, qの組み合わせで4モードある
- ◎ p=0かつq=1のとき、Tr : onとなり、 $R3//r3$ が小(\betaが小) さくなる



LEDホタルの回路図：type A1

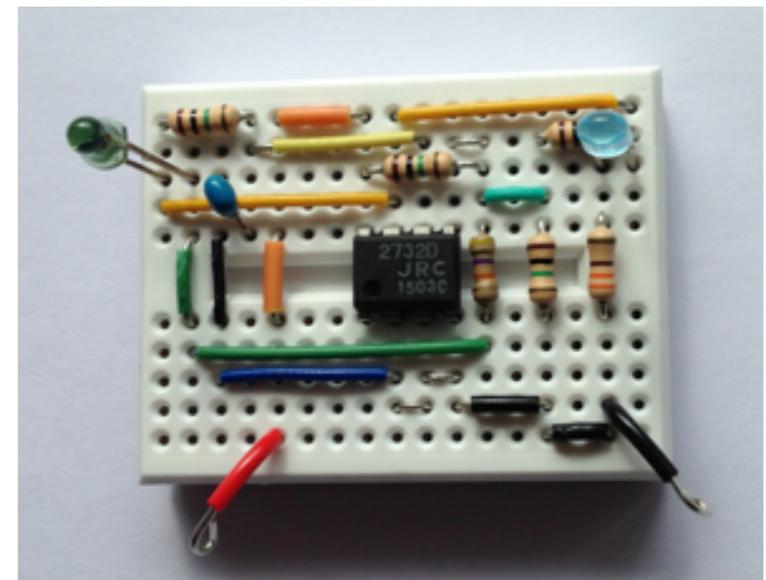
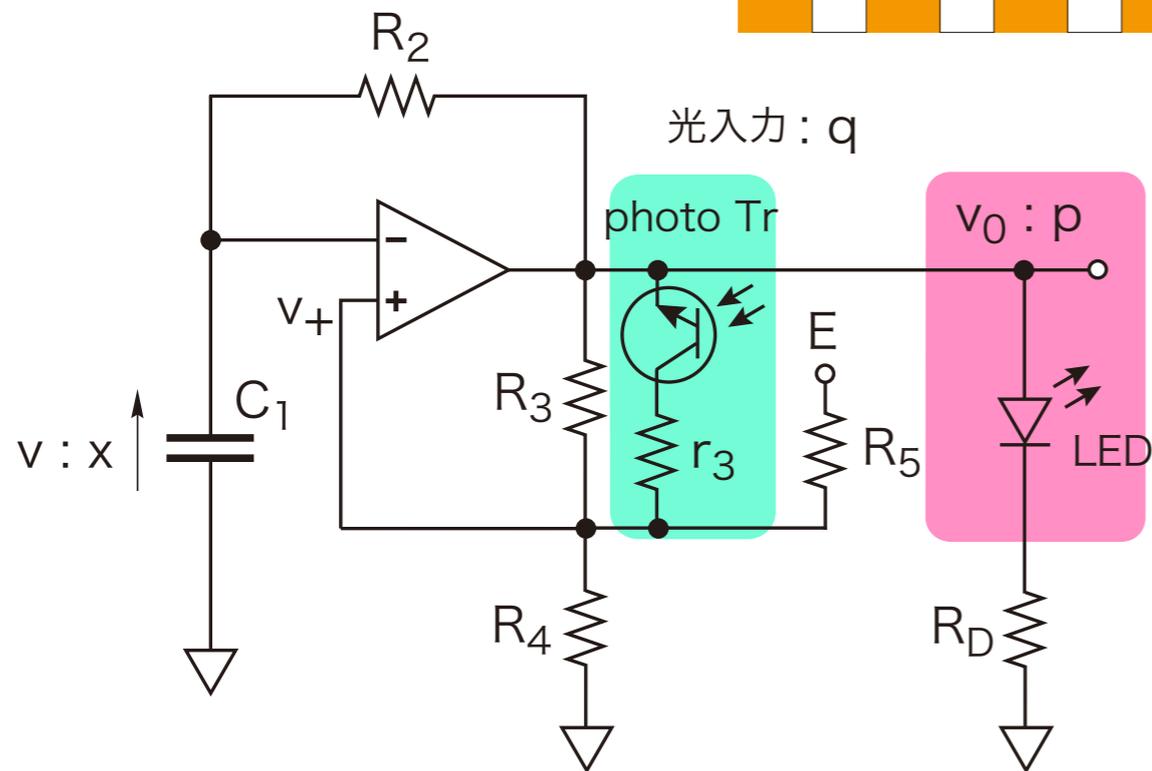


