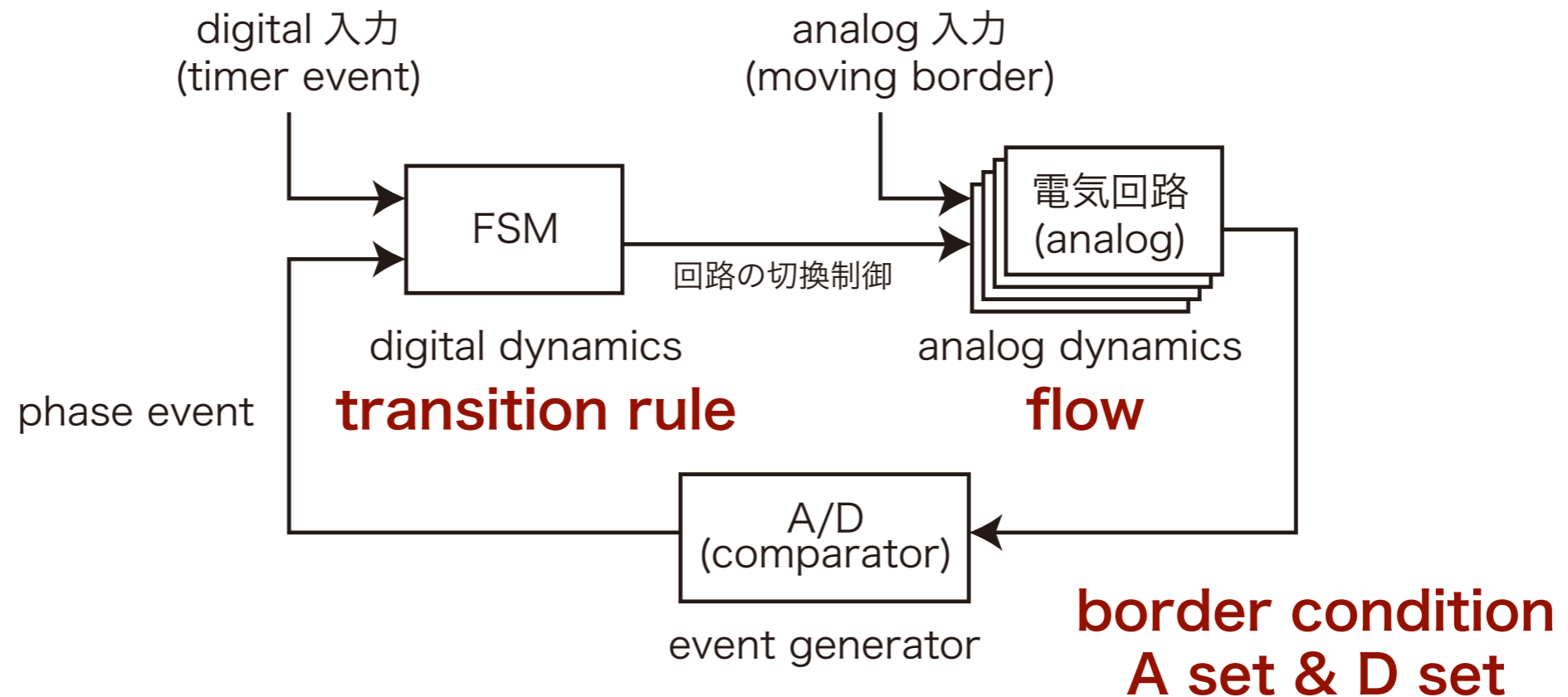


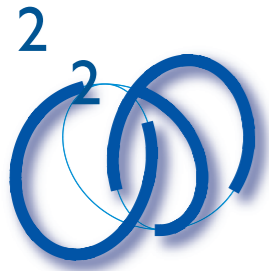


Hybrid 回路の定性論 (2)



川上 博

2014(H26).01.27



話の流れ

1. Hybrid回路の例

- ◎ スイッチ, コンパレータ, 論理素子を含むRLC回路
- ◎ これまでに研究した回路との関連性

2. Hybrid回路の数学モデル

- ◎ FSMとODE混合系の定式化法

3. Hybrid回路の解析法

- ◎ 貼合わされた(FSM)相空間の流れ(ODE)に関する幾何学

4. Hybrid回路の数値計算

- ◎ FSMと力学系を実現するdesign patternをつくる

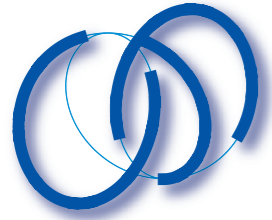


今日の内容

3. Hybrid回路の定性的解析法

1. 数学モデルの定式化
2. 貼合せ多様体をつくる
3. 貼合せ多様体上にPoincaré断面を定義する
4. Poincaré断面上で運動の定性的解析

◎ 貼合わされた(FSM)相空間の流れ(ODE)に関する幾何学



ハイブリッド回路の回路方程式

Hybrid回路の数学モデル：ODE+FSM

		時 間	
		連続	離散
状態	連続	微分方程式 ODE	差分方程式 Poincaré map
	離散	FSM in continuous time (Comparator)	Finite State Machine FSM

ODE : ordinary differential equations: vector field

FSM : finite state machine(有限状態機械)