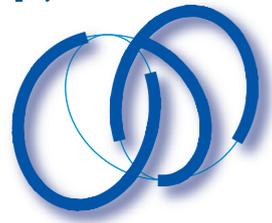


2. 引き込み現象：強制同期

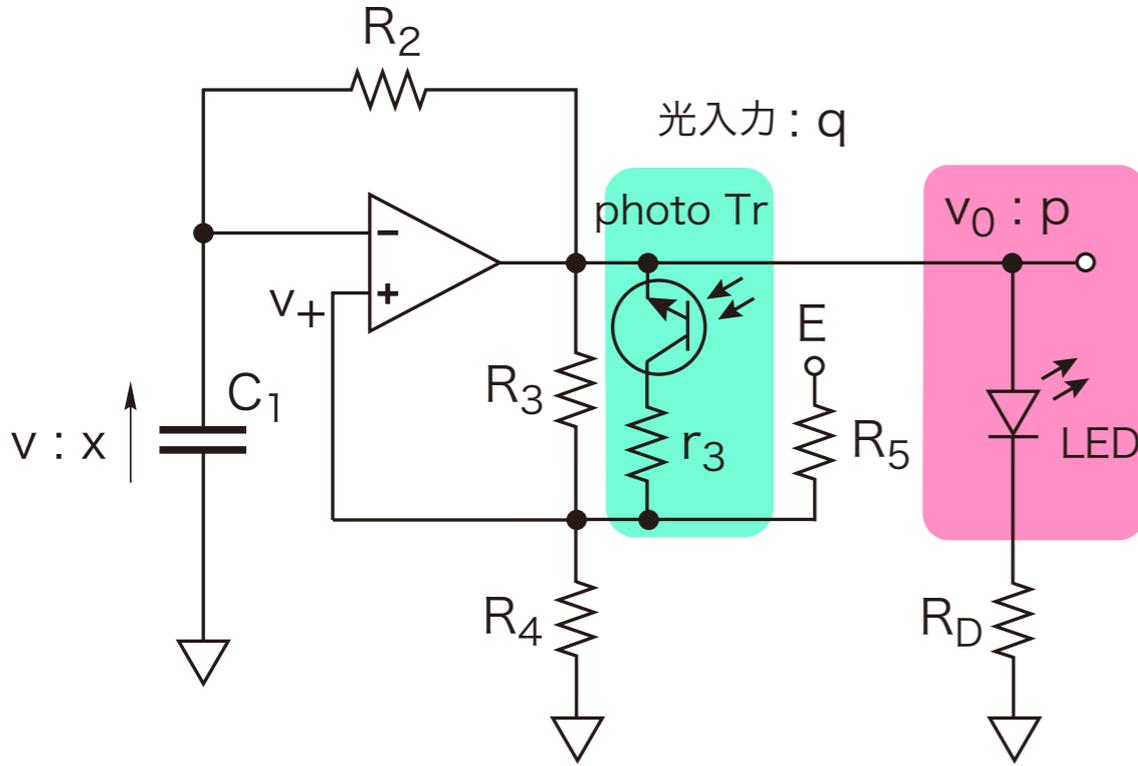
◎ LEDホタルを光方形波信号で強制振動させる

受光  発光 





LEDホタルの回路例 : type A



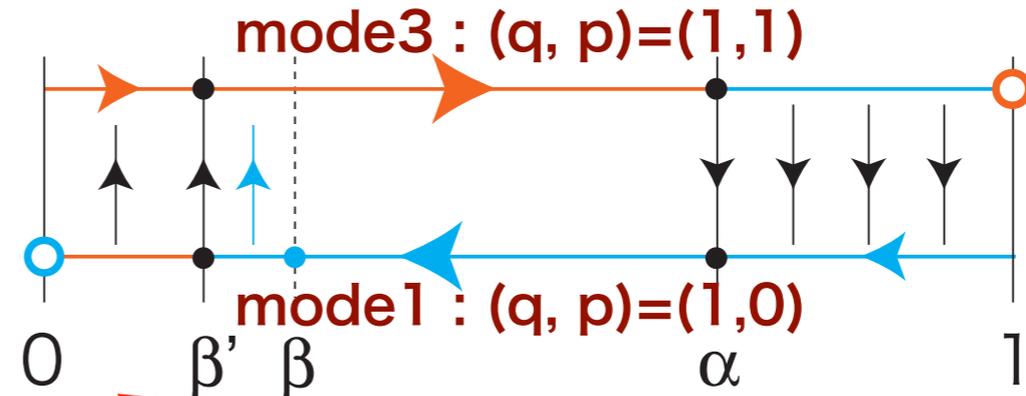
受光



発光



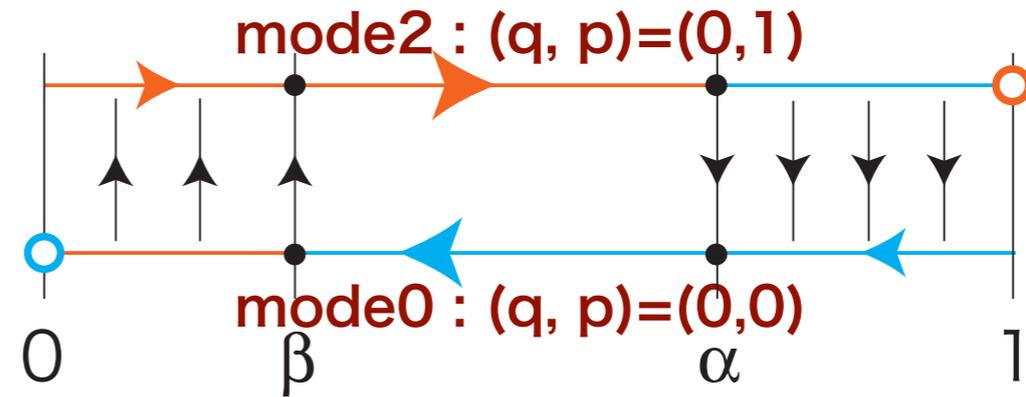
光の照射あり : $q=1$



$p=1$ の運動

$p=0$ の運動

光の照射なし : $q=0$



$p=1$ の運動

$p=0$ の運動

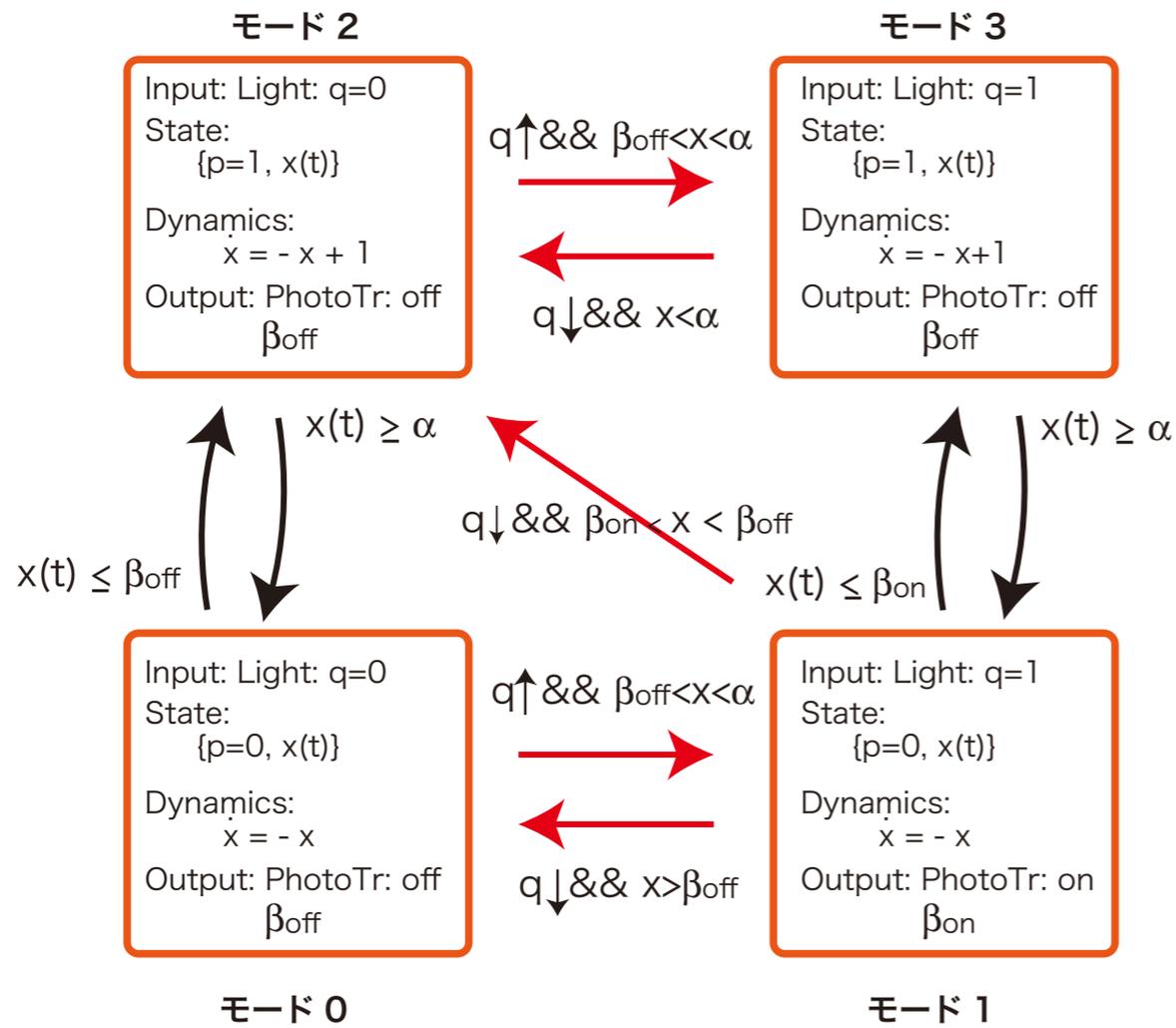
$x \longrightarrow$

$q=1$ かつ $p=0$ のとき, $Tr : on$ となり,
 $R3//r3$ が小(β が小) さくなる

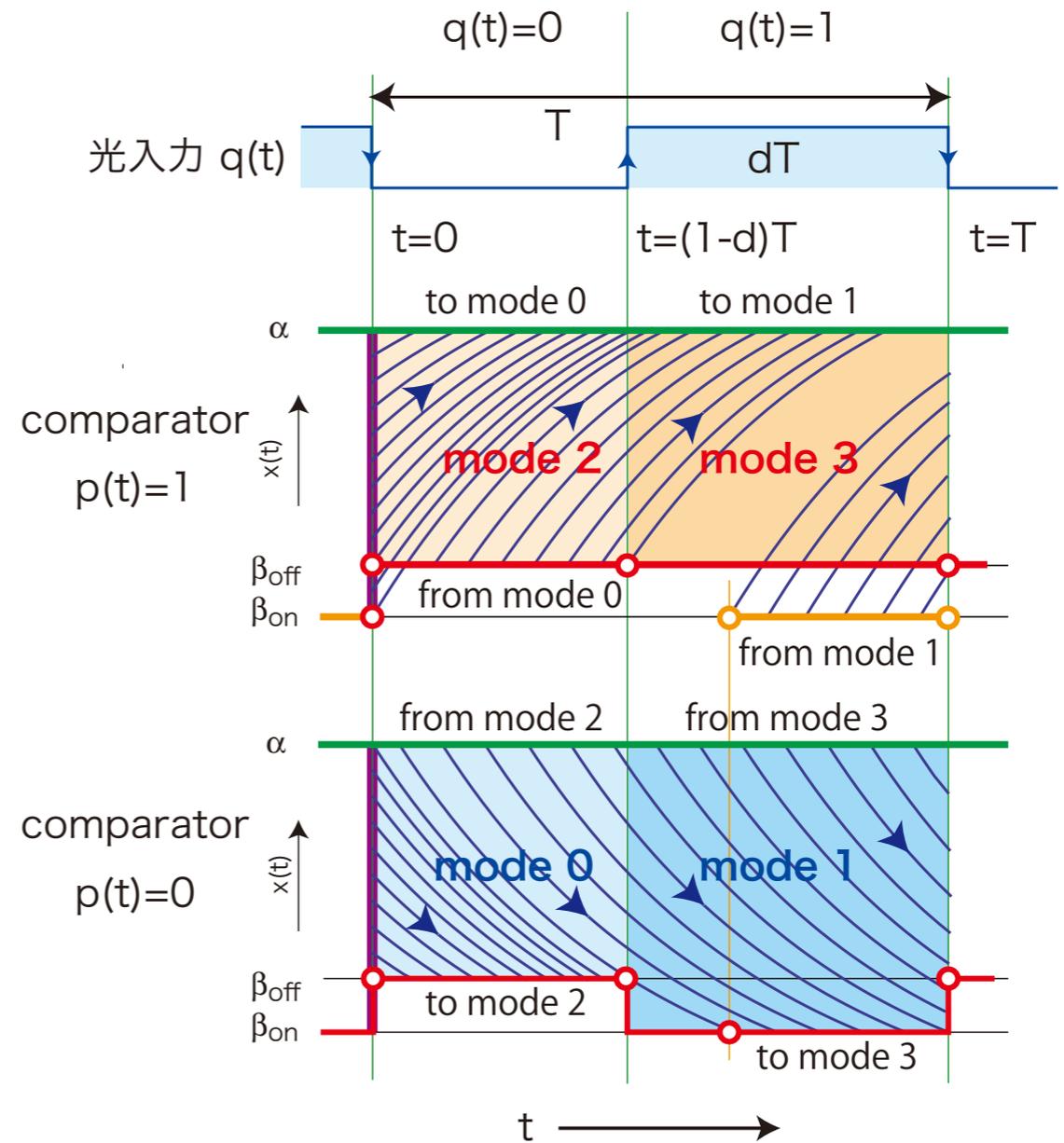


モード遷移図と相図

mode transition diagram

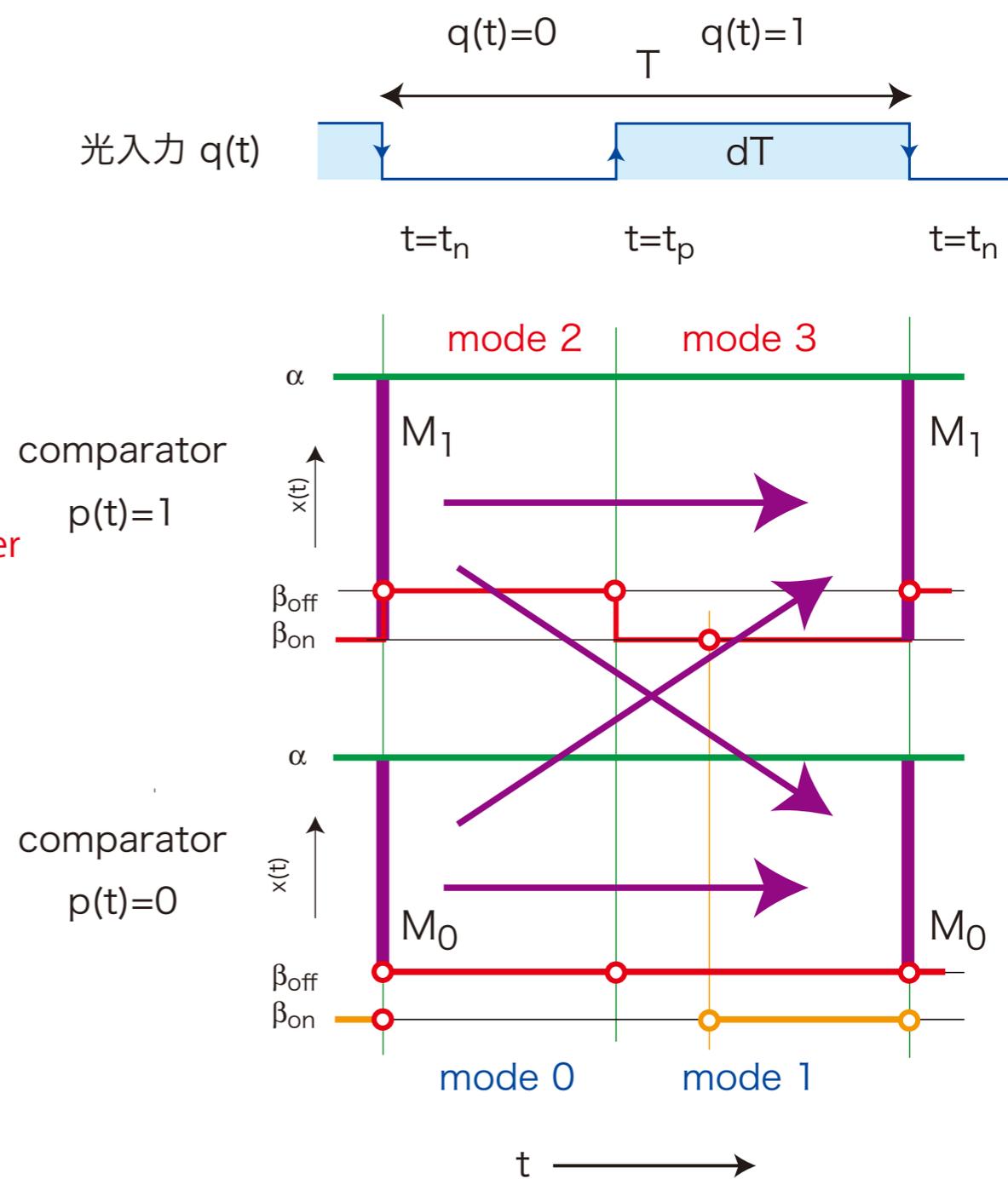
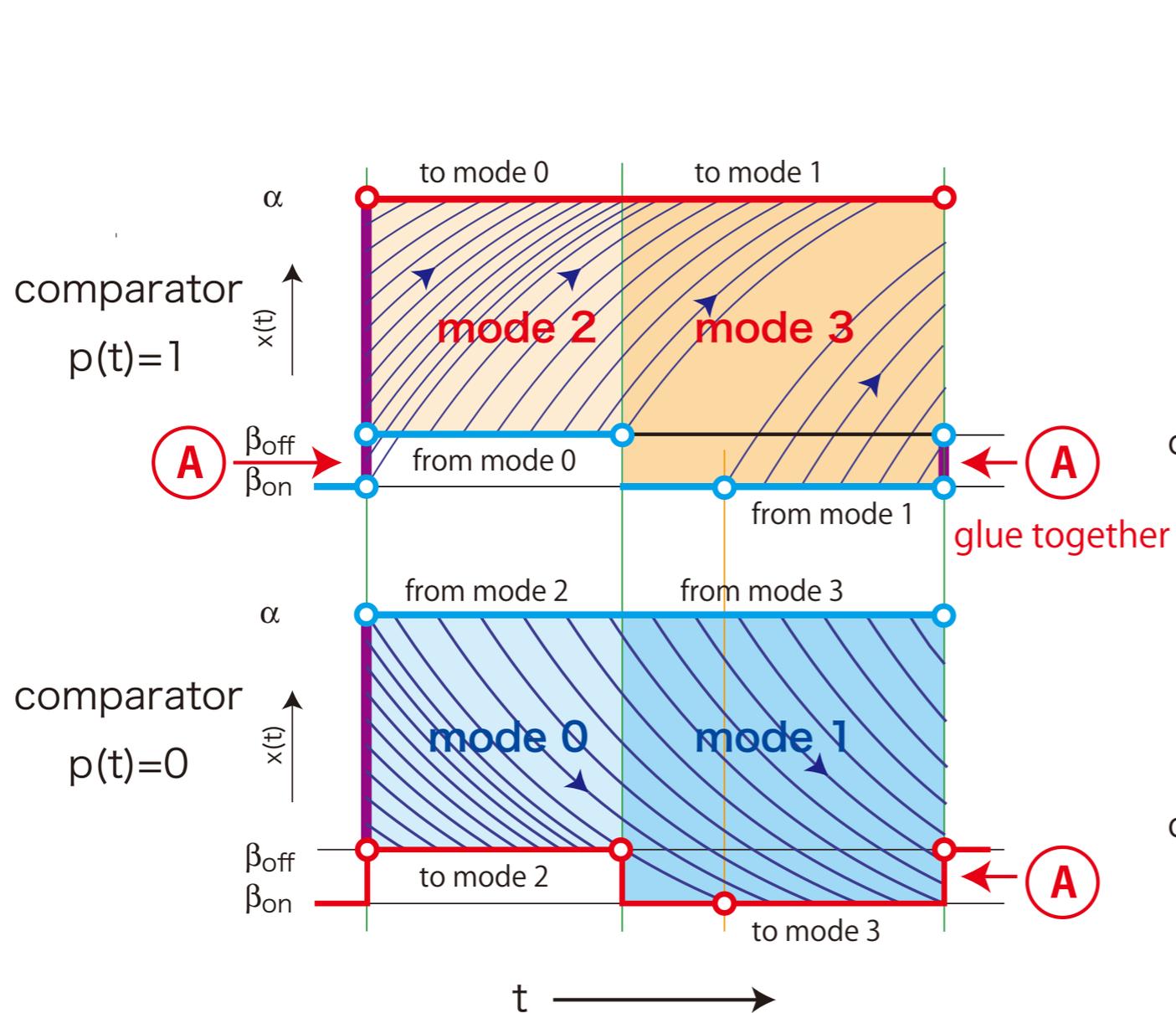


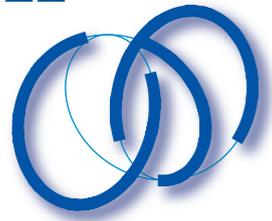
phase diagram



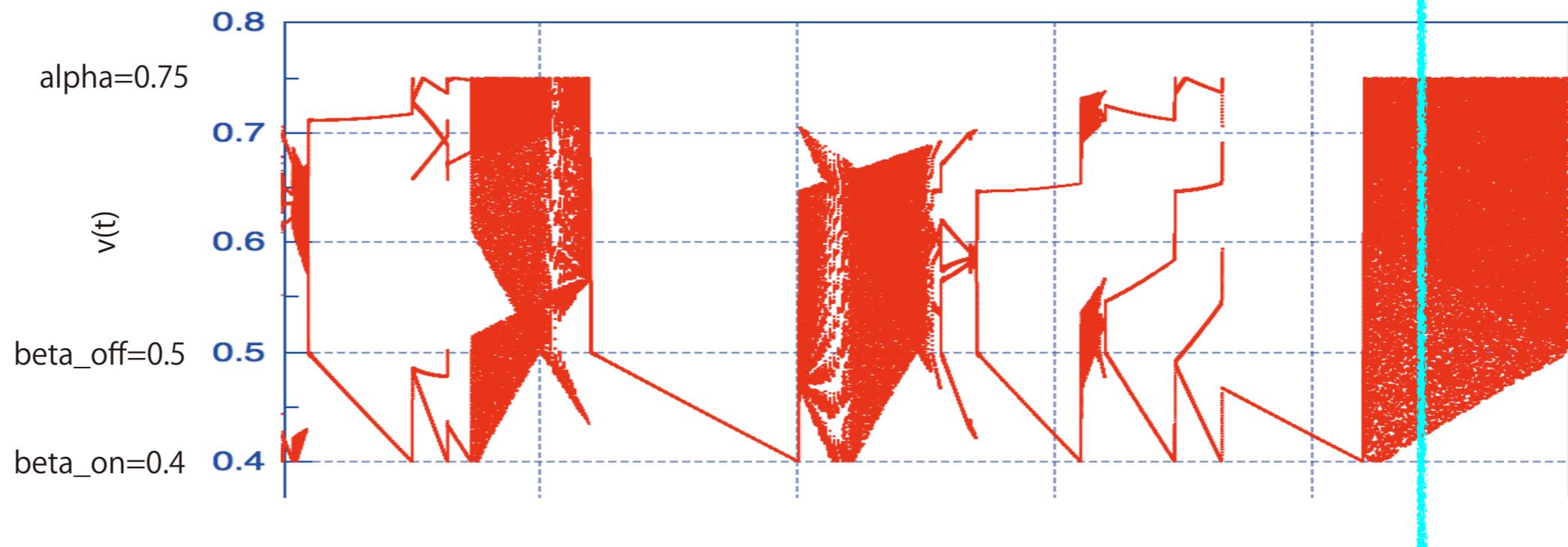
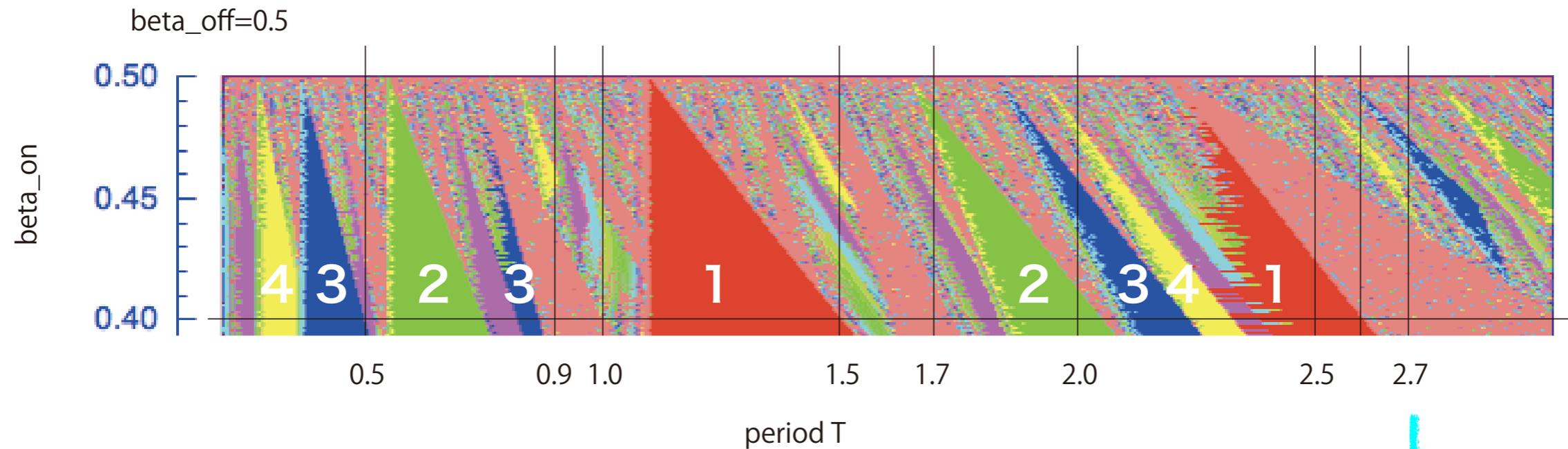