説明に使うハイブリッド回路

1. 高坂系

1. 基本BC回路, border時変 (方形波, ノコギリ波)
2. 積分器を用いた基本BC回路

2. LEDホタル系

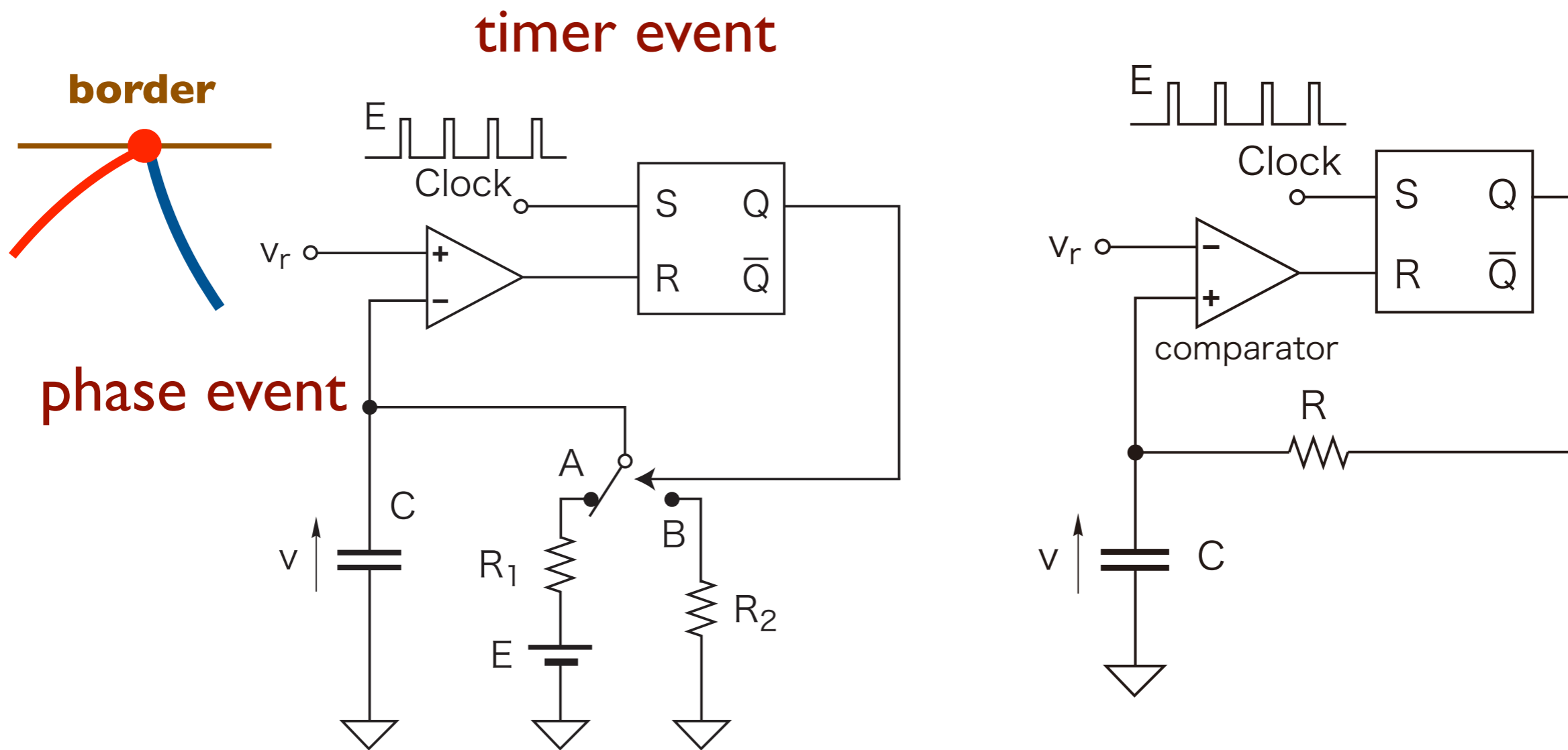
1. 方形波による強制系
2. 2個結合系



説明に使うハイブリッド回路

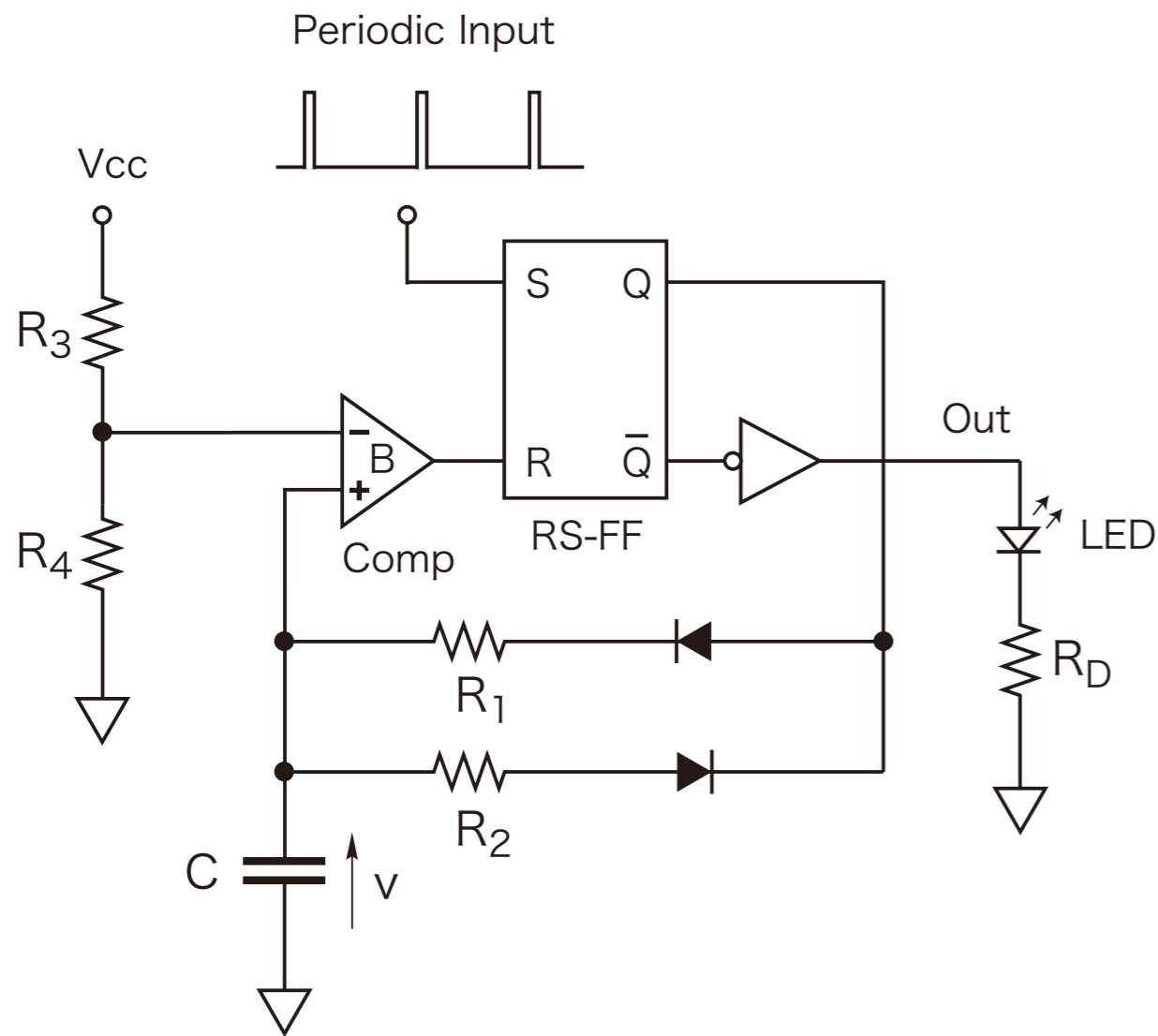


Kousaka's BC Circuit

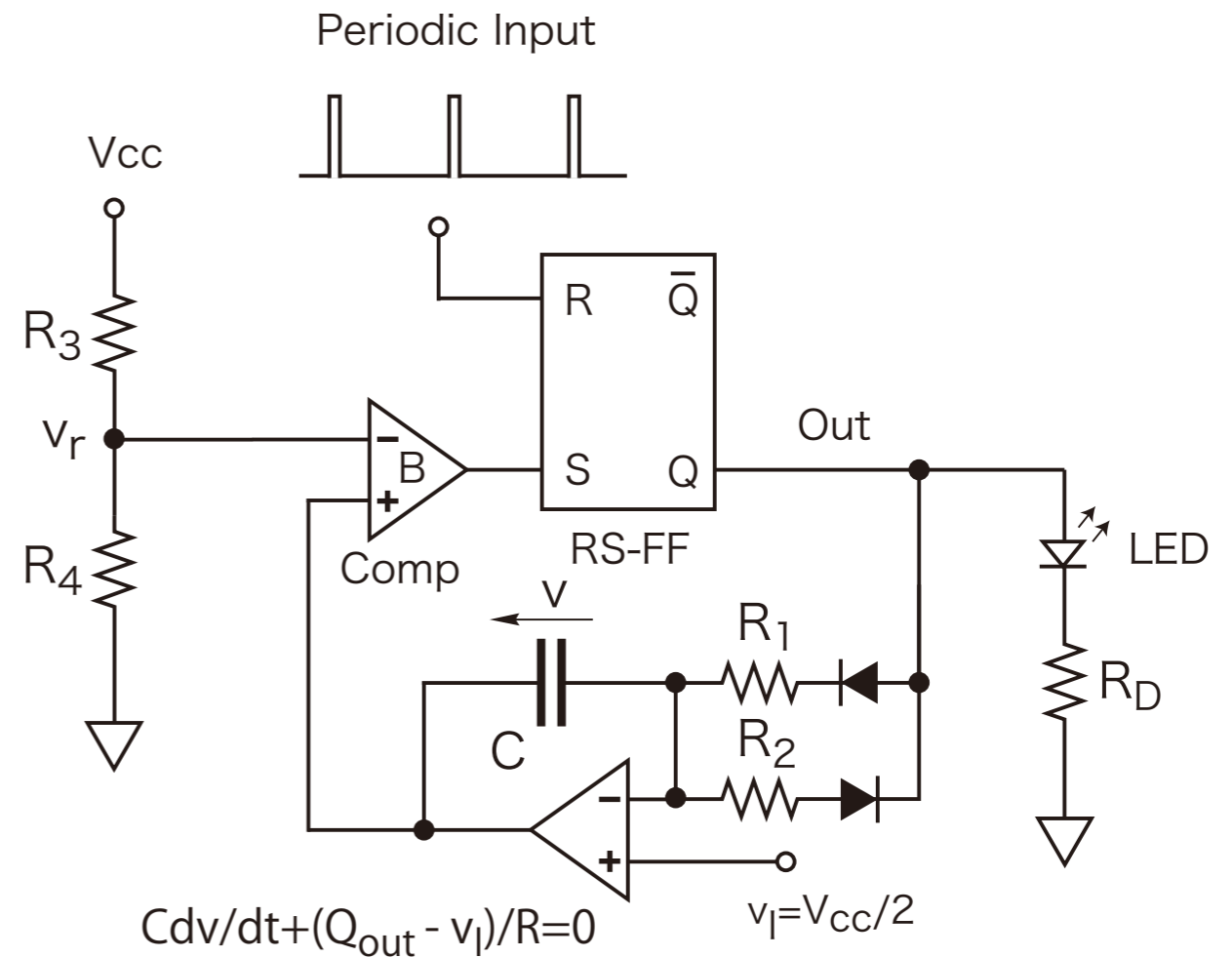




RC circuit vs Integrator



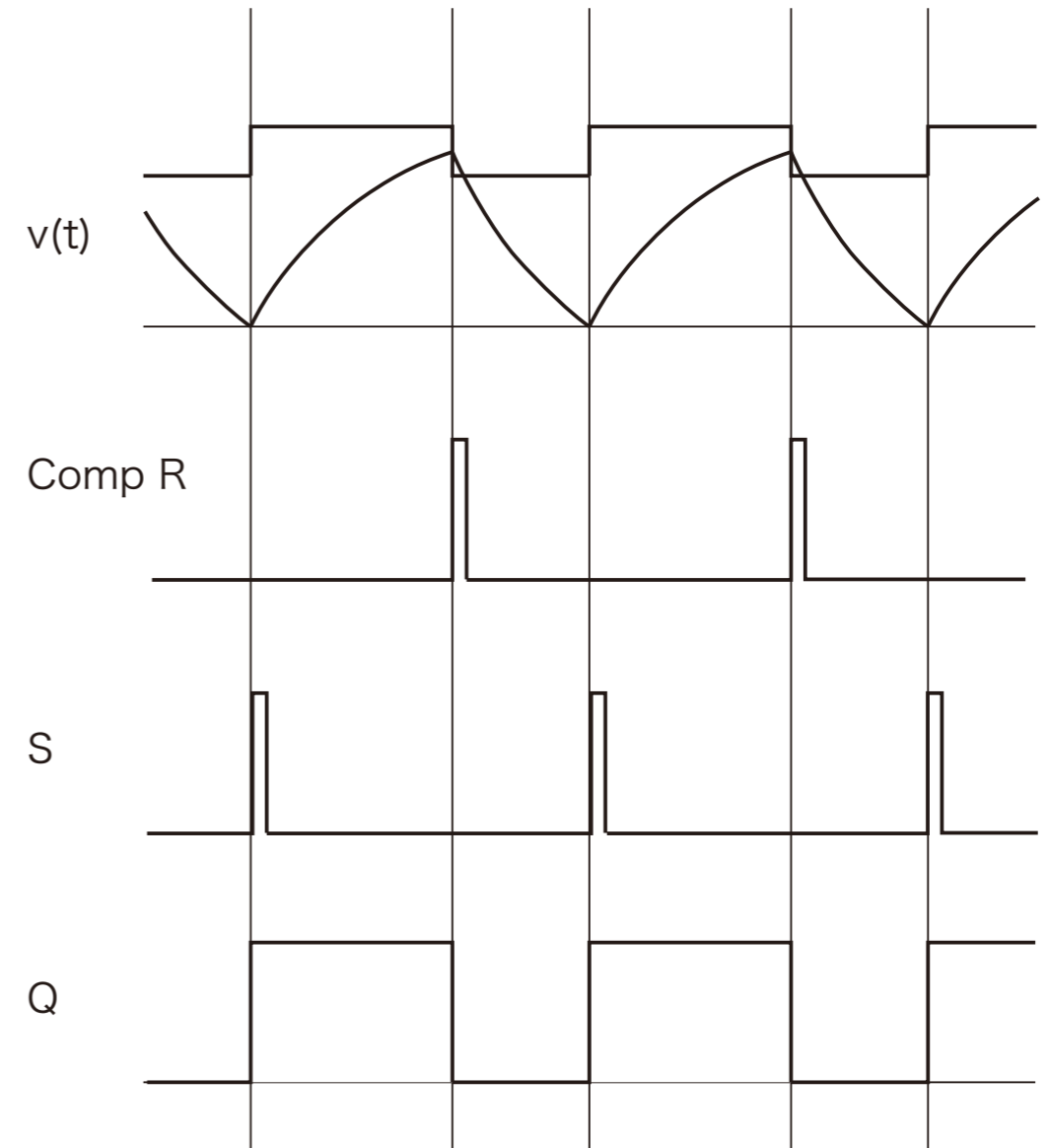
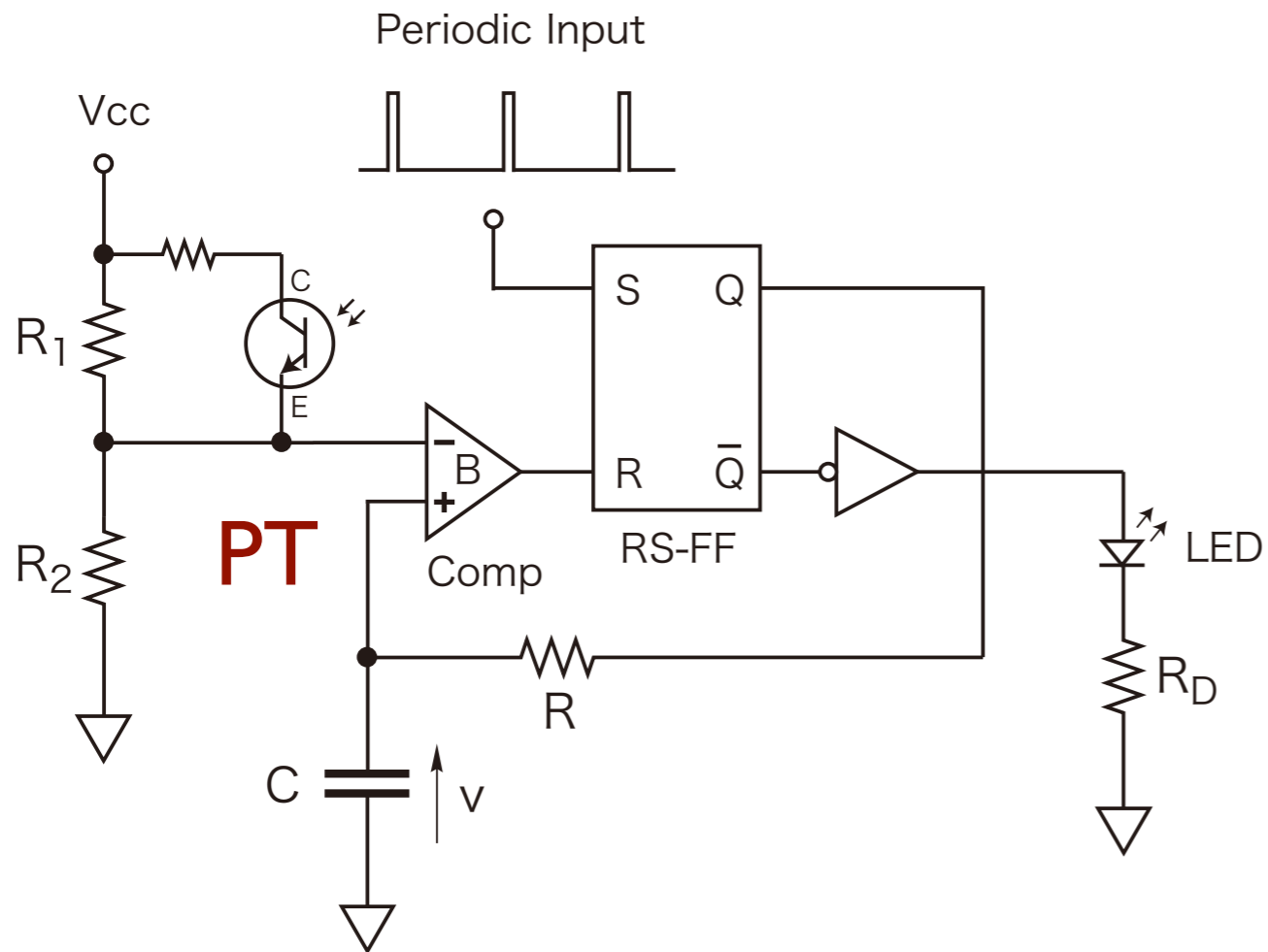
線形RC回路



積分器回路



Forced KBC Circuit

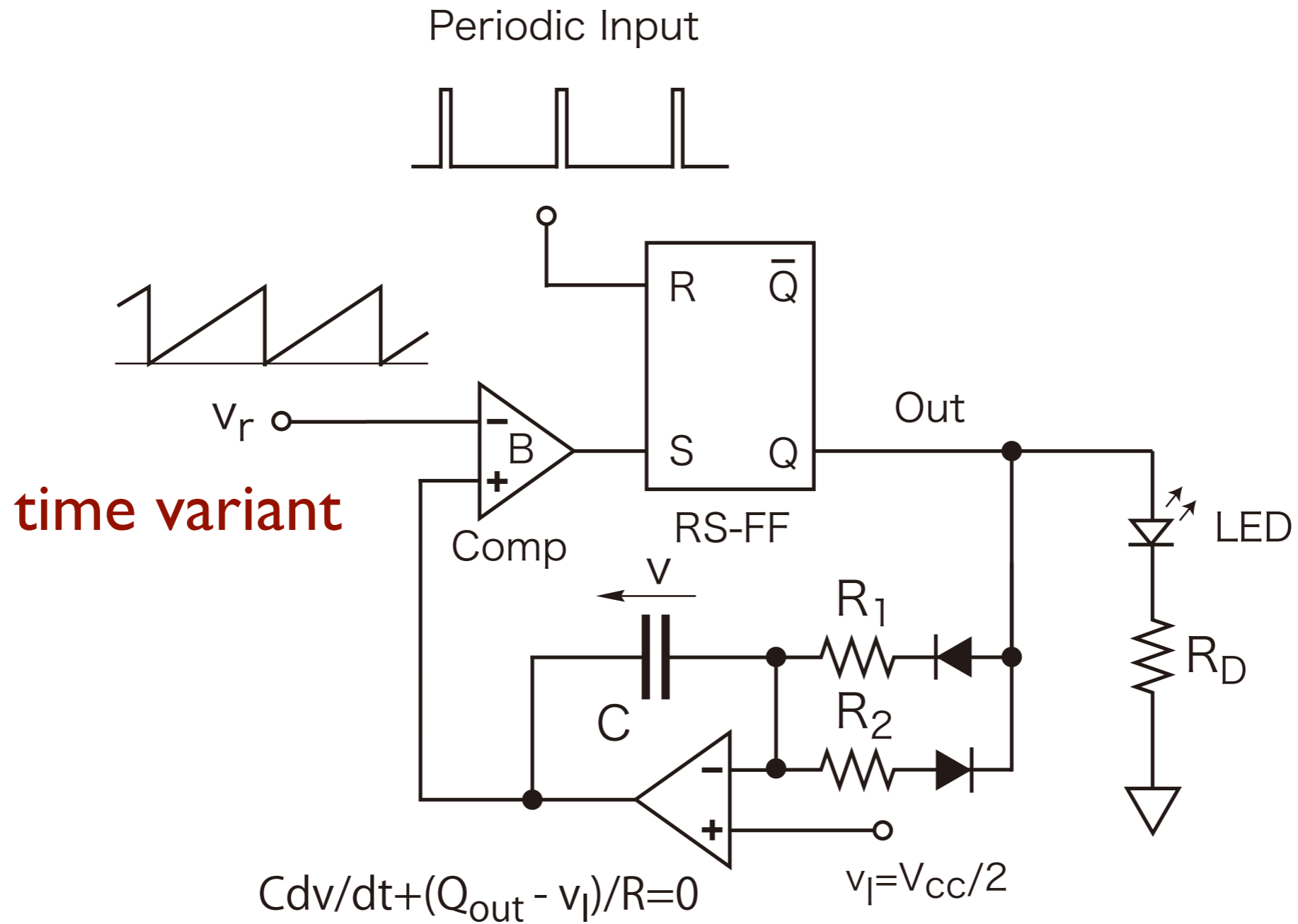


PTの制御法は？

square wave, or LED output

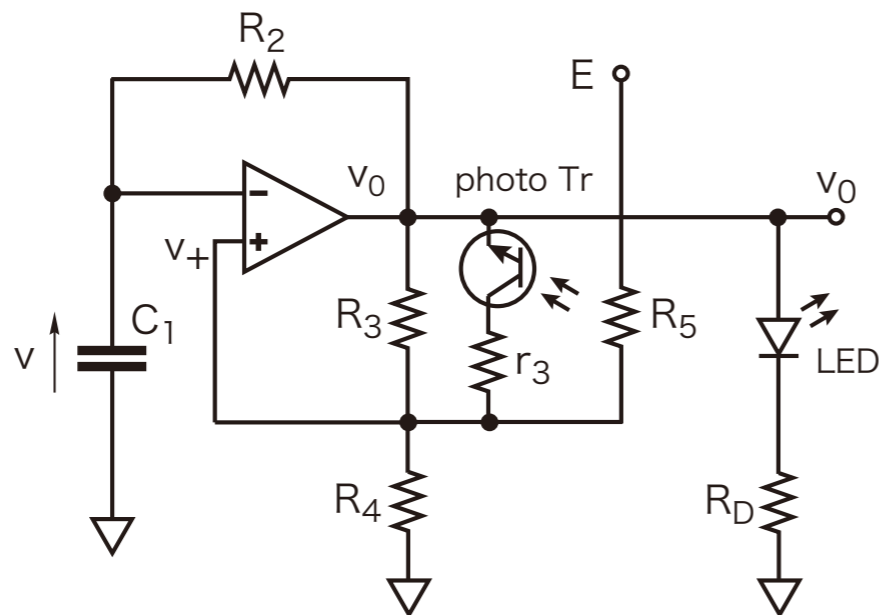


BC Circuit with time variant border

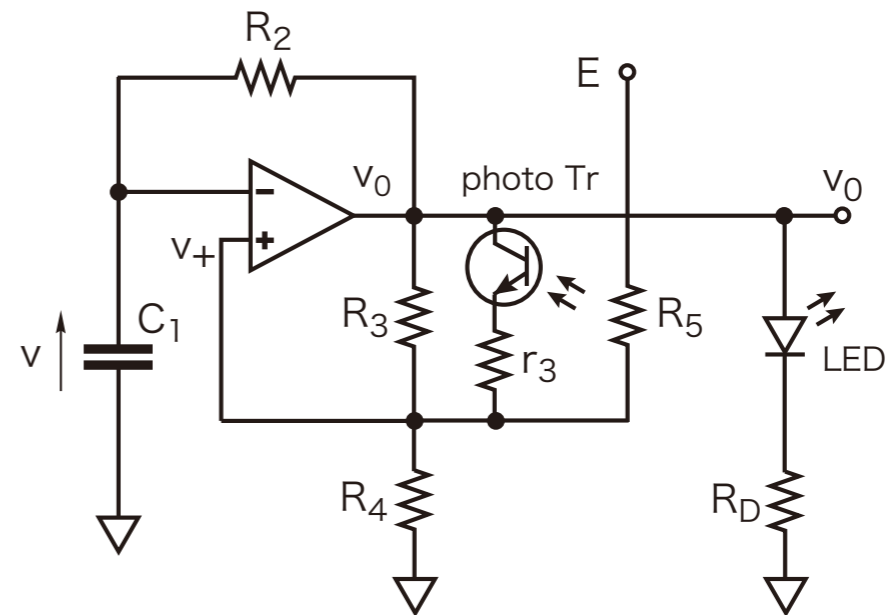




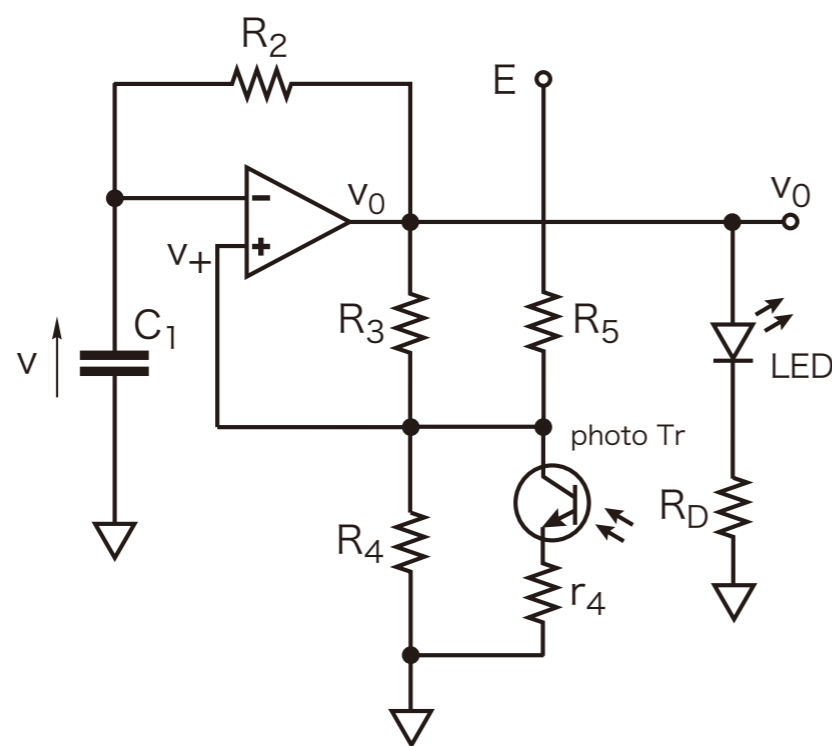
LEDホタルの回路



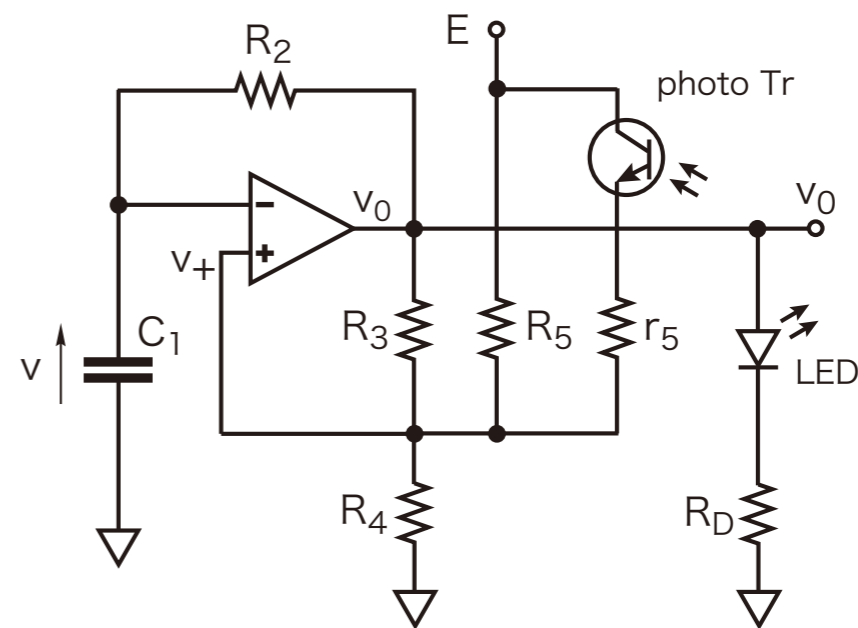
(1) 回路 a1: $v_0=0[v]$ && 光あり : $\beta_{on} < \beta_{off}$



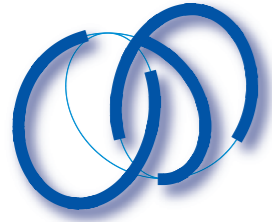
(2) 回路 b1: $v_0=V_{cc}[v]$ && 光あり : $\alpha_{off} < \alpha_{on}$