Poincaré 断面と写像





bifurcation diagrams: duty cycle=0.5



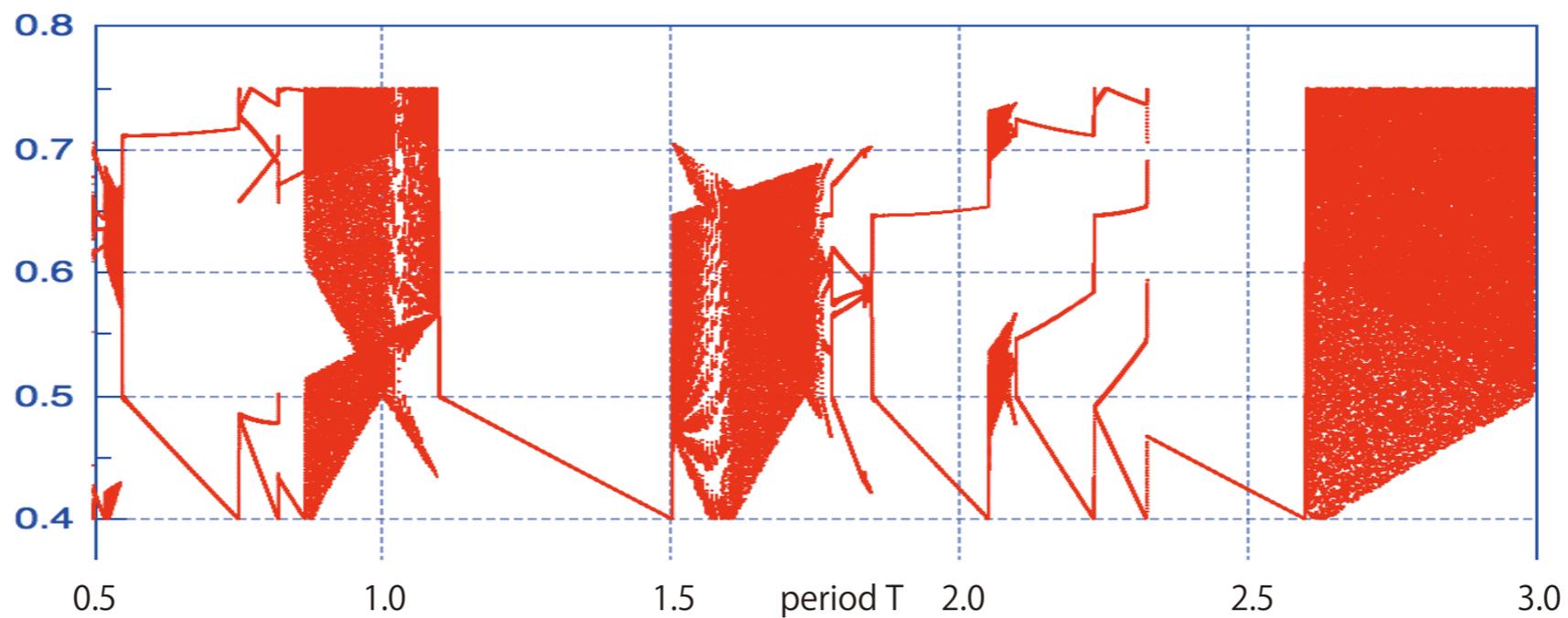


$\alpha=0.75$

$v(t)$

$\beta_{\text{off}}=0.5$

$\beta_{\text{on}}=0.4$

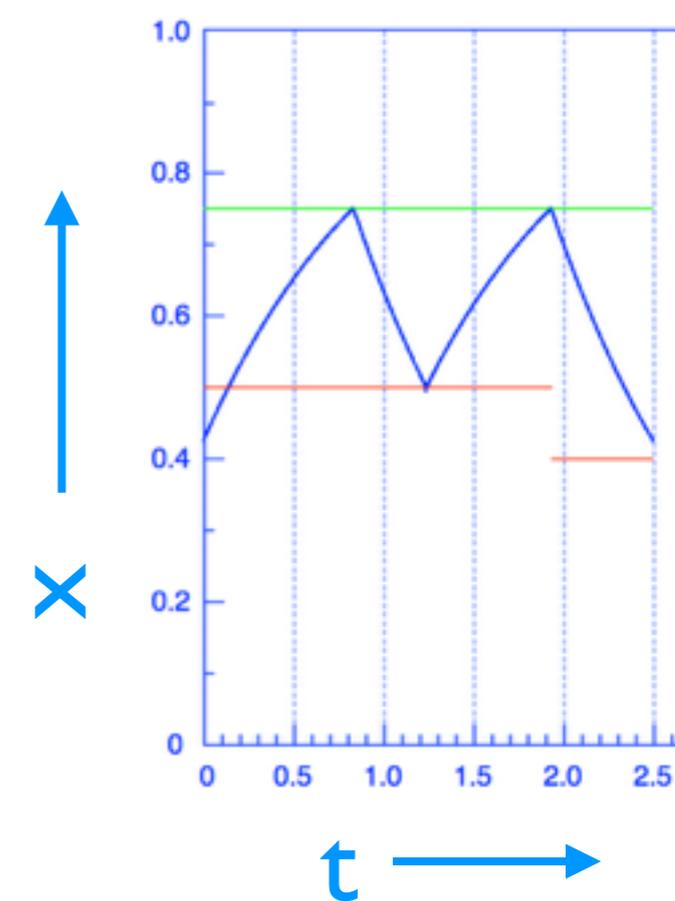
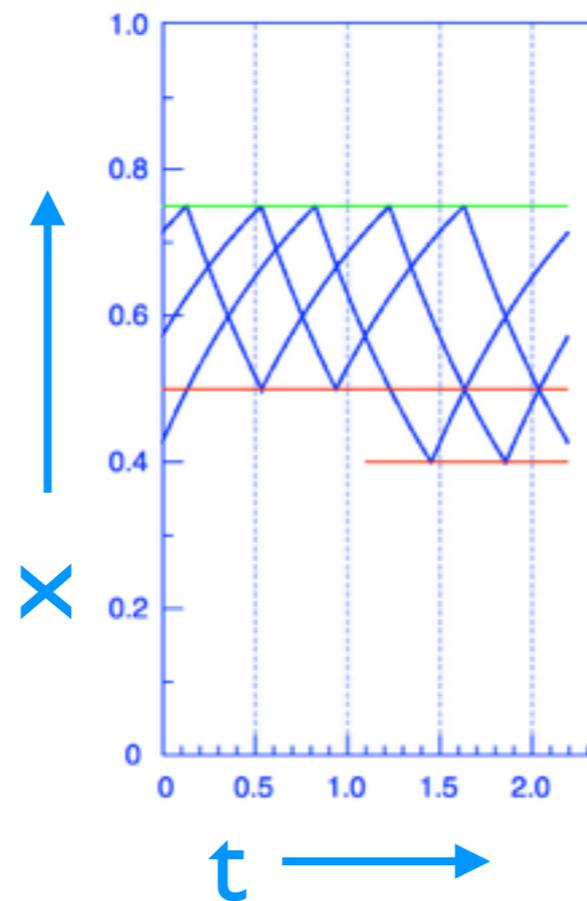
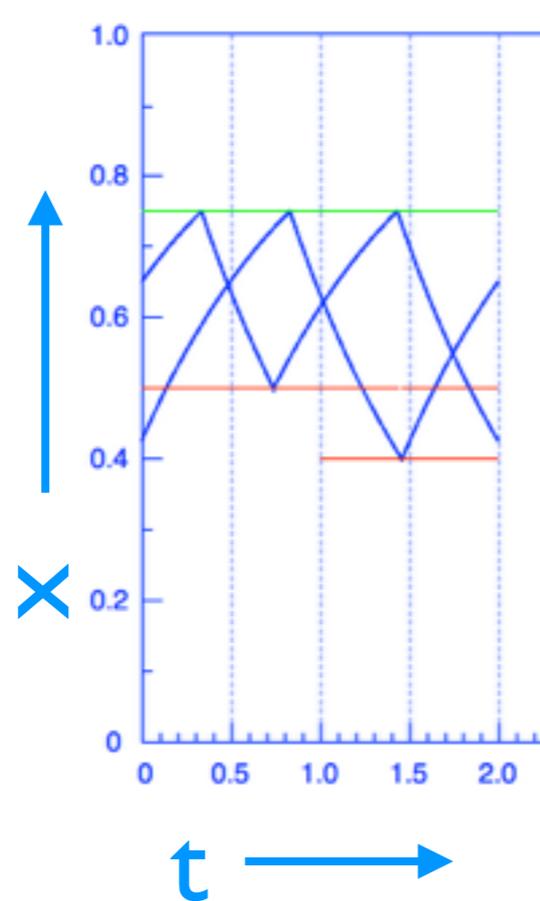
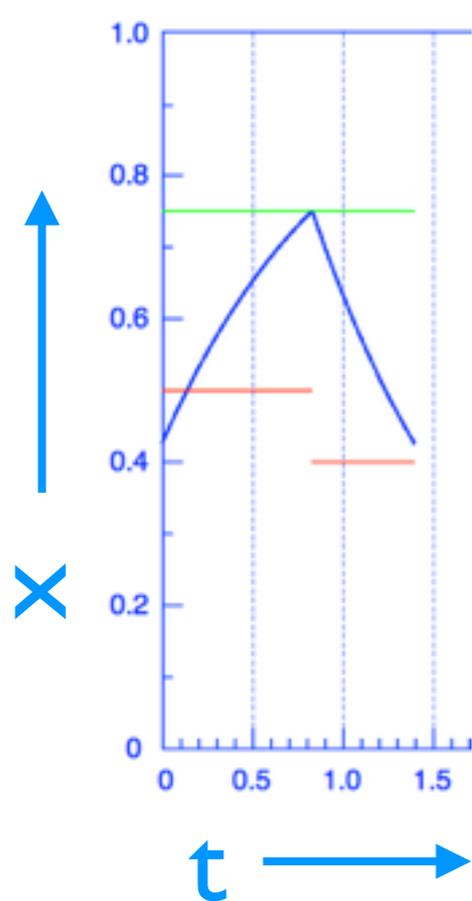