(3) 回路 c1: 光あり : $\beta_{on} < \beta_{off}, \alpha_{on} < \alpha_{off}$



(4) 回路 d1: 光あり : $\beta_{off} < \beta_{on}, \alpha_{off} < \alpha_{on}$



LEDホタルの回路

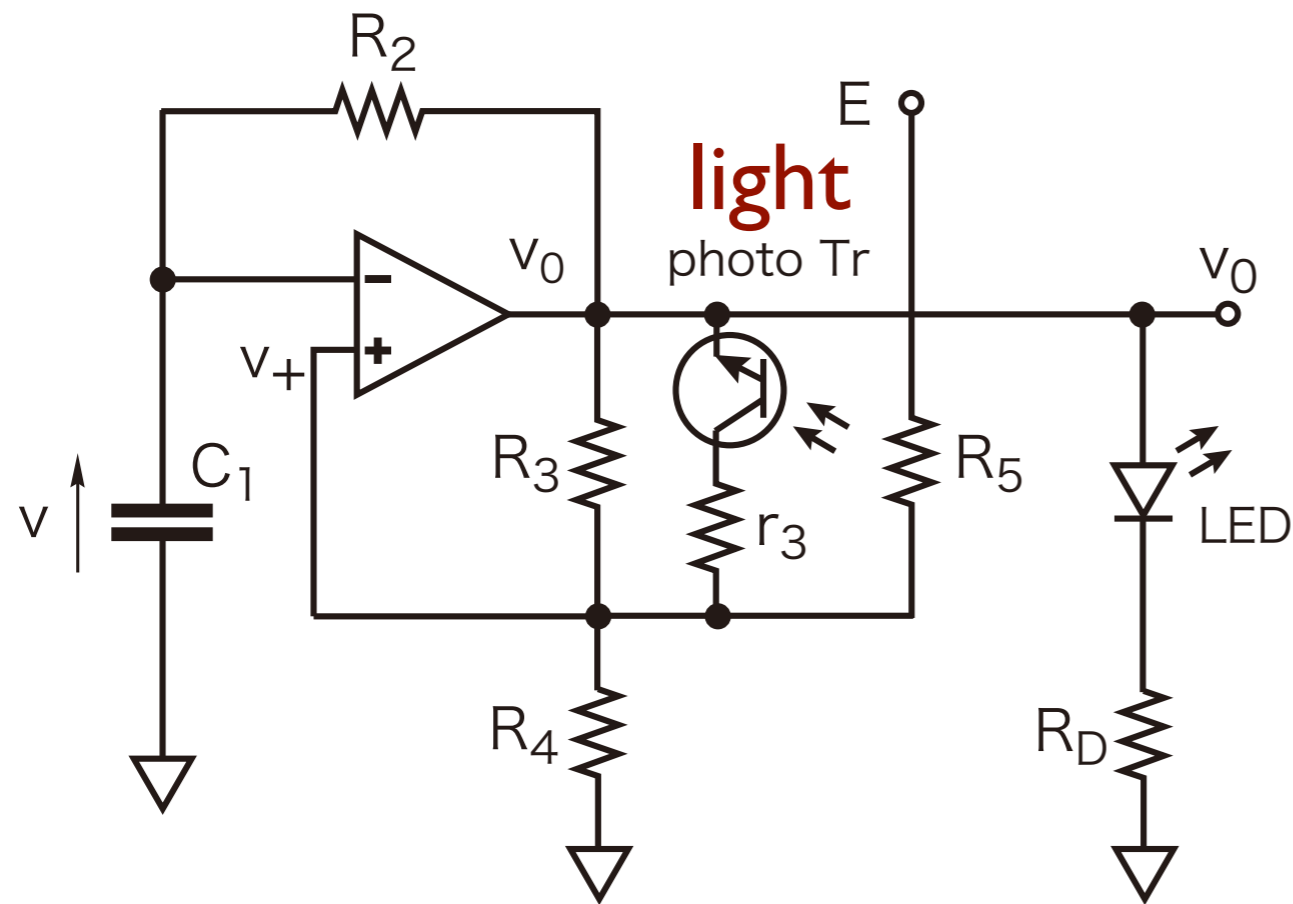


photo Tr	light on	light off
$v_0 = 0$	on	off
$v_0 = E$	off	off

$$C_1 R_2 \frac{dv}{dt} + v = E$$

 α

 β_{on} β_{off}

$$C_1 R_2 \frac{dv}{dt} + v = 0$$



LEDホタルの回路：2個結合

mode

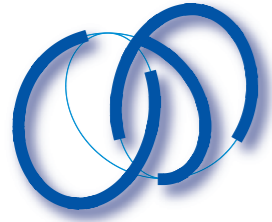
0

1

2

3

Osc	モード	出力電圧	LED	Photo Tr	Parameter	Dynamics
発振器 1	0	0	off	off	\beta _{off}	$v'1 = -v1$
発振器2	0	0	off	off	\beta _{off}	$v'2 = -v2$
発振器 1	1	V _{cc}	on	off	\beta _{off}	$v'1 = -v1 + 1$
発振器2	0	0	off	on	\beta _{on}	$v'2 = -v2$
発振器 1	0	0	off	on	\beta _{on}	$v'1 = -v1$
発振器2	1	V _{cc}	on	off	\beta _{off}	$v'2 = -v2 + 1$
発振器 1	1	V _{cc}	on	off	\beta _{off}	$v'1 = -v1 + 1$
発振器2	1	V _{cc}	on	off	\beta _{off}	$v'2 = -v2 + 1$

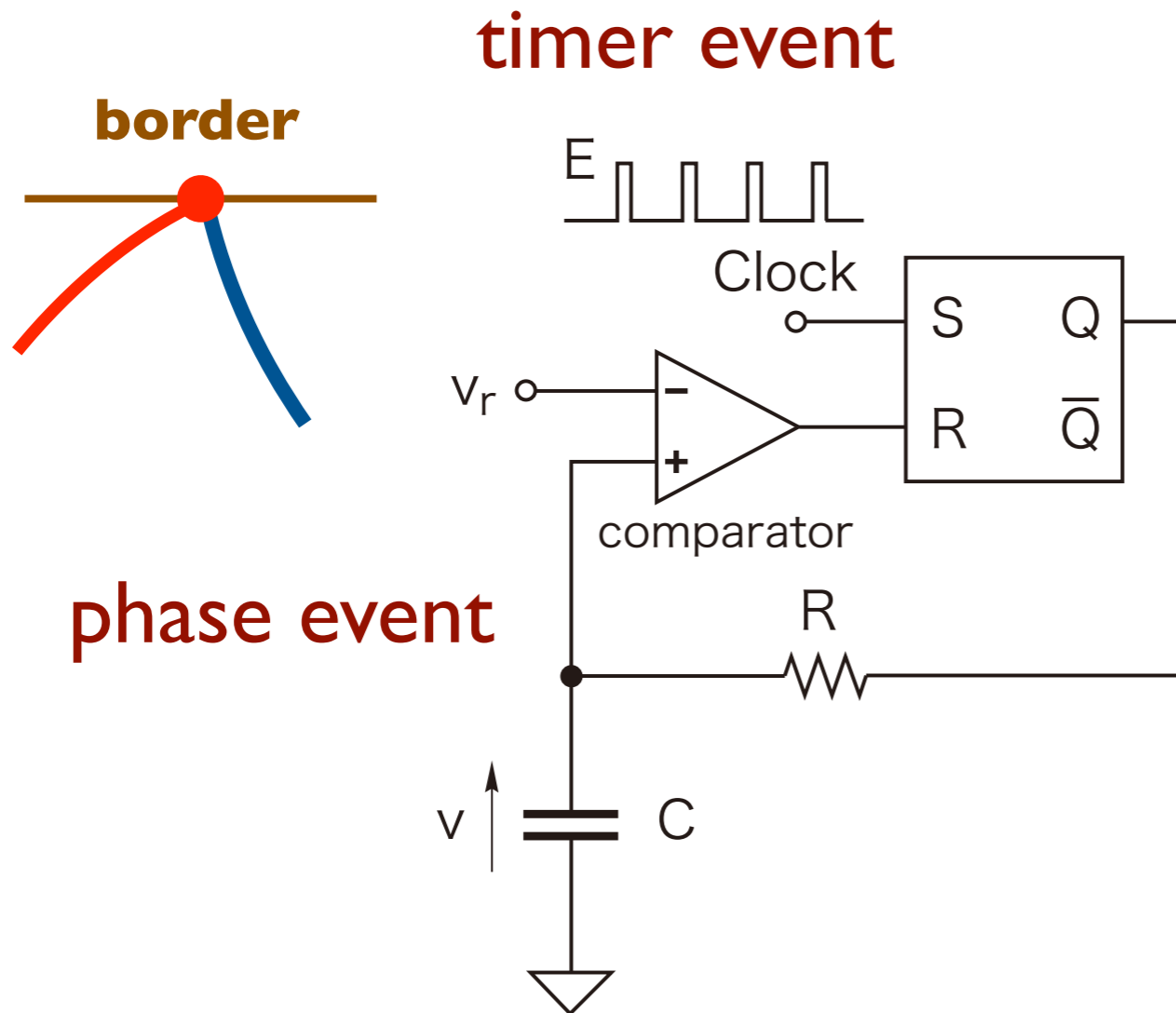


Hybrid回路の定性的解析法

1. 数学モデルの定式化
2. 貼合せ多様体をつくる
3. 貼合せ多様体上にPoincaré断面を定義する
4. Poincaré断面上で運動の定性的解析



Kousaka's BC circuit



1. digital part -> modes

$Q=0$ (mode 0)

$Q=1$ (mode 1)