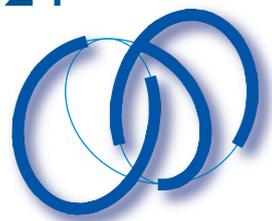
T=1.4

T=2.0

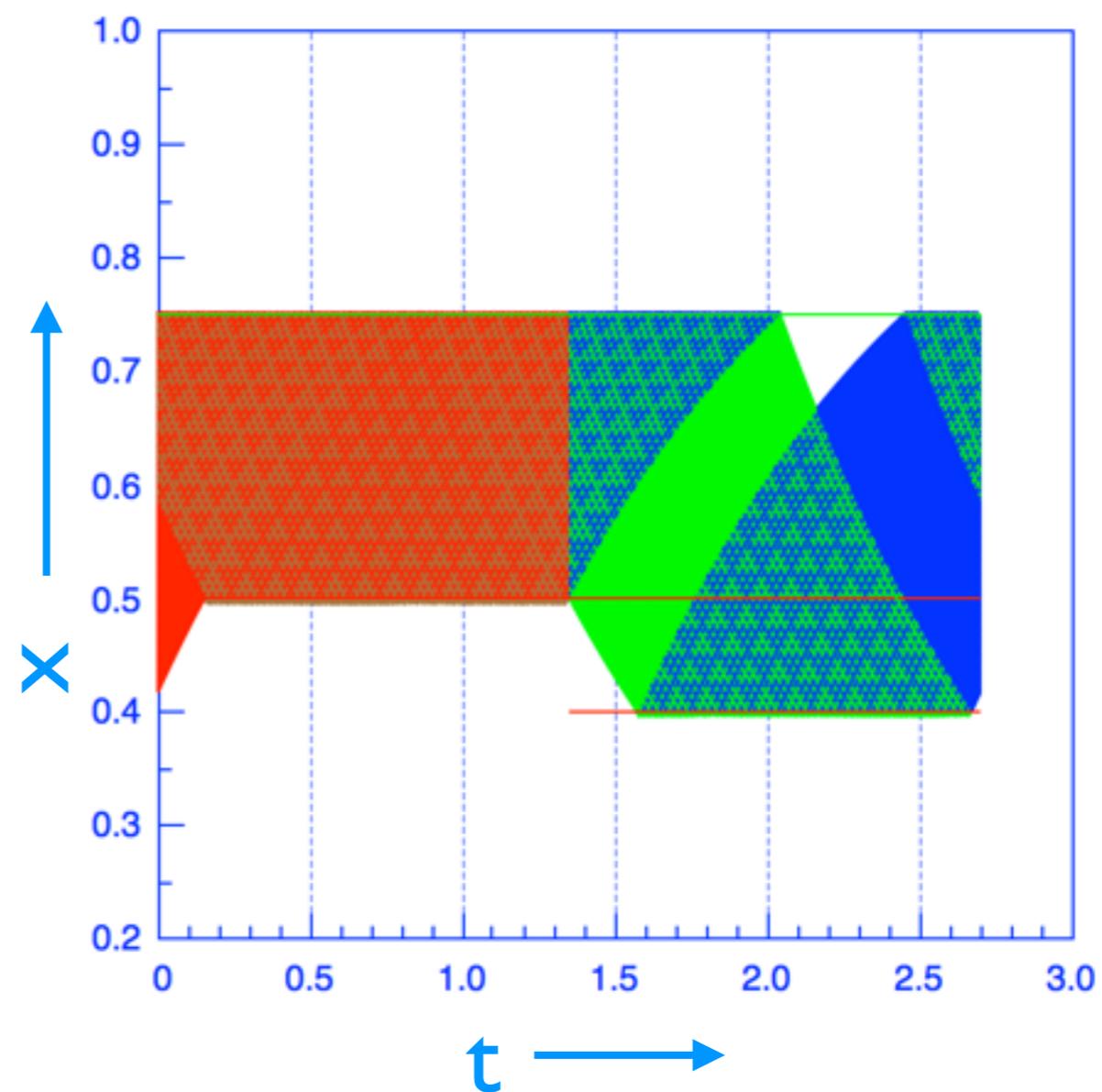
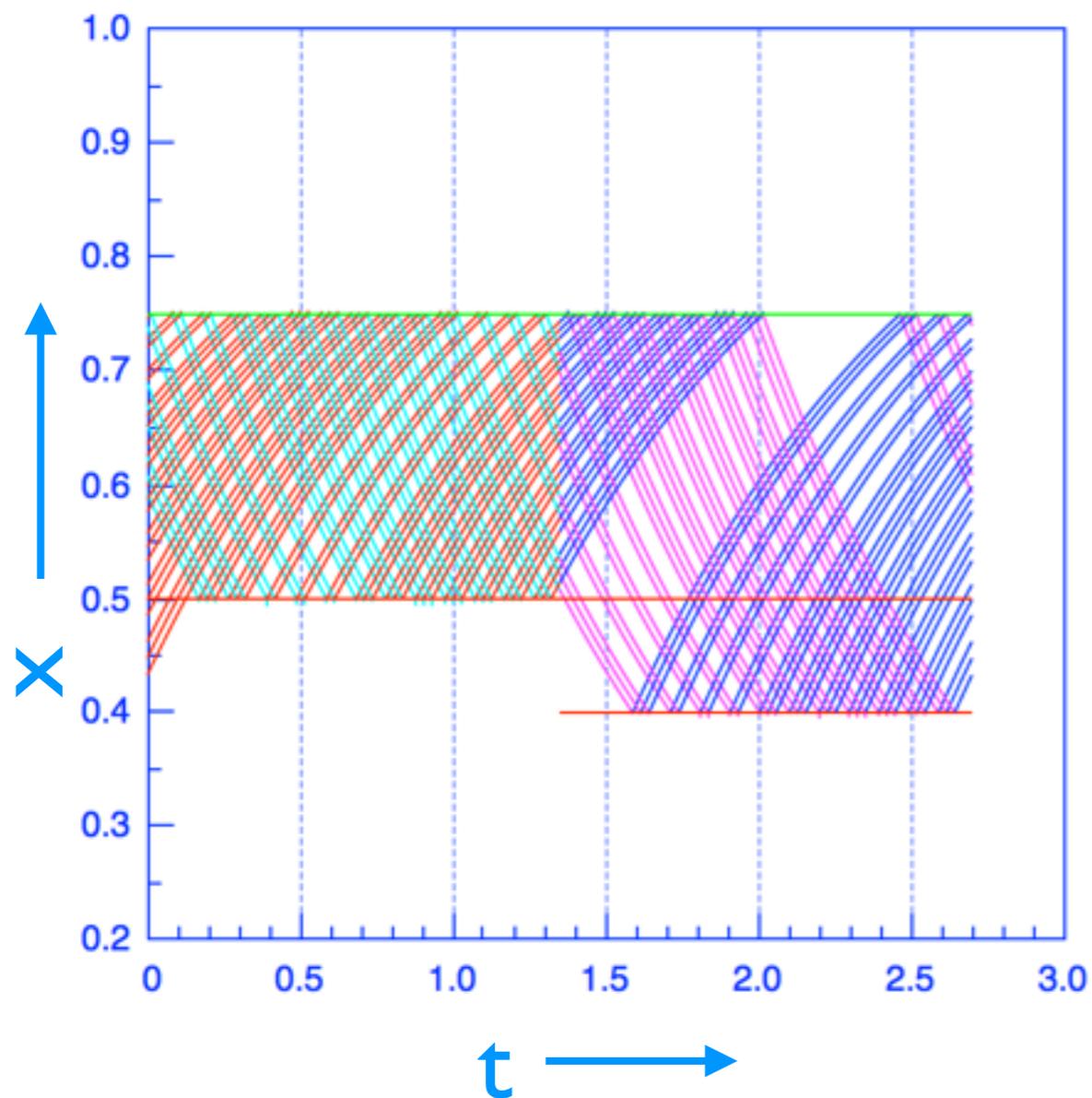
T=2.2

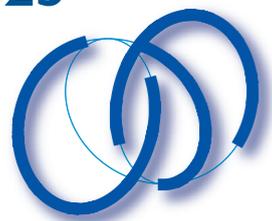
T=2.5



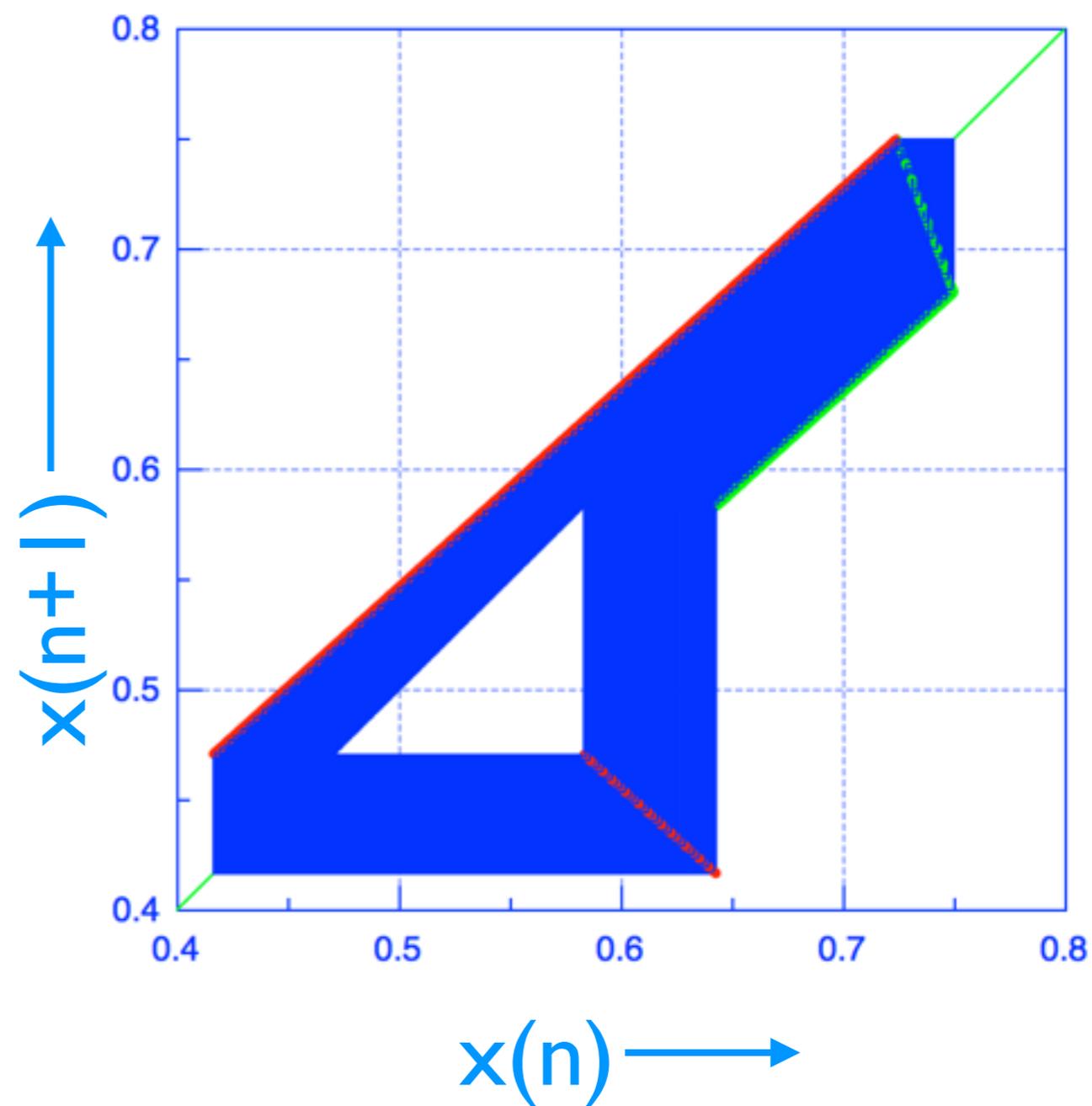
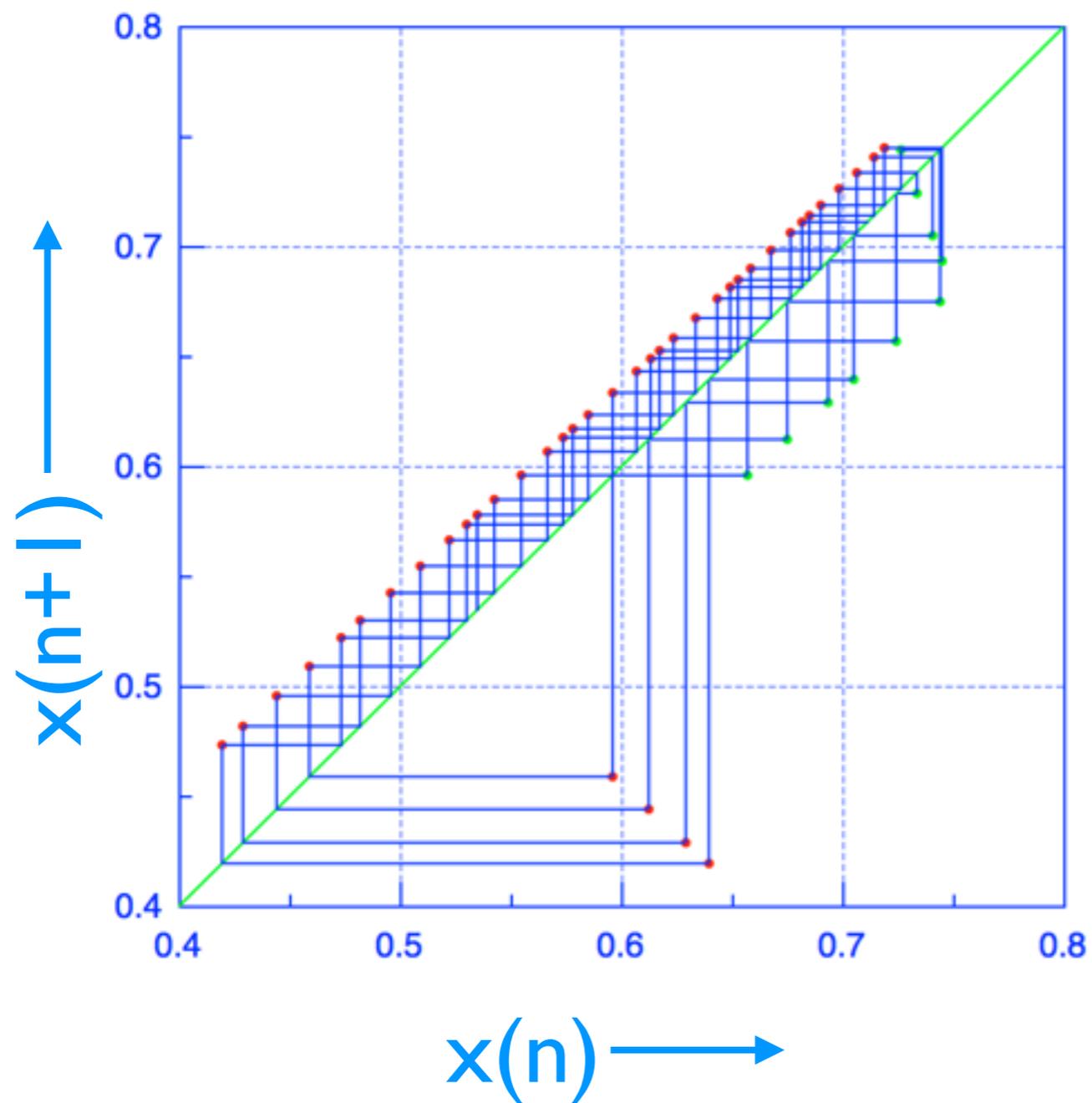