2. analog part -> ode

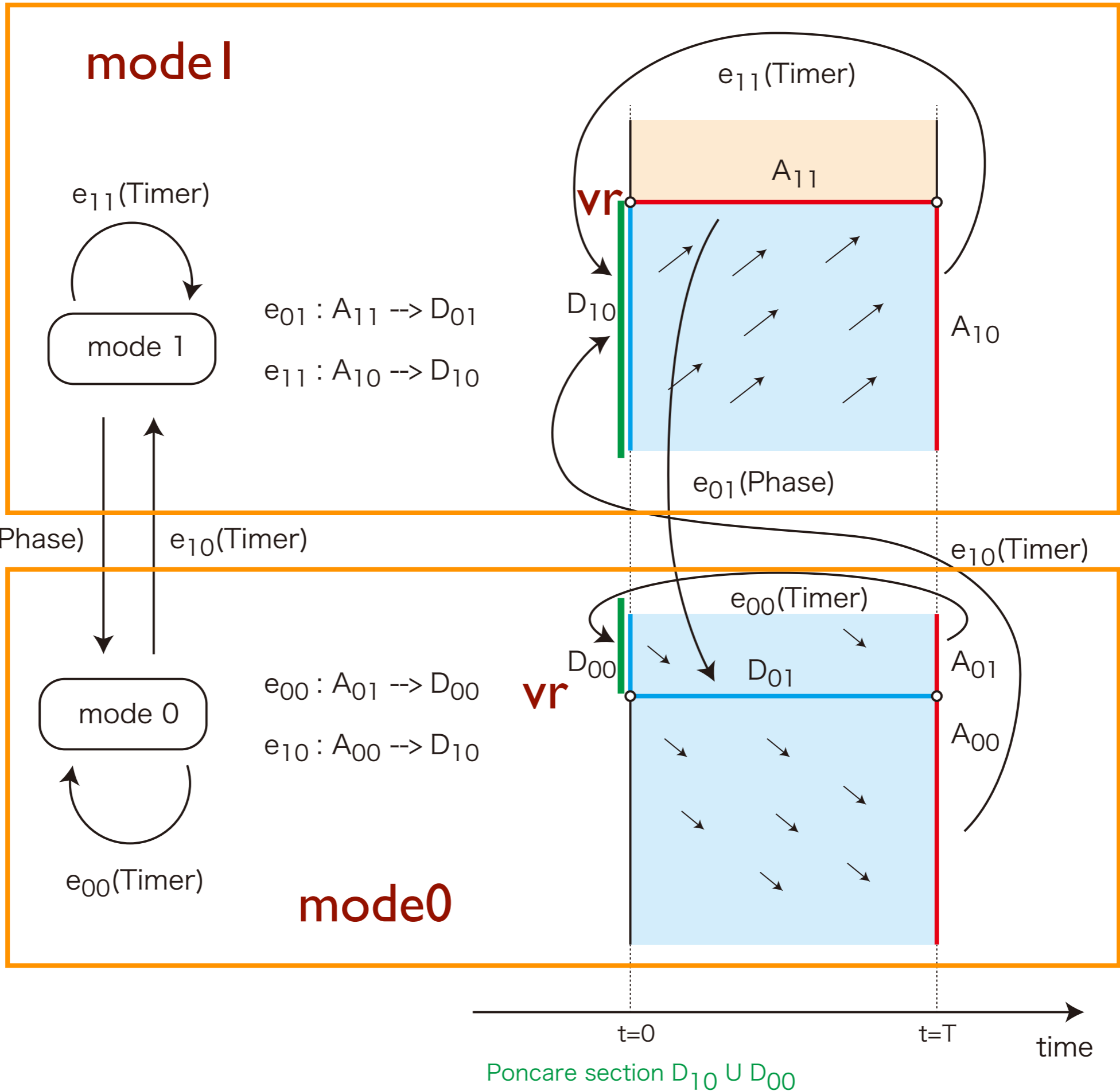
mode 0 : $dx/dt + x = 0$

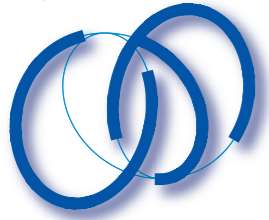
mode 1 : $dx/dt + x = 1$

3. ADC part

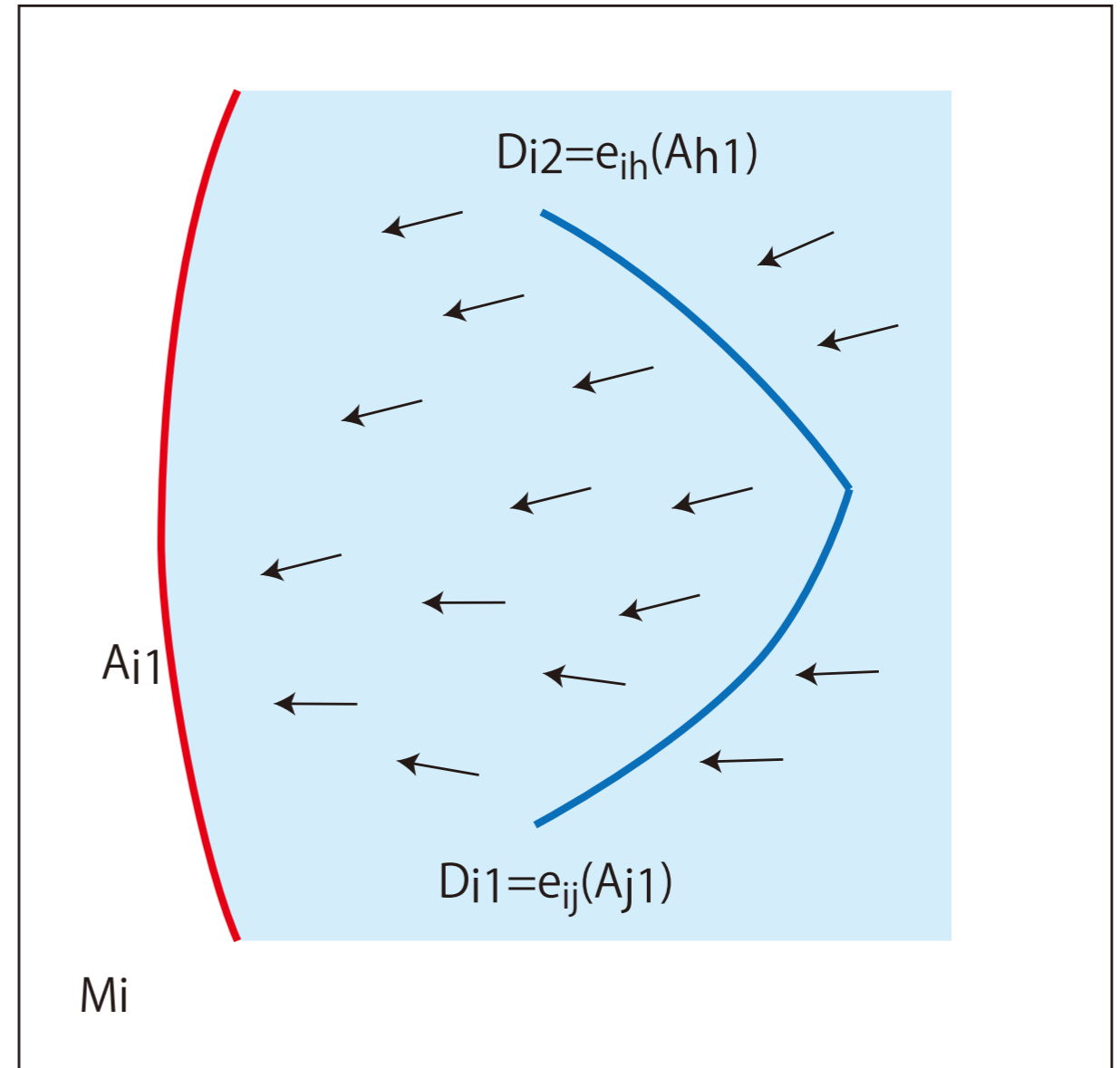
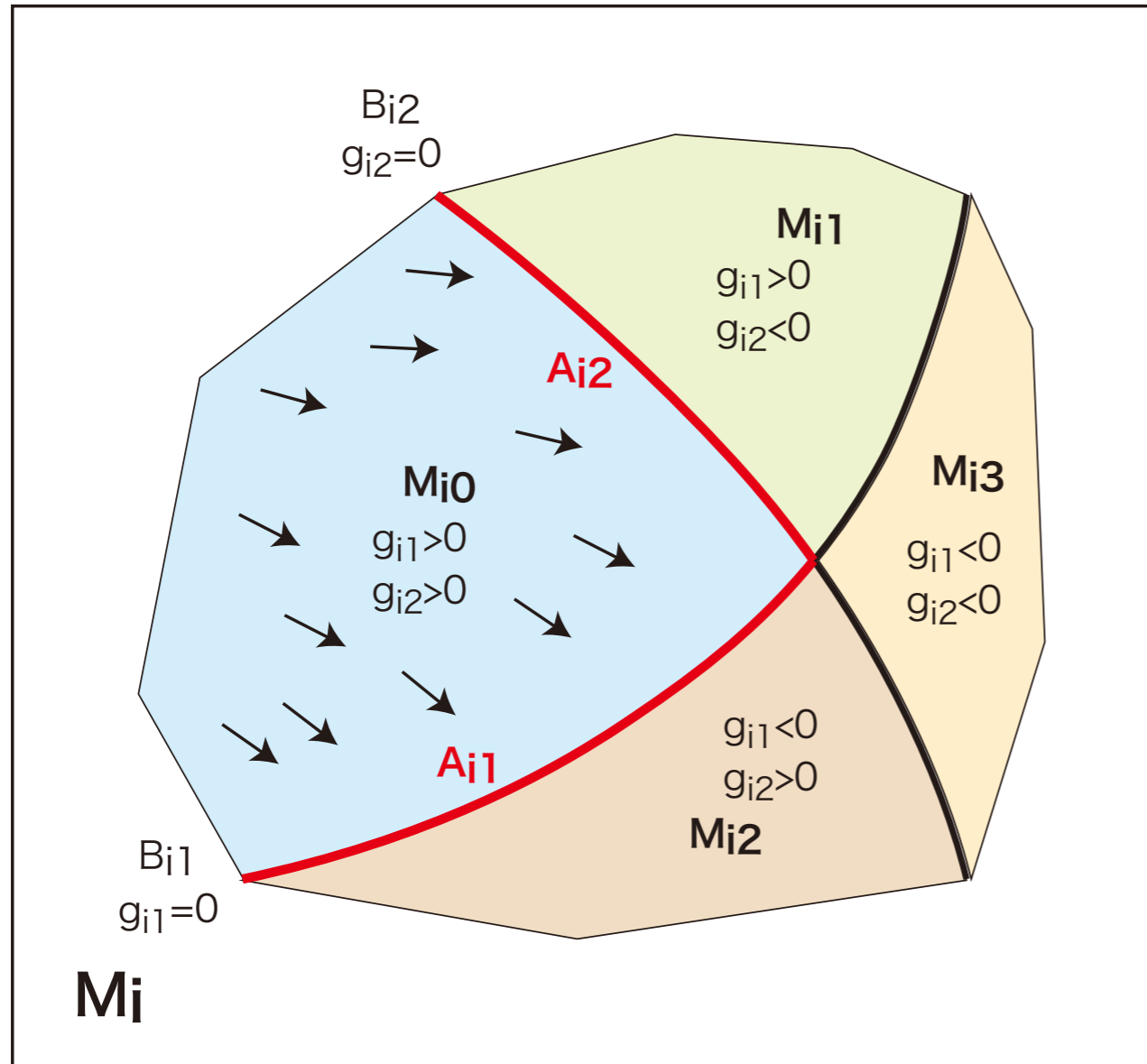
border: phase event