waveform for $T=2.7$



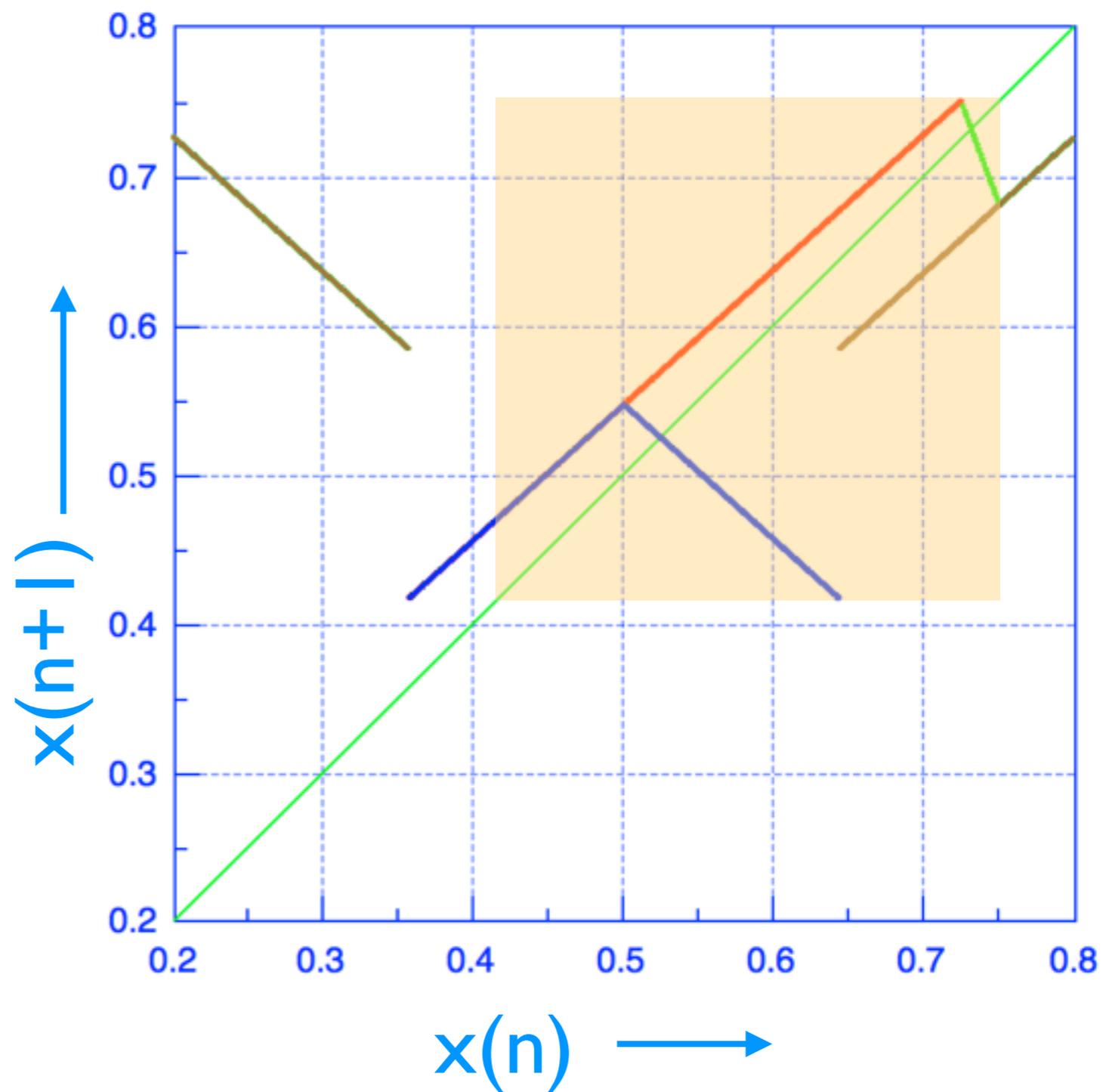
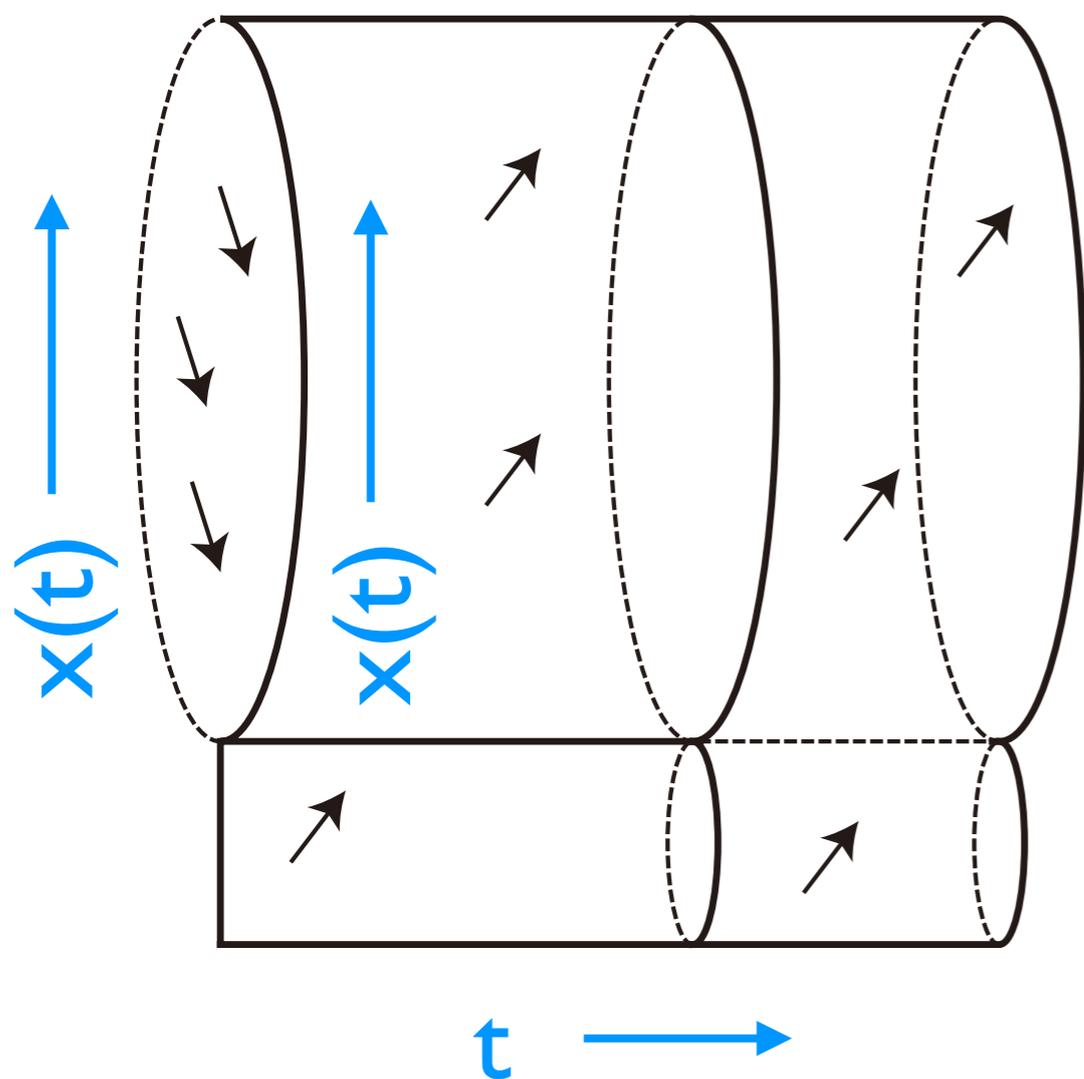


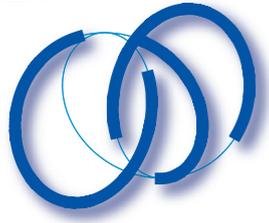
mapping trajectories for $T=2.7$





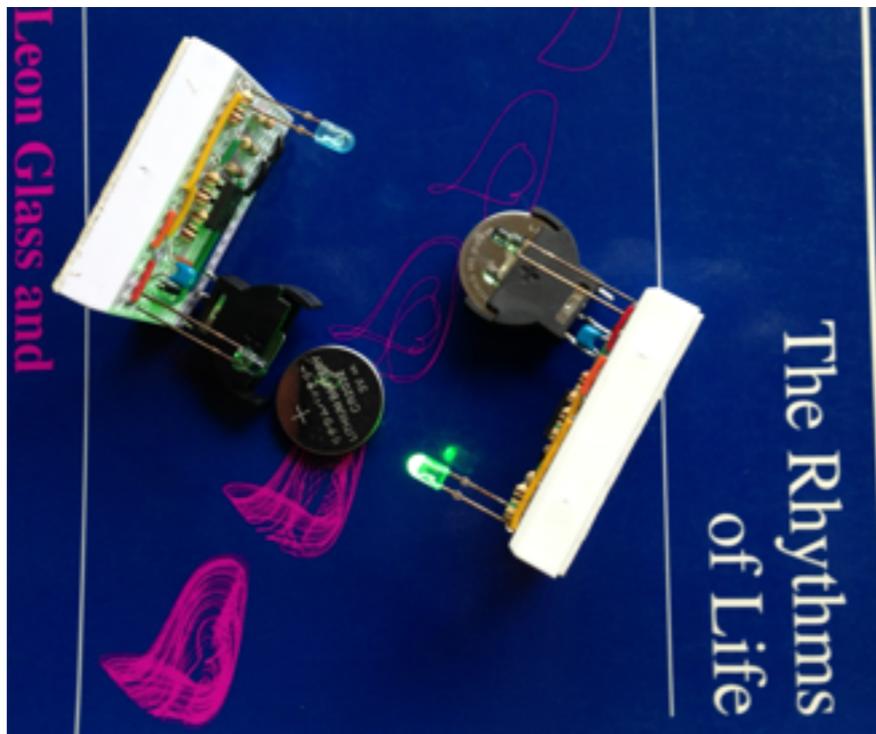
Poincaré map on torus-like surface





3. 同期現象：相互同期

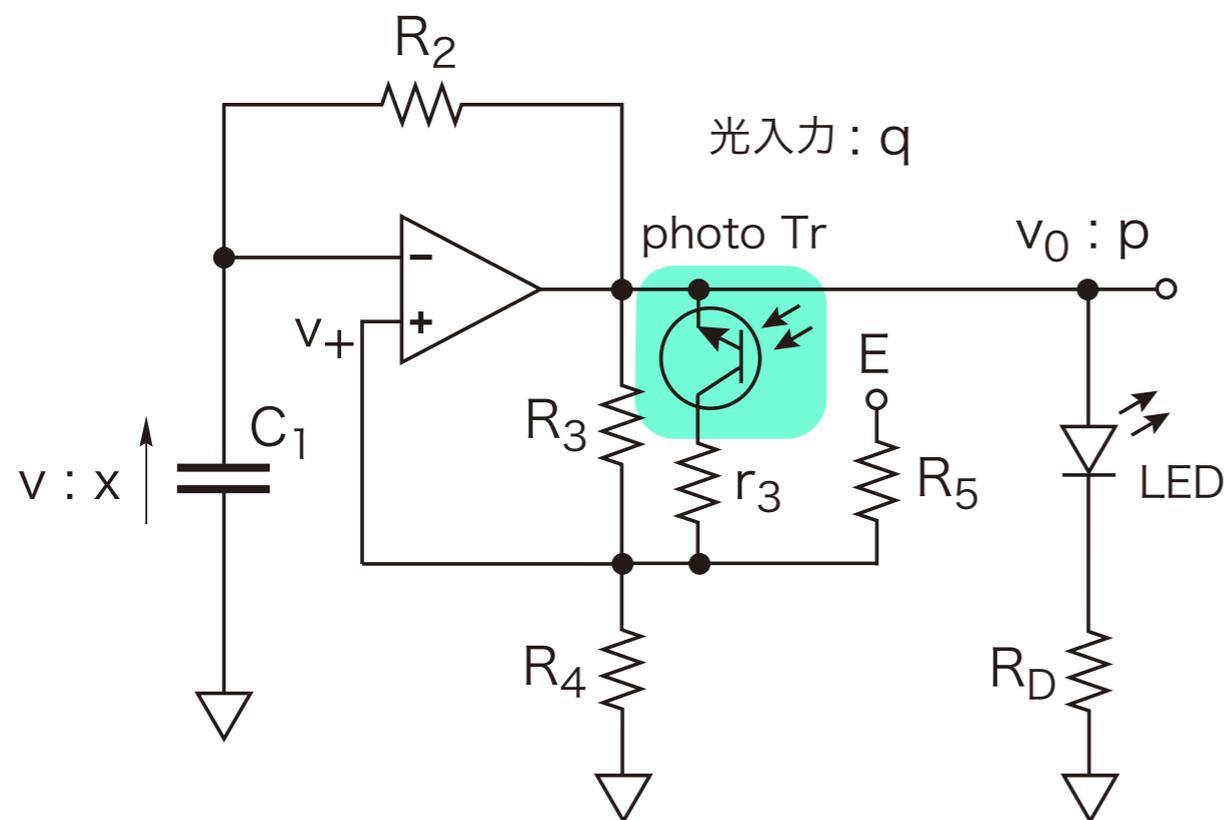
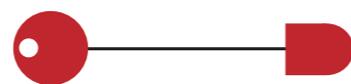
◎ LEDホタルを相互に結合する