transition rule



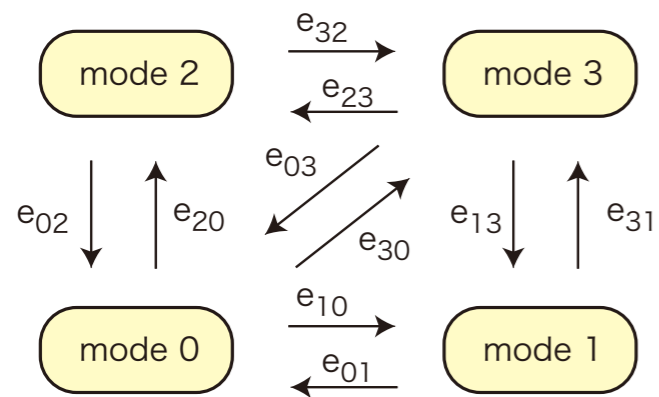


Arrival set and Departure set

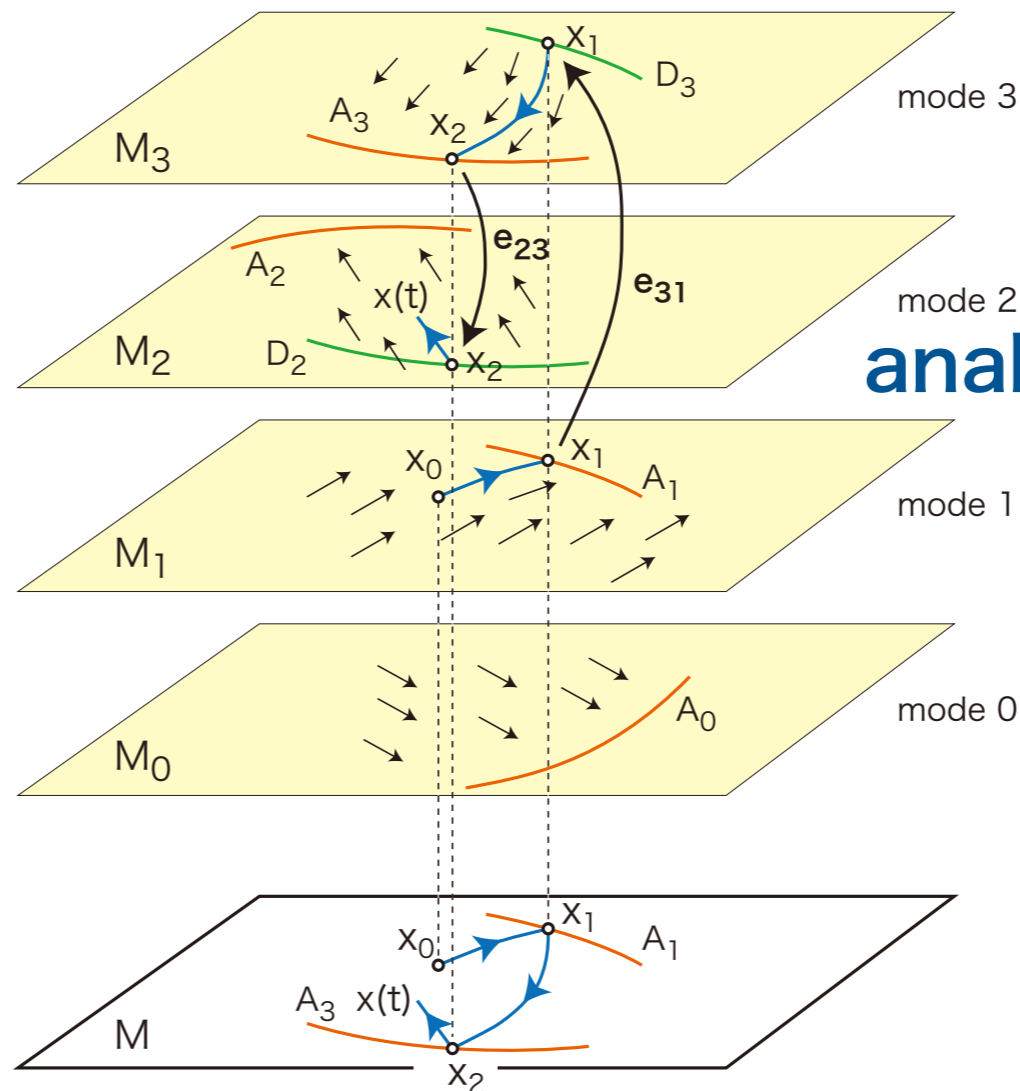




hybrid系の運動



digital dynamics

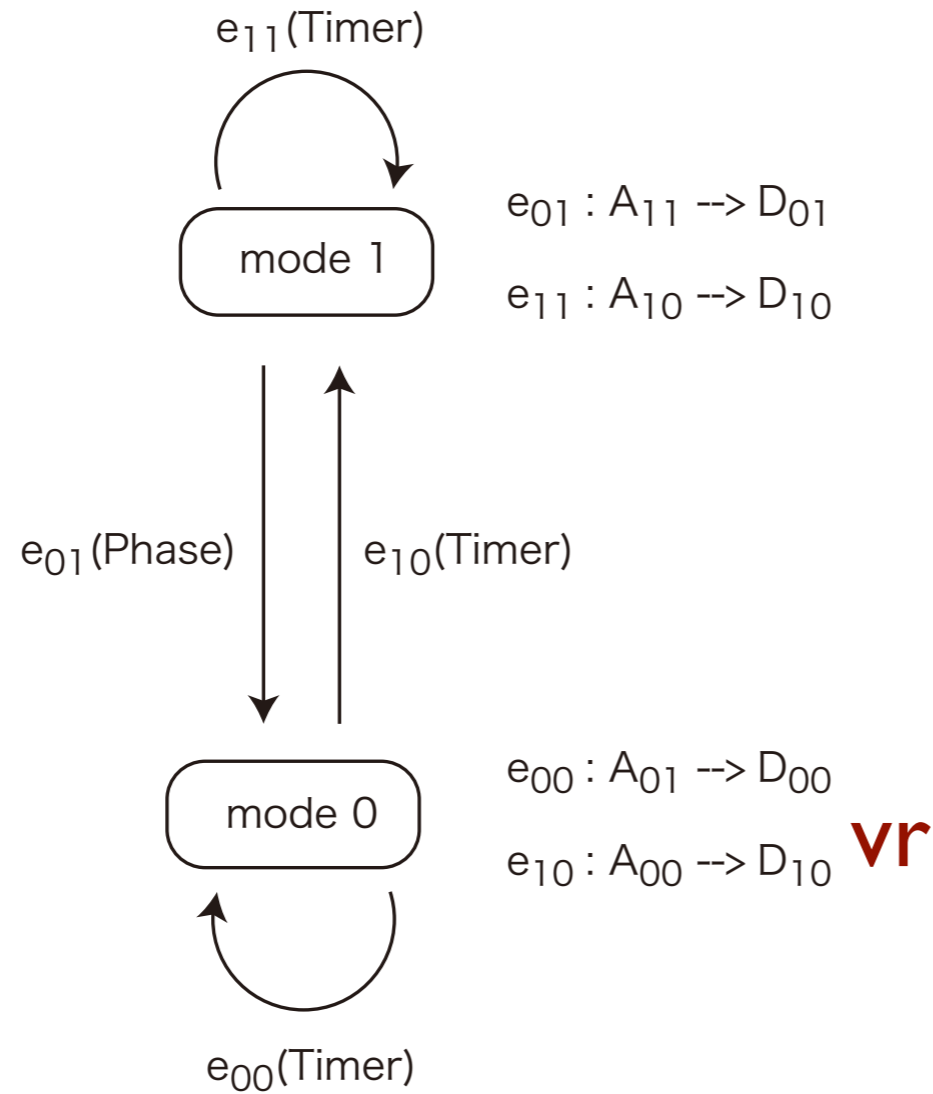


analog dynamics

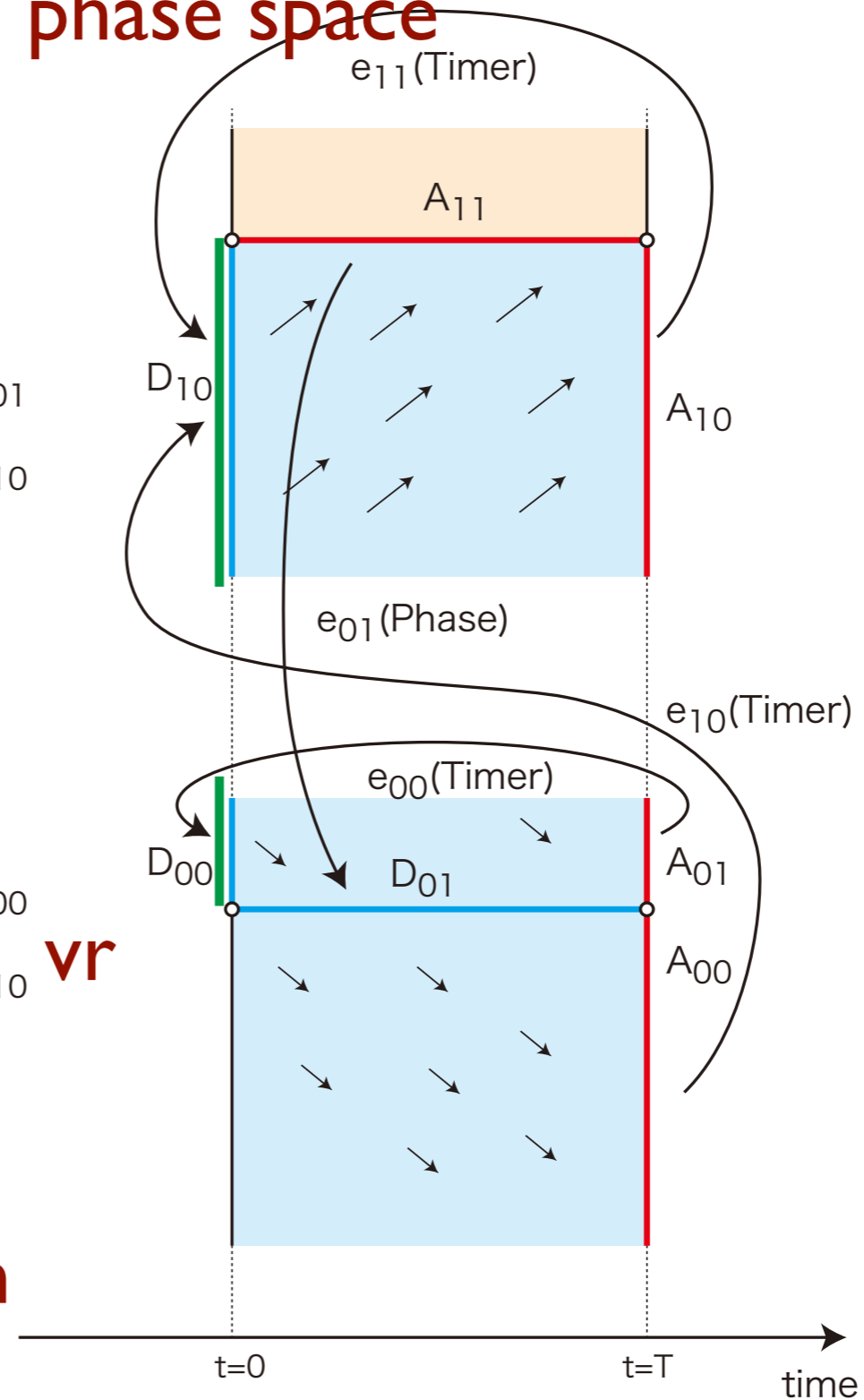
hybrid dynamics



arrive an arrival set \rightarrow transition \rightarrow leave a departure set
 collaged phase space



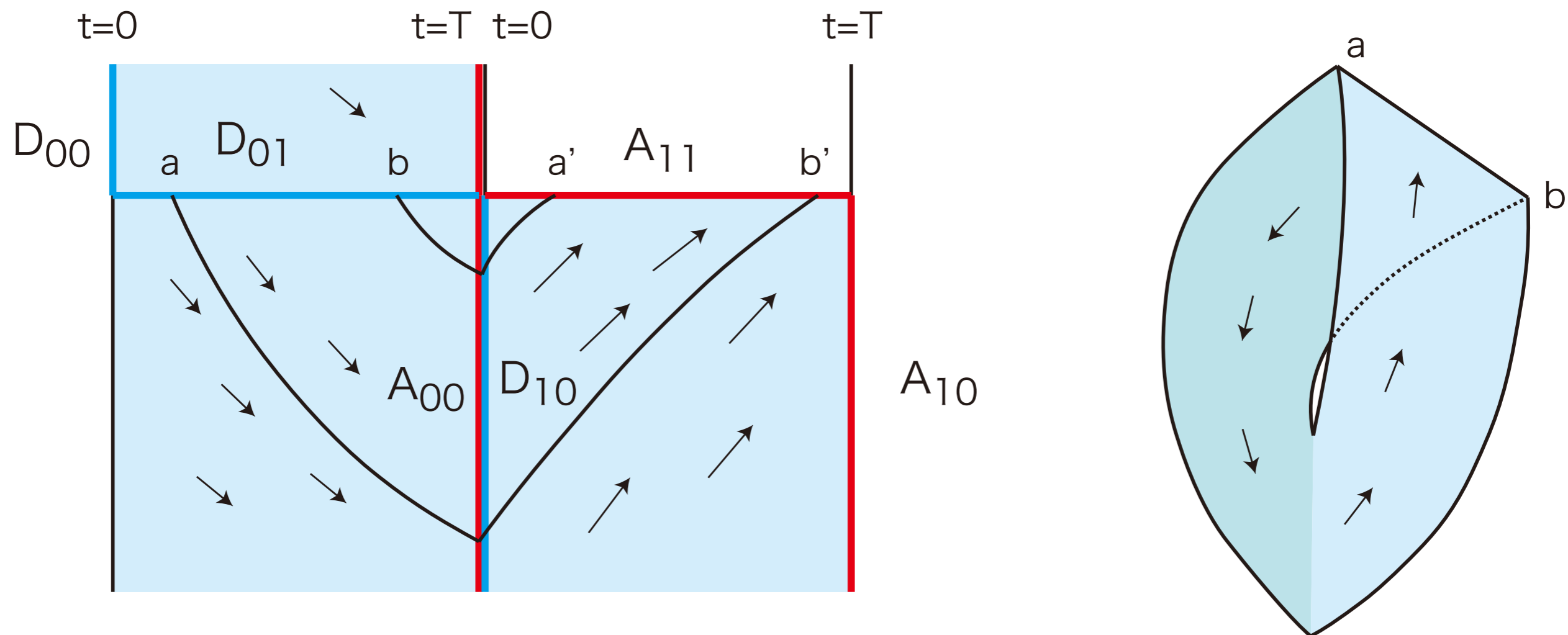
mode transition diagram



phase border diagram



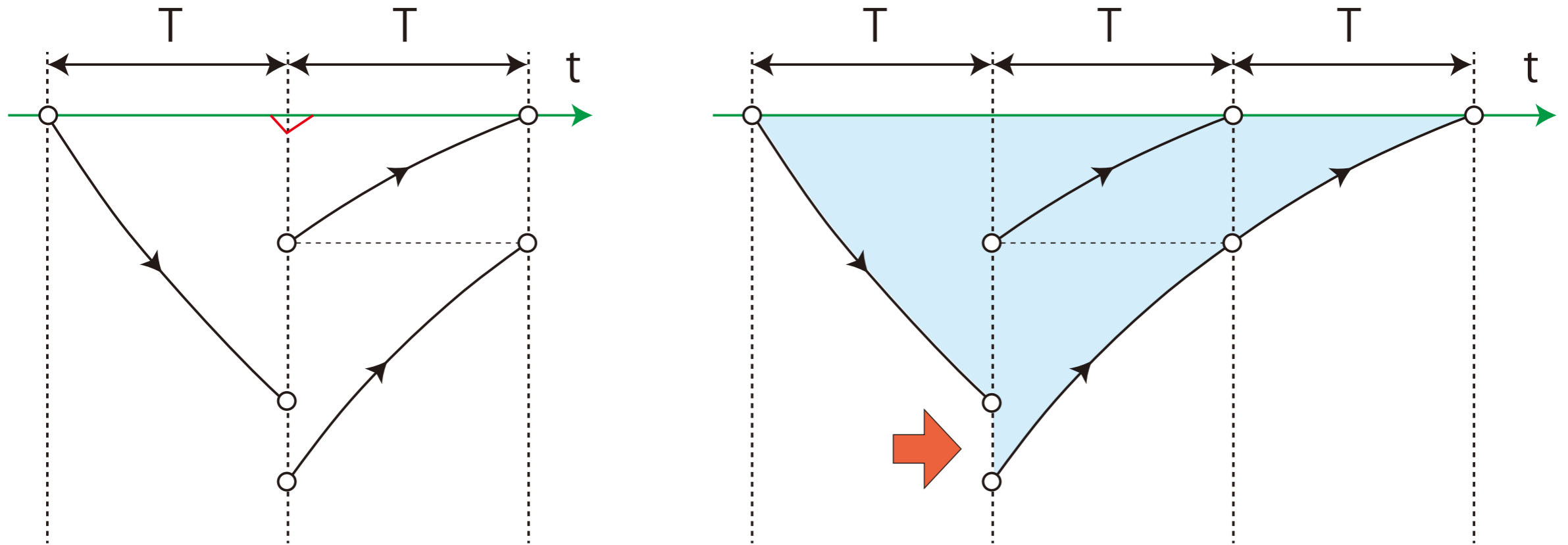
貼合せ多様体(collaged mfd)



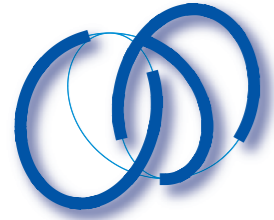
最も単純なflowを仮定して描いたcollaged mfd



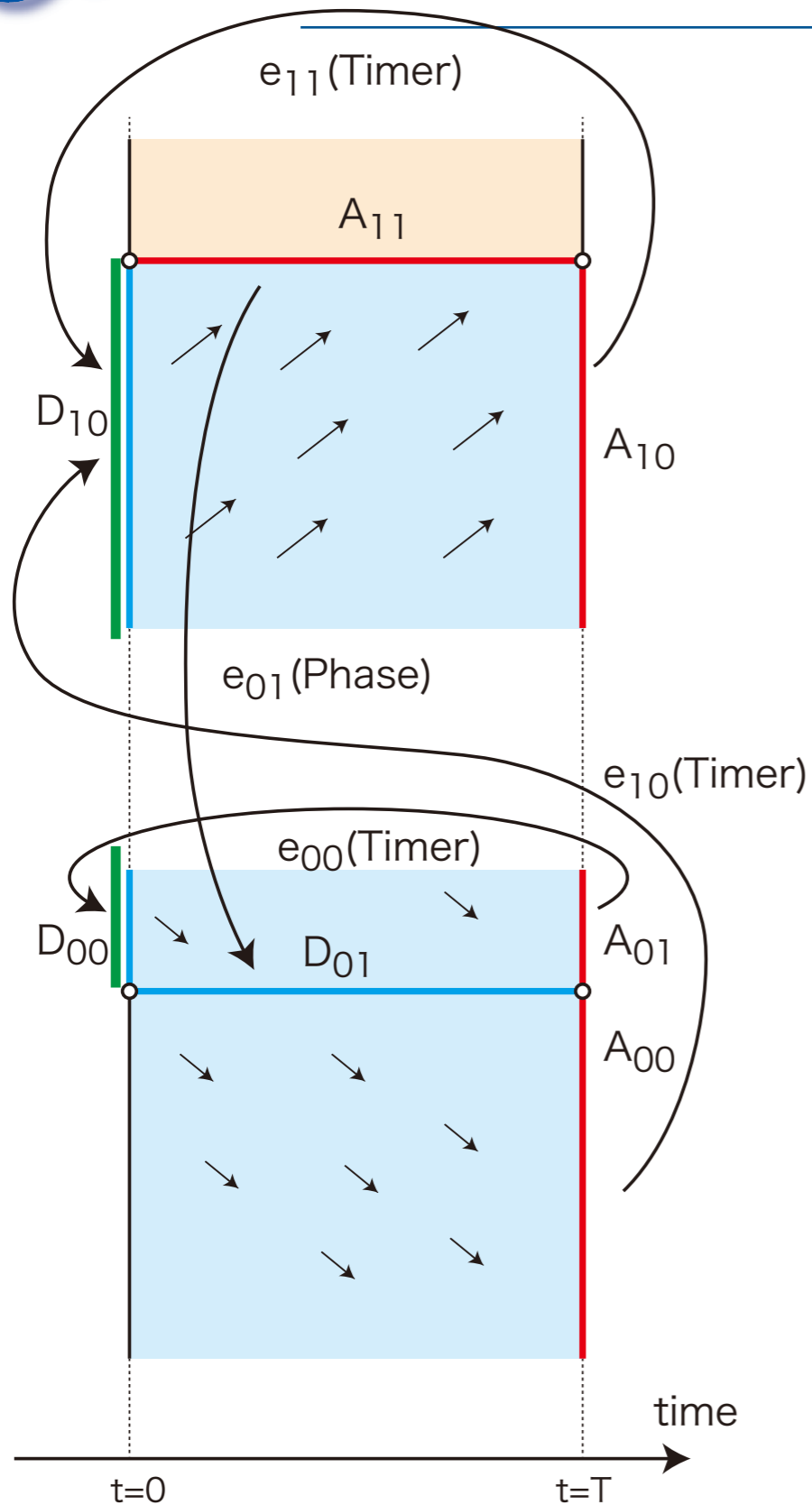
大域安定性 (Confinor)



系の運動は最終的にはblueの領域に吸い込まれる



Poincaré section



departure set の適当な集合を選ぶ

$$D_{00} \cup D_{10} \longrightarrow D_{10}$$

Poincaré map を定義する

$$1) P1: D_{10} \longrightarrow A_{10} \longrightarrow D_{10}$$

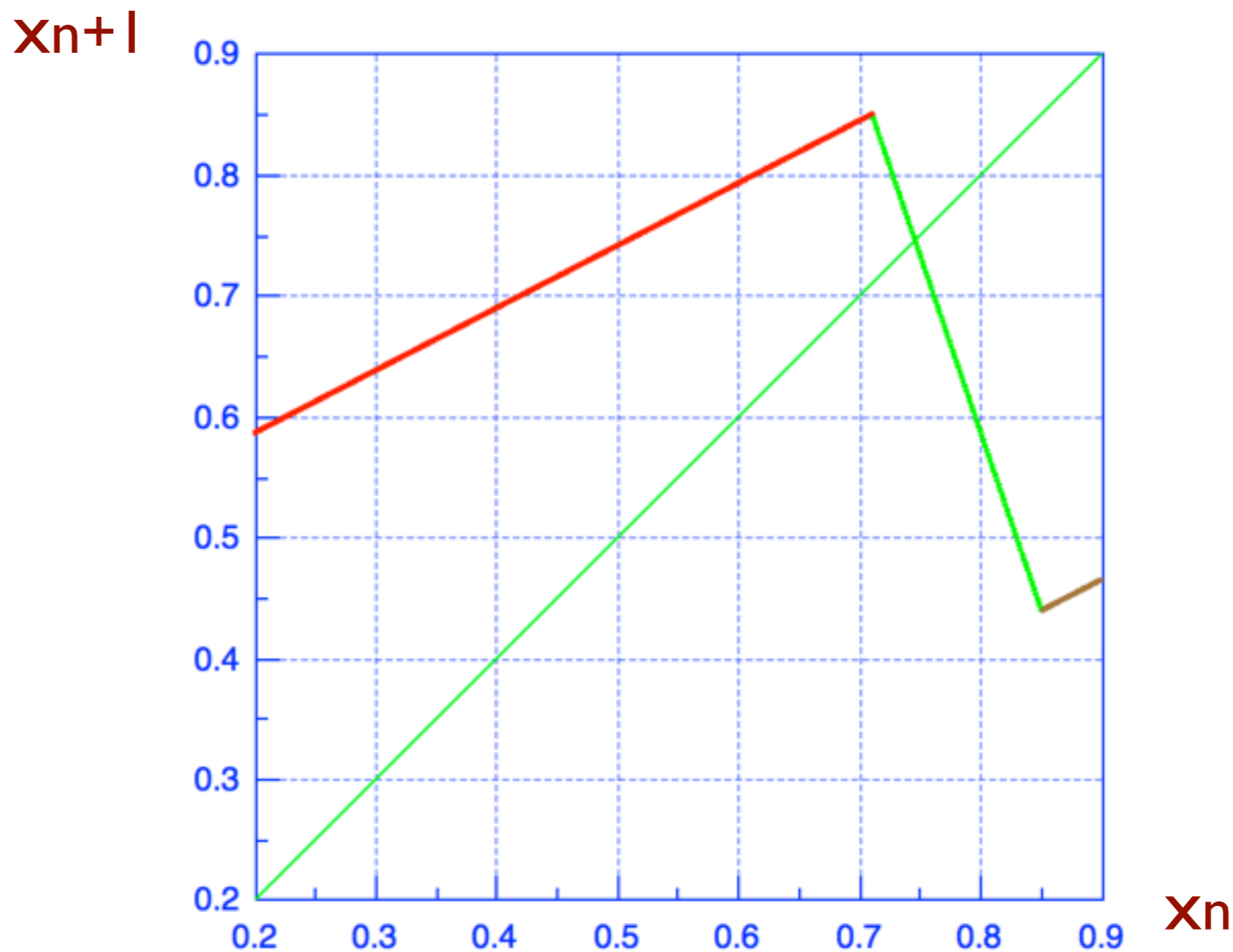
$$2) P2: D_{10} \longrightarrow A_{11} \longrightarrow D_{01} \longrightarrow A_{00} \longrightarrow D_{10}$$

$$3) P3: D_{00} \longrightarrow A_{01} \cup A_{00}$$



ここからは通常解析をどうぞ

alpha=0.85
T=0.66

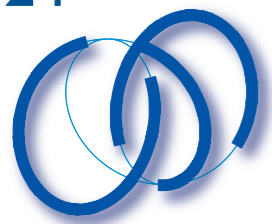


[4] N.N. Leonov; Map of the line onto itself, *Radiofisica*, 2(6), 1959, pp. 942-056

N.N. Leonov; Piecewise linear map, *Radiofisica*, 3(3), 1960, pp. 496-510

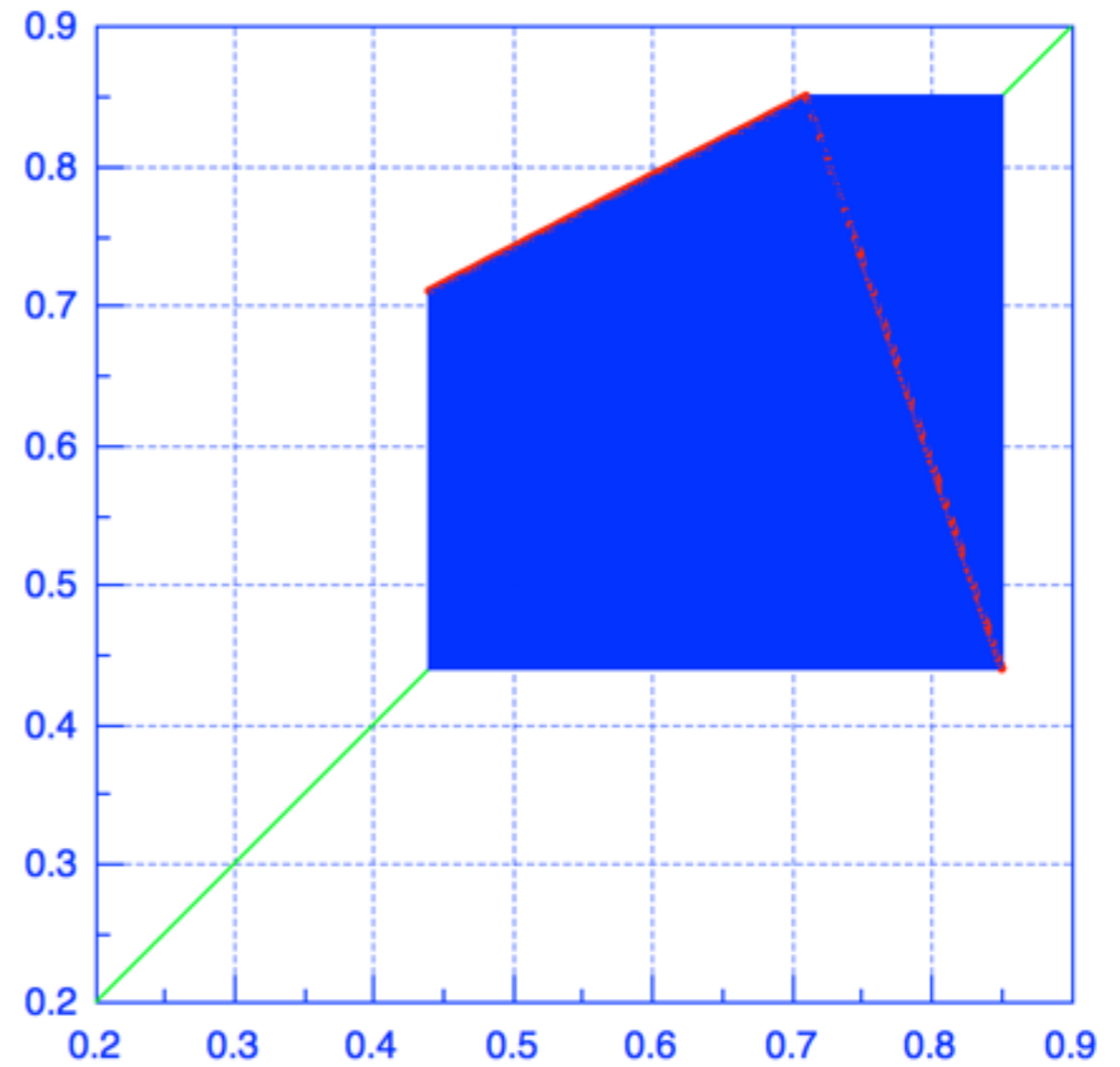
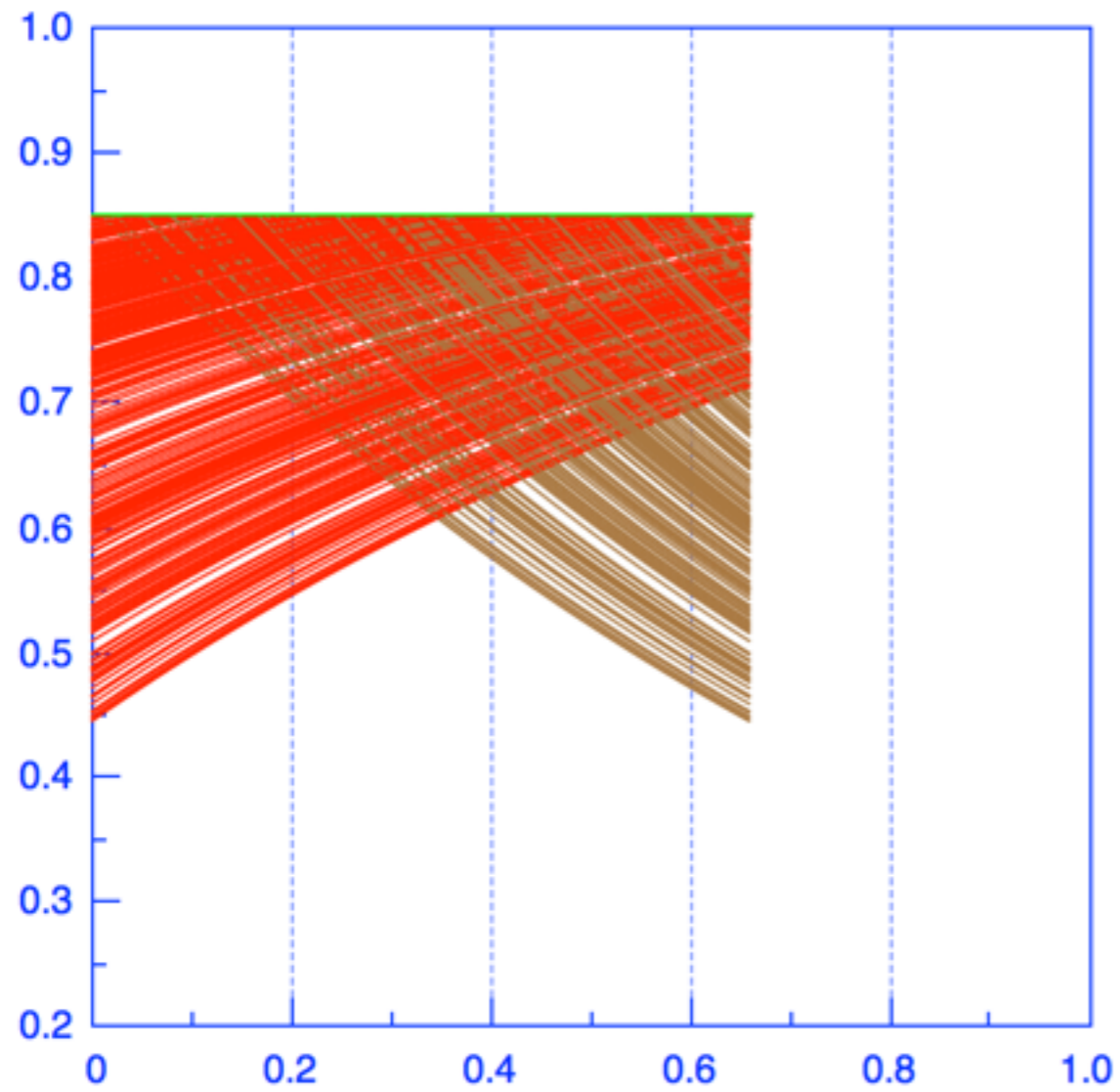
N.N. Leonov; Theory of discontinuous maps of the line, *Radiofisica*, 3(5), 1960, pp. 872-886