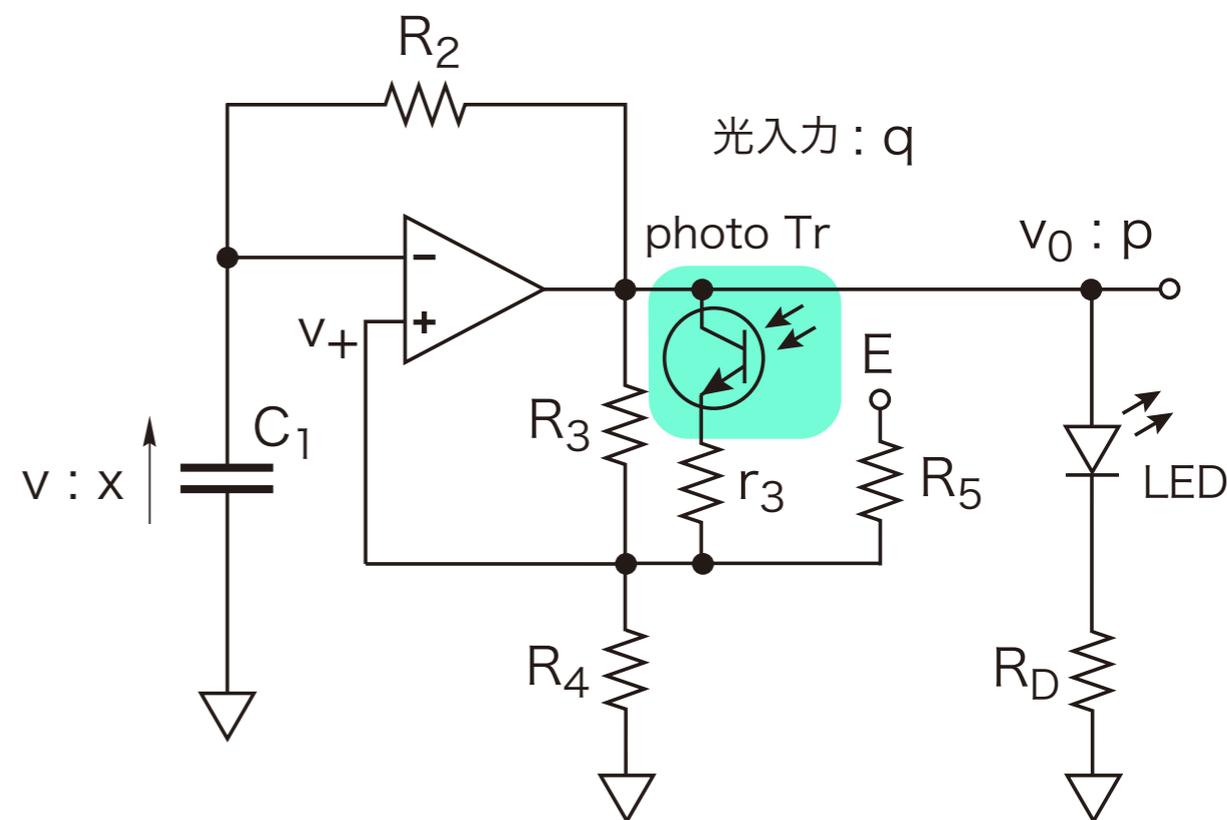
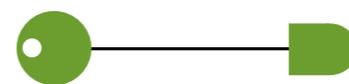
2つのLEDホタル : A1, B1

typeA1

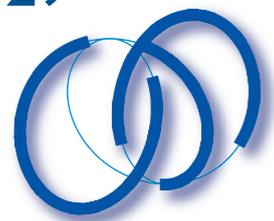


| photo Tr | | comparator : p | |
|----------|-------|----------------|-------|
| | | on:1 | off:0 |
| 光入力 : q | on:1 | off:0 | on:1 |
| | off:0 | off:0 | off:0 |

typeB1



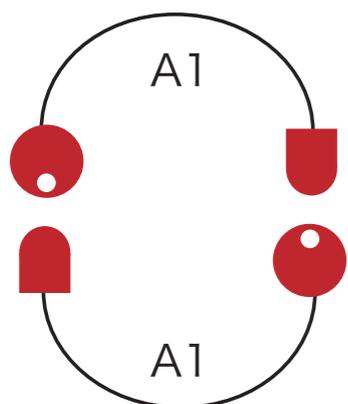
| photo Tr | | comparator : p | |
|----------|-------|----------------|-------|
| | | on:1 | off:0 |
| 光入力 : q | on:1 | on:1 | off:0 |
| | off:0 | off:0 | off:0 |



LEDホタルの結合

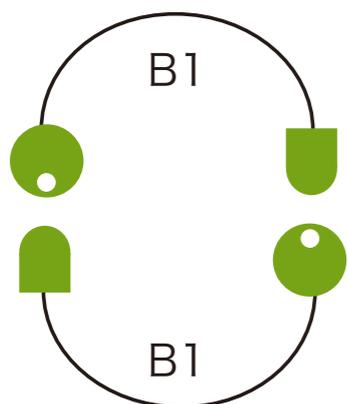
$$\frac{dx}{dt} + x = q, \quad \frac{dy}{dt} + y = p$$

x-axis



y-axis

x-axis



y-axis

mode0 : $(q, p) = (0, 0)$, $dx/dt+x = 0$, $dy/dt+y=0$

mode1 : $(q, p) = (1, 0)$, $dx/dt+x = 1$, $dy/dt+y=0$, $\backslash\beta_{yon}$

mode2 : $(q, p) = (0, 1)$, $dx/dt+x = 0$, $dy/dt+y=1$, $\backslash\beta_{xon}$

mode3 : $(q, p) = (1, 1)$, $dx/dt+x = 1$, $dy/dt+y=1$

mode0 : $(q, p) = (0, 0)$, $dx/dt+x = 0$, $dy/dt+y=0$

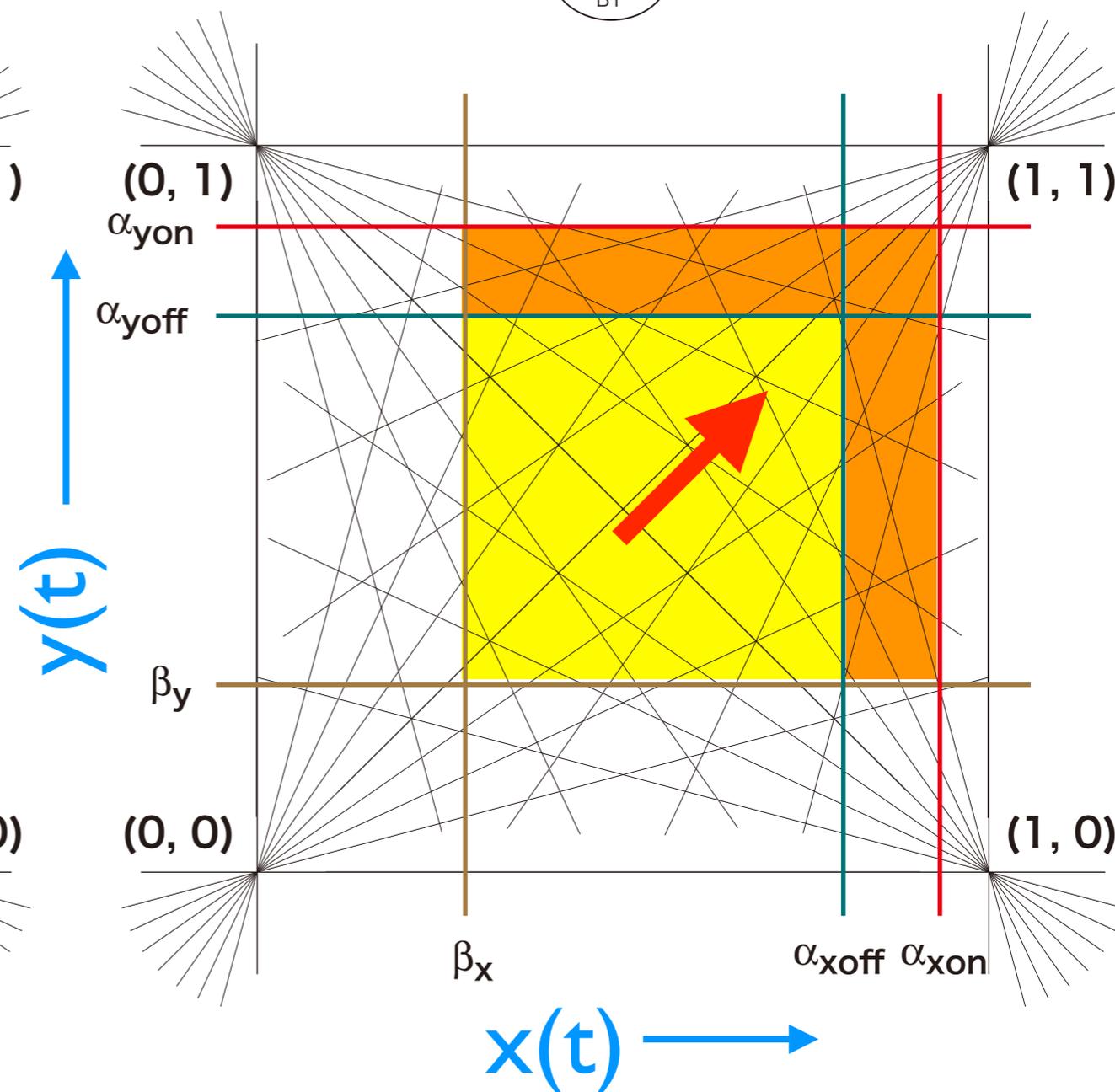
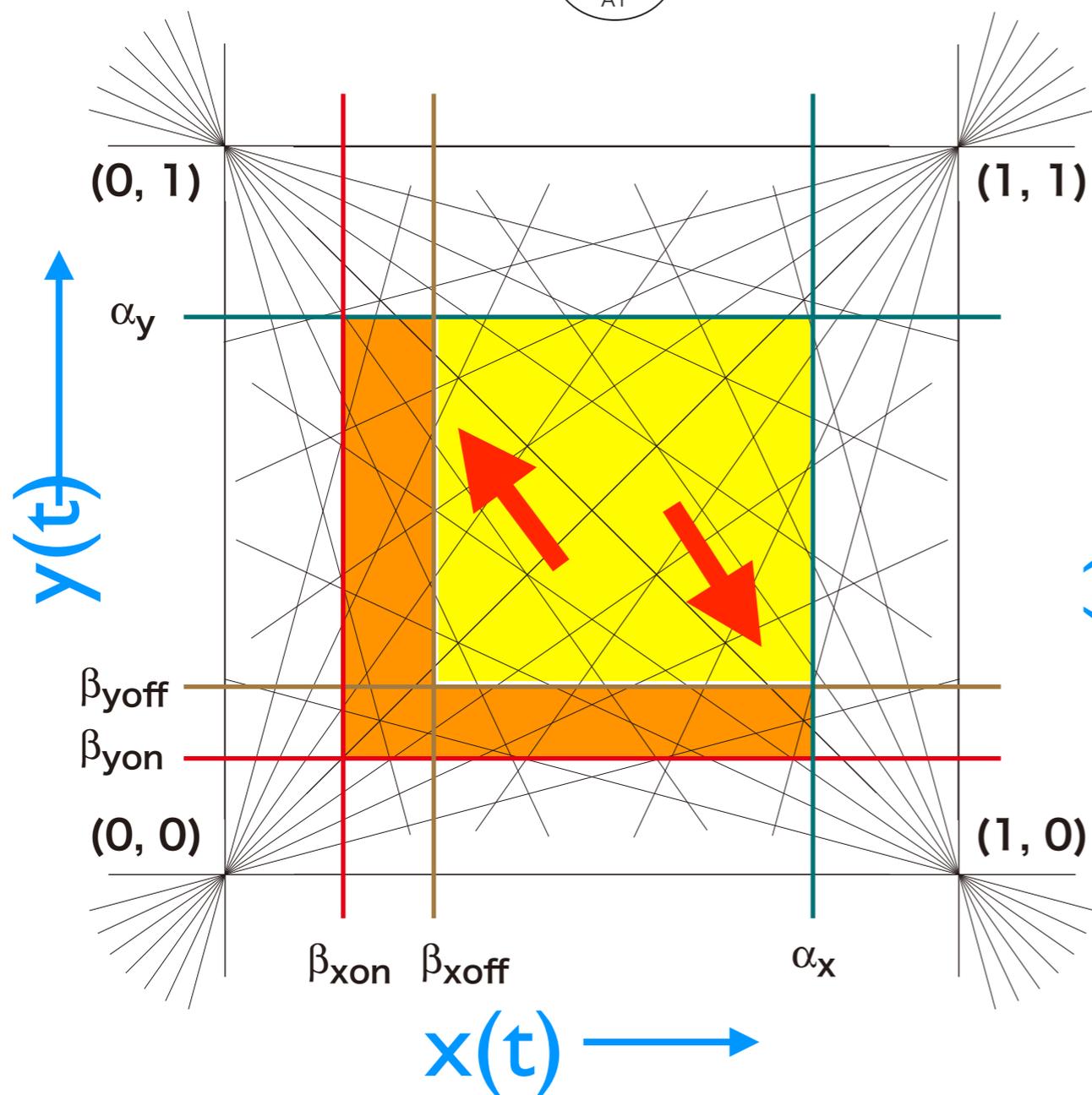
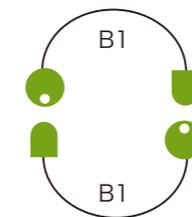
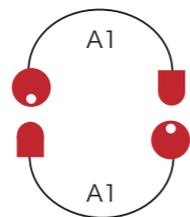
mode1 : $(q, p) = (1, 0)$, $dx/dt+x = 1$, $dy/dt+y=0$

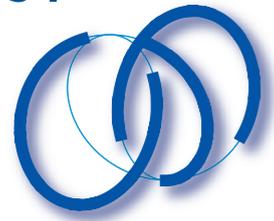
mode2 : $(q, p) = (0, 1)$, $dx/dt+x = 0$, $dy/dt+y=1$

mode3 : $(q, p) = (1, 1)$, $dx/dt+x = 1$, $dy/dt+y=1$, $\backslash\alpha_x, yon$

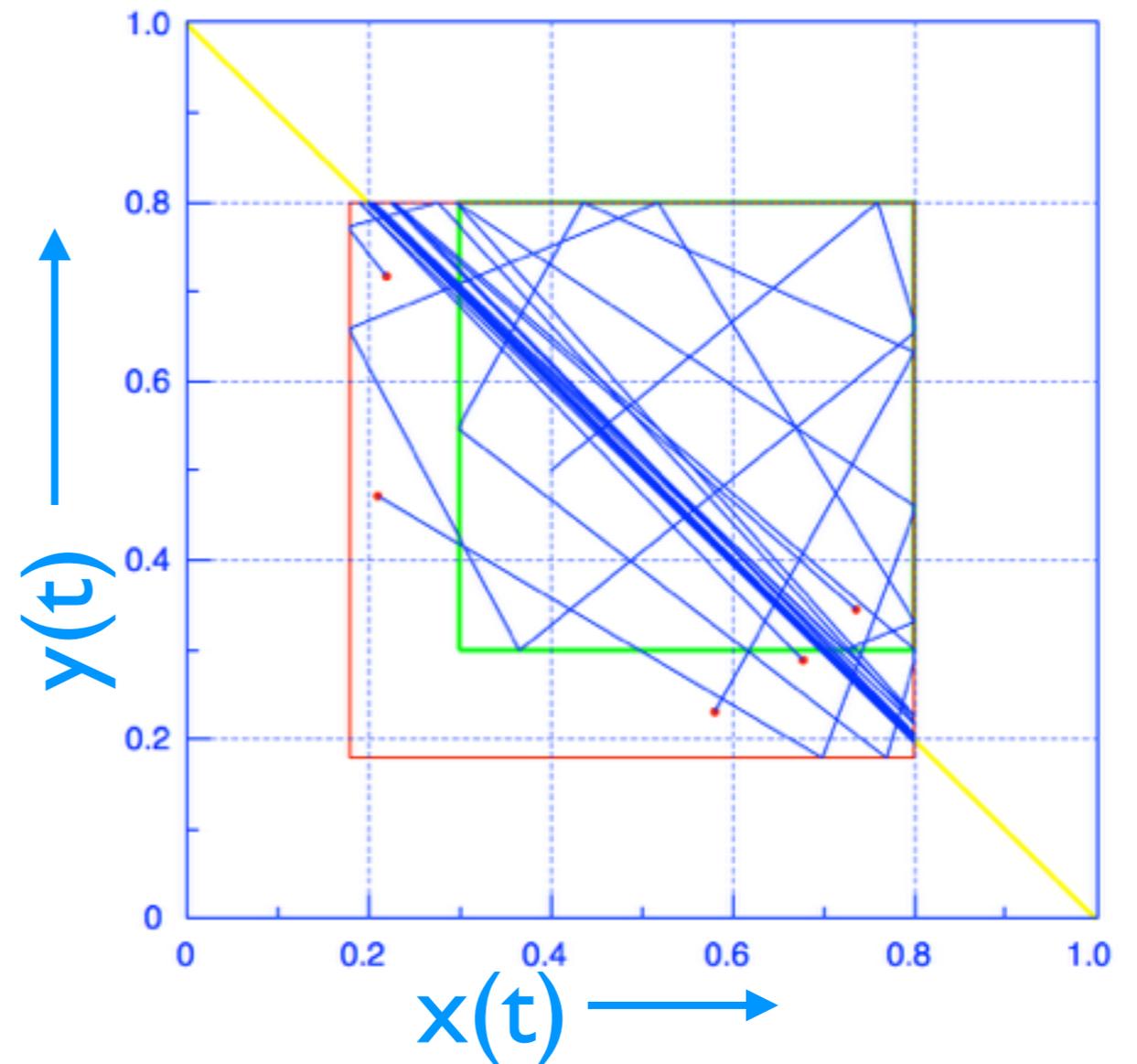
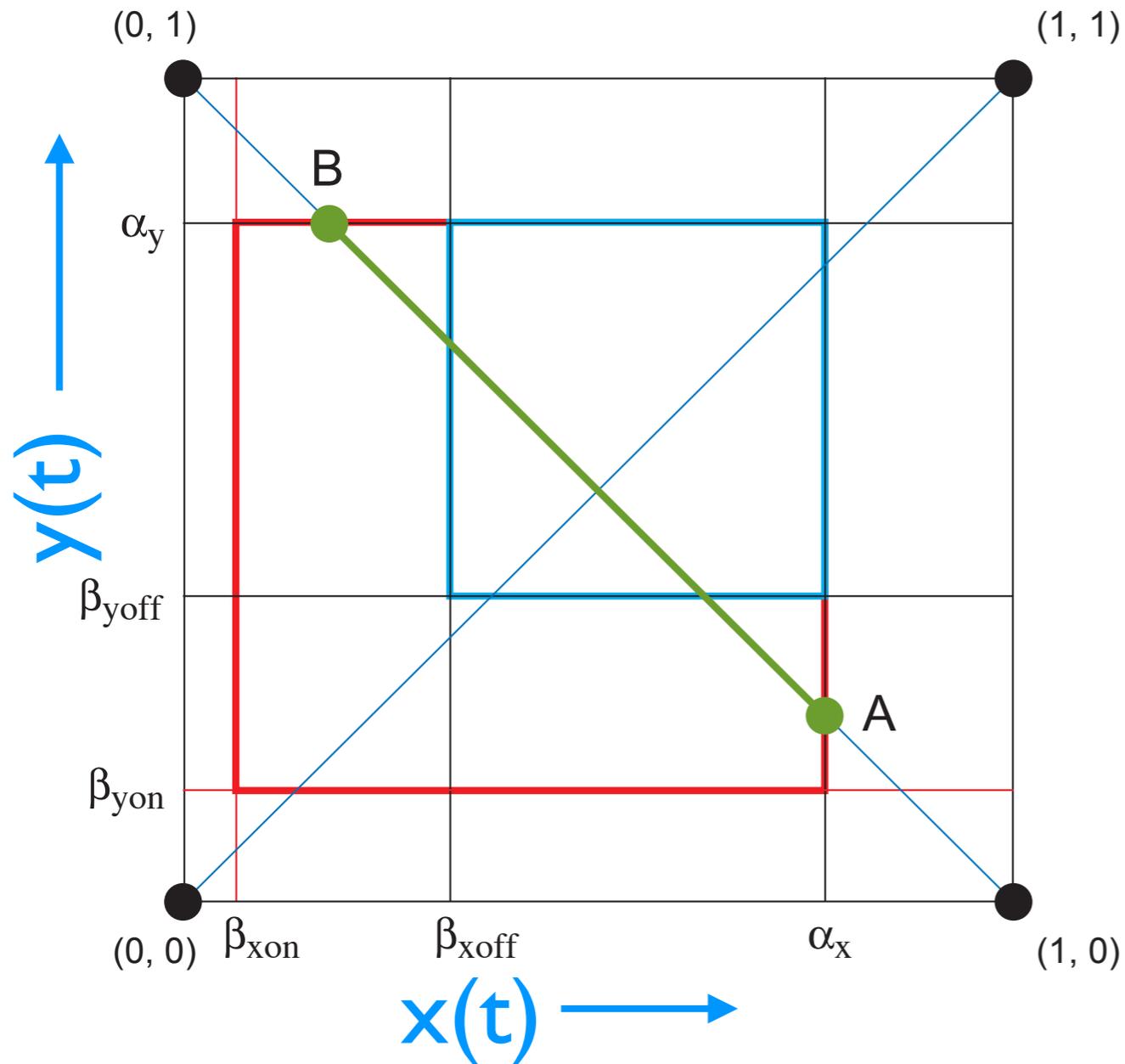


phase portraits

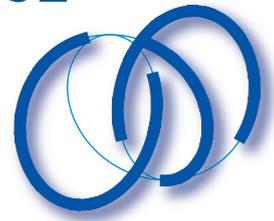




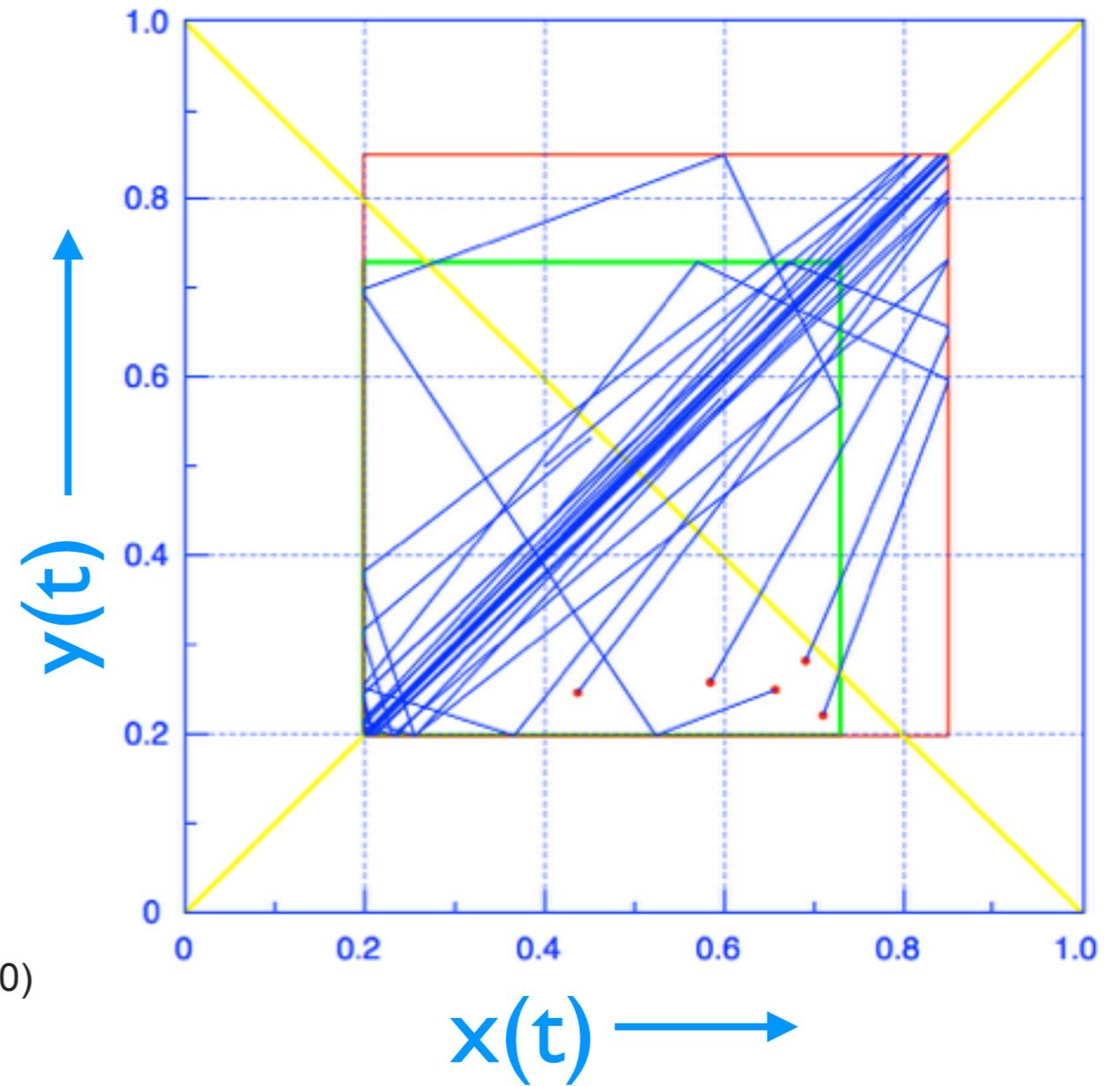
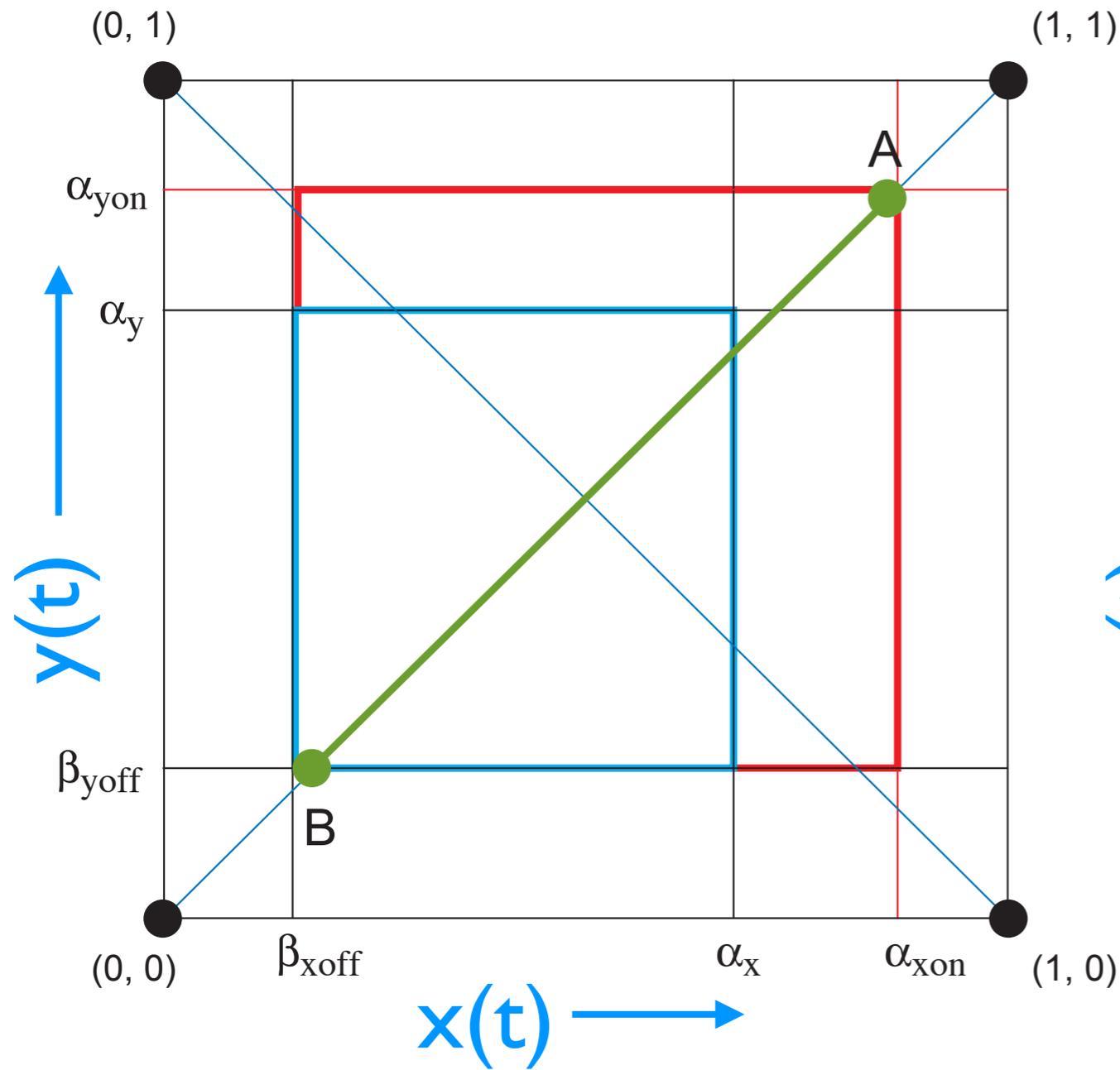
LEDホタルA1の同期：逆相



$$\begin{aligned} \alpha_x &= \alpha_y = 0.8 \\ \beta_x &= \beta_y = 0.3 \\ \beta_{xon} &= \beta_{yon} = 0.18 \end{aligned}$$



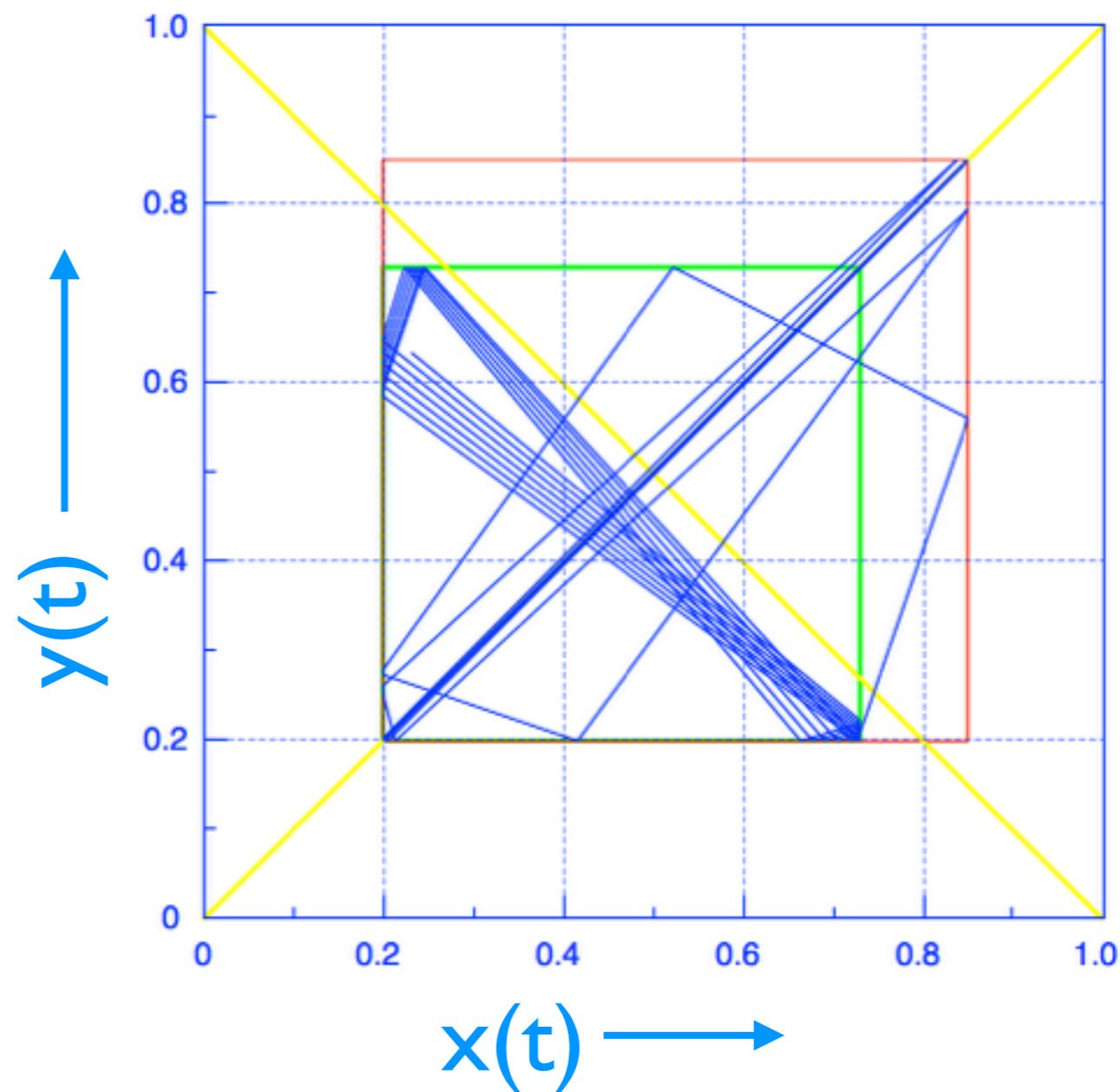
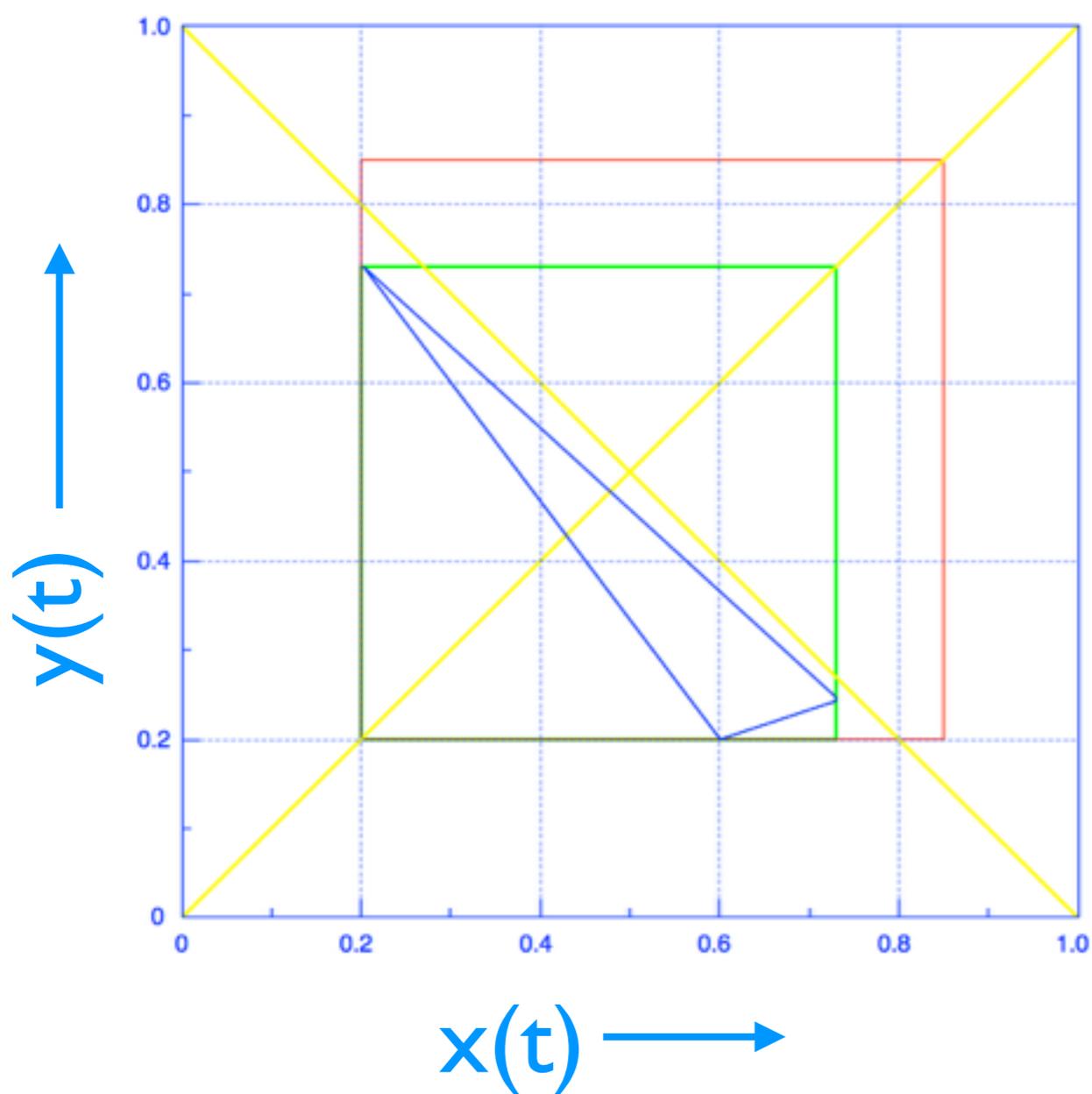
LEDホテルB1の同期：同相



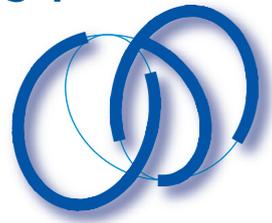
$$\begin{aligned} \alpha_x &= \alpha_y = 0.73 \\ \alpha_{xon} &= \alpha_{yon} = 0.85 \\ \beta_x &= \beta_y = 0.2 \end{aligned}$$



LEDホテルB1の同期：逆相



$$\begin{aligned} \alpha_x &= \alpha_y = 0.73 \\ \alpha_{xon} &= \alpha_{yon} = 0.85 \\ \beta_x &= \beta_y = 0.2 \end{aligned}$$



4. 解析：幾何学的アプローチ

◎ hybrid系：FSM+力学系を解析する



ハイブリッド回路の回路方程式

Hybrid回路の数学モデル：ODE+FSM

| | | 時 間 | |
|----|----|---------------------------|------------------------------------|
| | | 連続 | 離散 |
| 状態 | 連続 | 微分方程式 ODE | 差分方程式 Poincaré map |
| | 離散 | FSM in continuous time | Finite State Machine FSM |

ODE : ordinary differential equations: vector field

FSM : finite state machine(有限状態機械)



Hybrid回路の数学モデル

- ◎ 1つのFSM (Finite State Machine: 有限状態機械)と
このFSMのモード数と同じ数の力学系からなる複合系

mode — FSM の状態 state — 力学系の状態

- ◎ 系の運動 — FSMのeventと力学系のflowで時間発展する

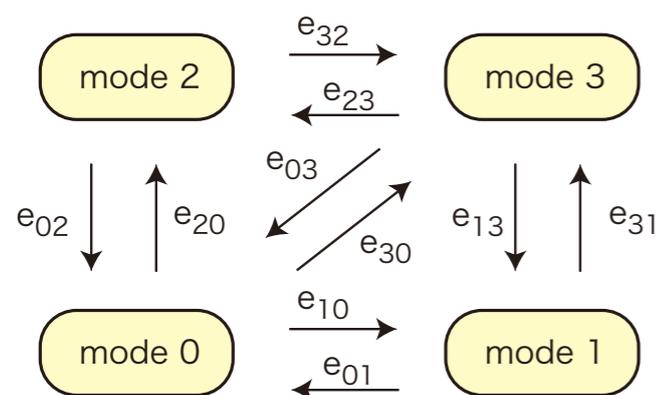
event — phase event, timer event, mixed event

mode遷移図
(graph)

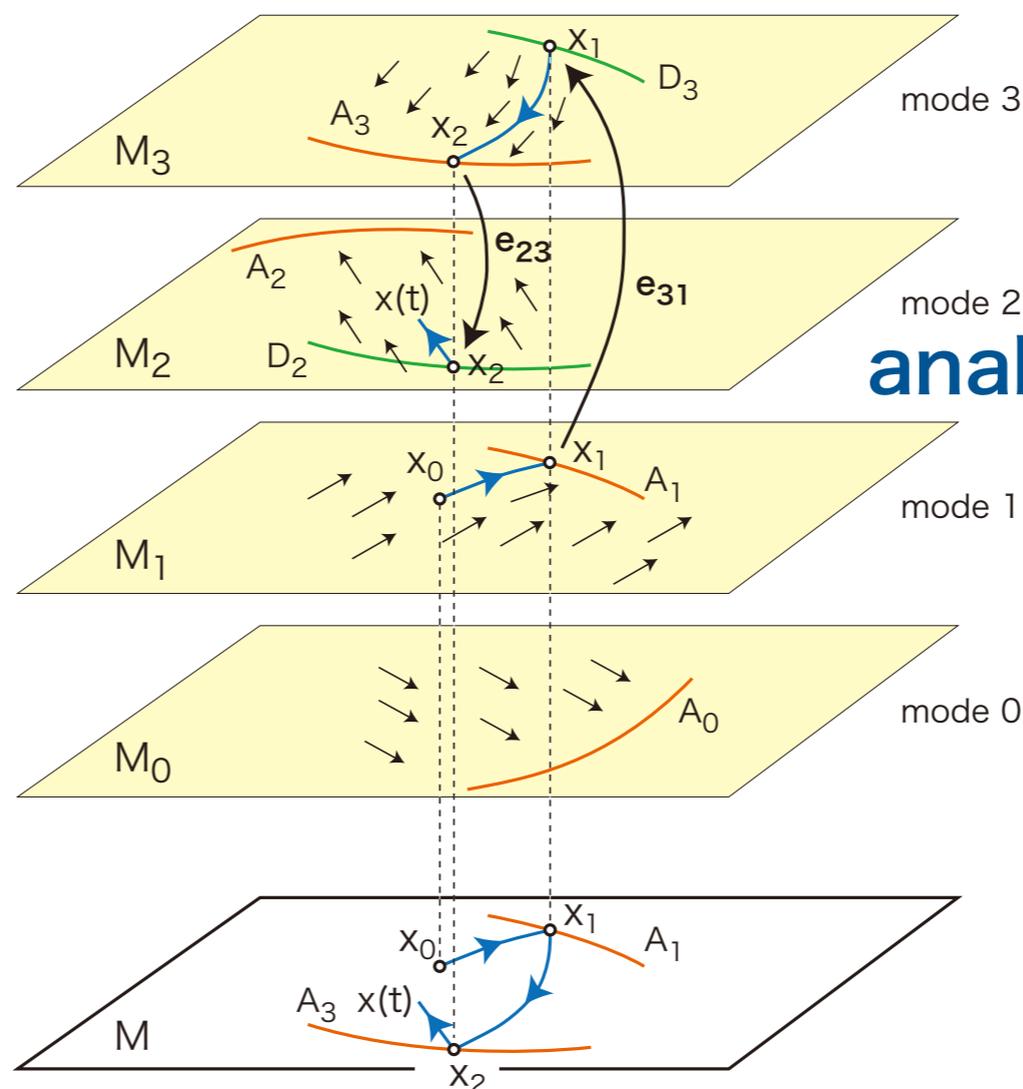
vector場
(ODE)



hybrid系の運動



digital dynamics



analog dynamics

hybrid dynamics



日本科学未来館：メディアラボ第13期展示



日本科学未来館：メディアラボ第13期展示
「1たす1が2じゃない世界 — 数理モデルのすすめ」

2014年2月19日(水)～9月1日(月)

メディアラボ (Miraikan) 第13期展覧会
@ Laboratory for New Media (Miraikan), The 13th Exhibition

$\frac{dx_0}{dt} = w_{00}x_0(t)$ (for $x_0 \leq \dots$)
 $\frac{dx_1}{dt} = w_{10}x_0(t) + w_{11}x_1(t)$
 $\frac{d\phi_b}{dt} = \alpha \sin(\phi_{bp} - \phi_b) + \beta \sin(\phi_{bp} - \phi_b) + \dots$

1たす1が2じゃない世界

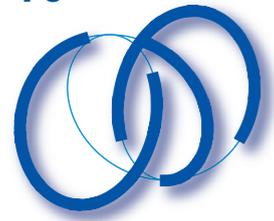
数理モデルのすすめ
The World where "1+1≠2" - An Encouragement of Mathematical Modelling

$x(t+1) = kx(t) - d \frac{1}{1 + \exp(-x(t)/\epsilon)} + a$



2014.2.19(wed.)-**9.1**(mon.)
FIRST 合原最先端数理モデルプロジェクト
FIRST, Aihara Innovative Mathematical Modelling Project

<http://www.miraikan.jst.go.jp/info/1401291716264.html>



ミッション7 《ホタルとシンクロせよ》



一人ひとりがバラバラにふるまっていたはずが、気がつくと同じリズムで行動していた、という経験は誰しもあるでしょう。こうした同期現象は、例えばホタルの集団発光でも見られます。ホタルを模した電子回路で同期を起こす仕組みを解明します。



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