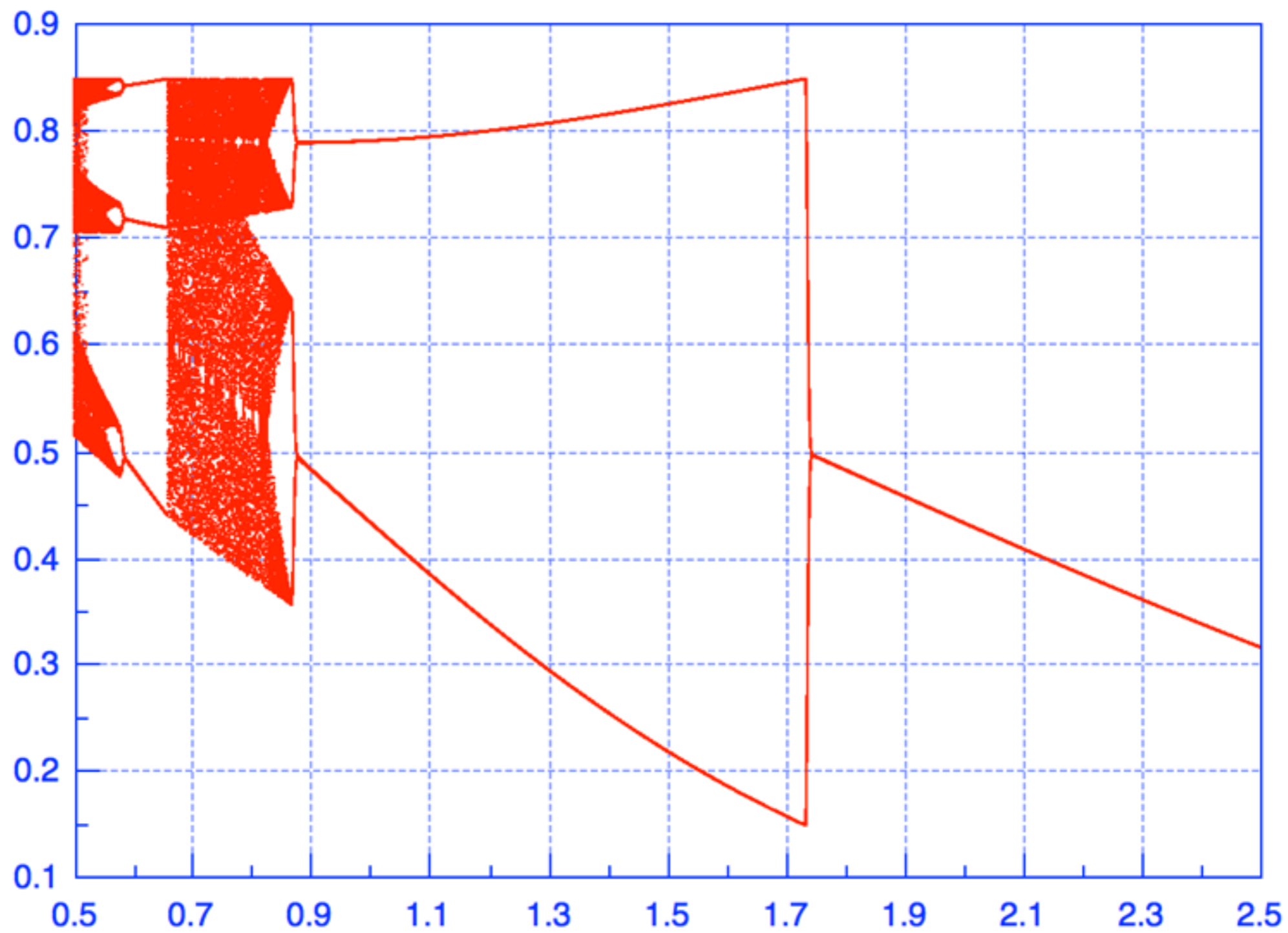
N.N. Leonov; Discontinuous map of the straight line, *Dokl. Akad. Nauk. SSSR*, 143(5) 1962, pp. 1038-1041



$\alpha=0.85$

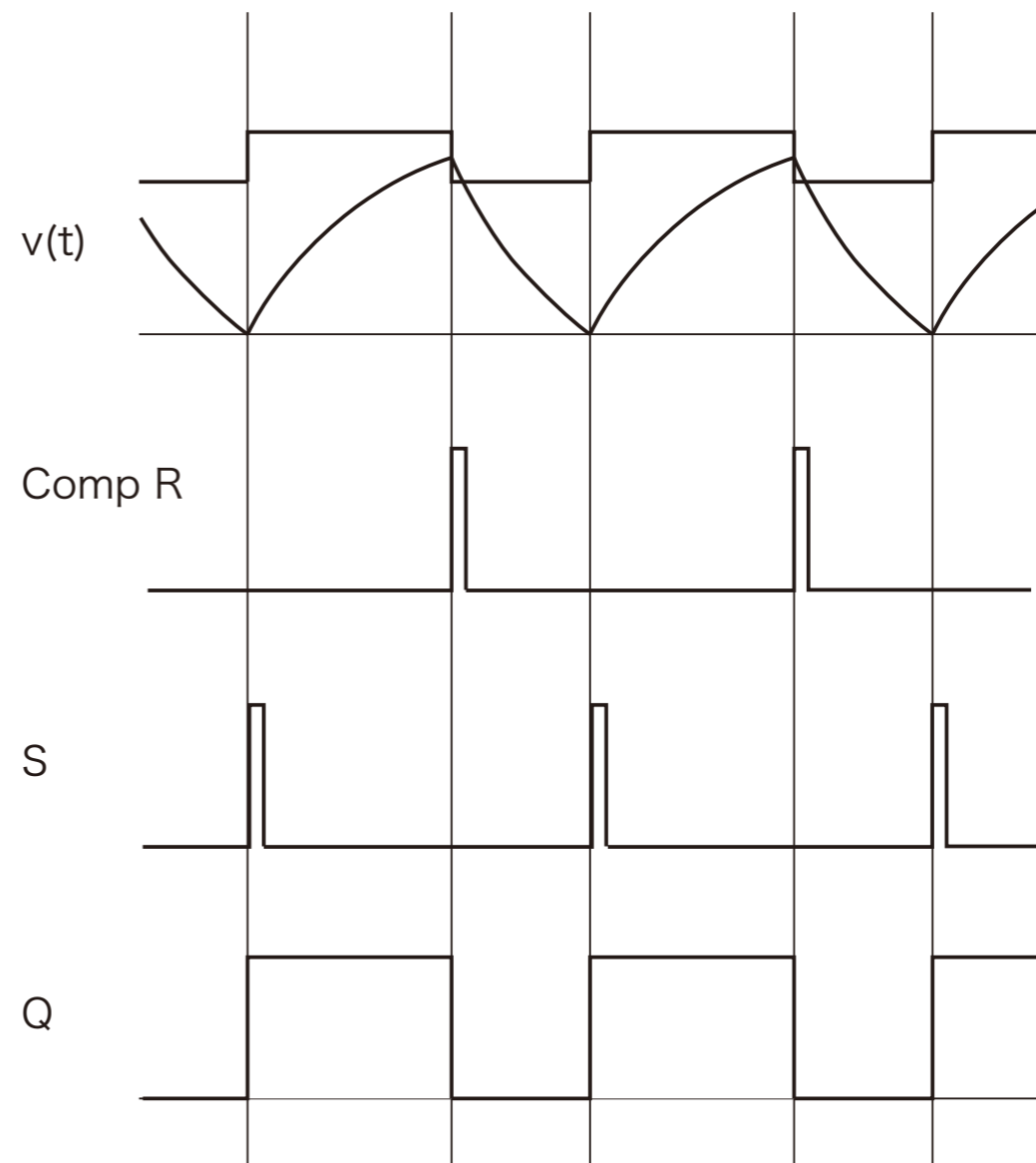
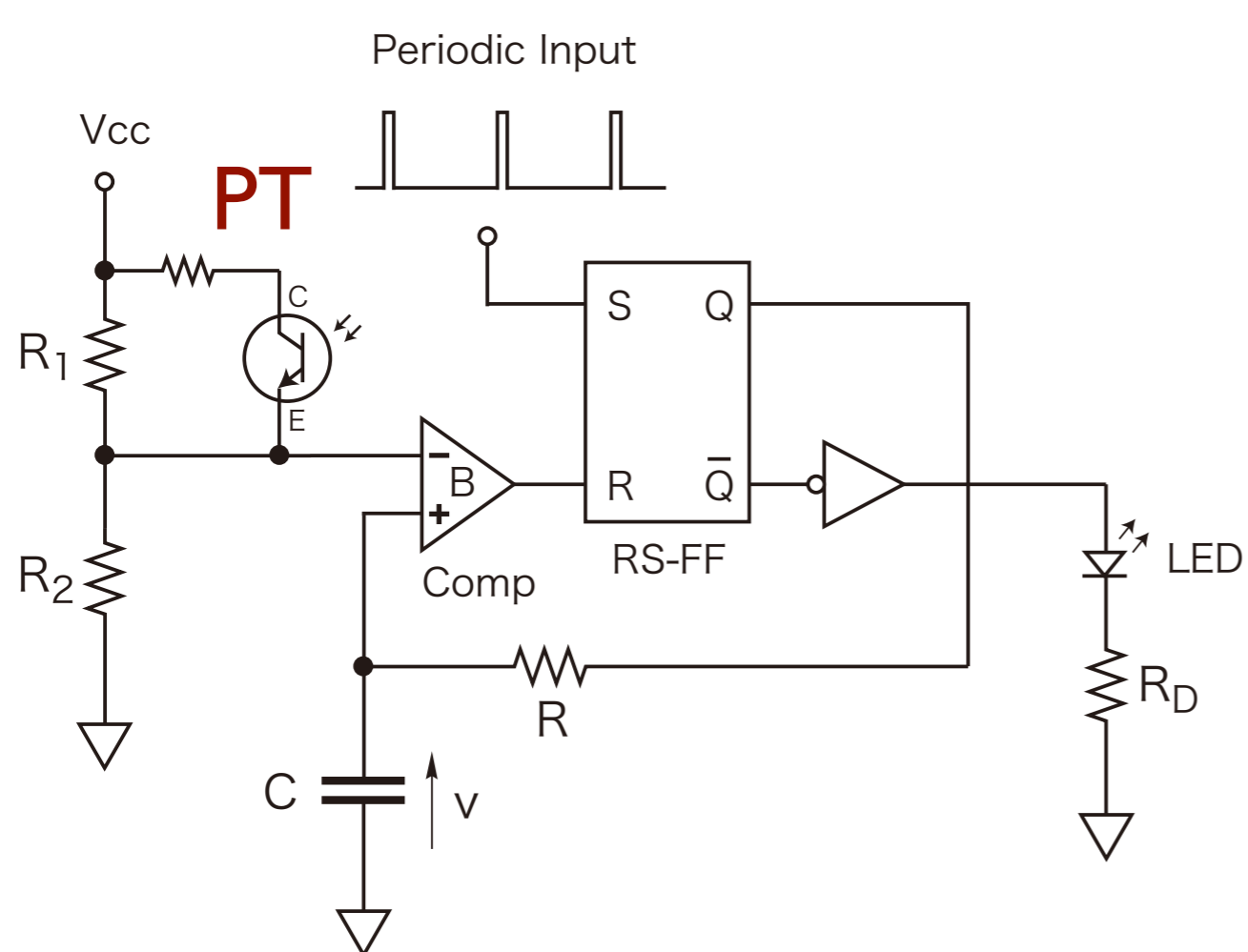
$T=0.66$







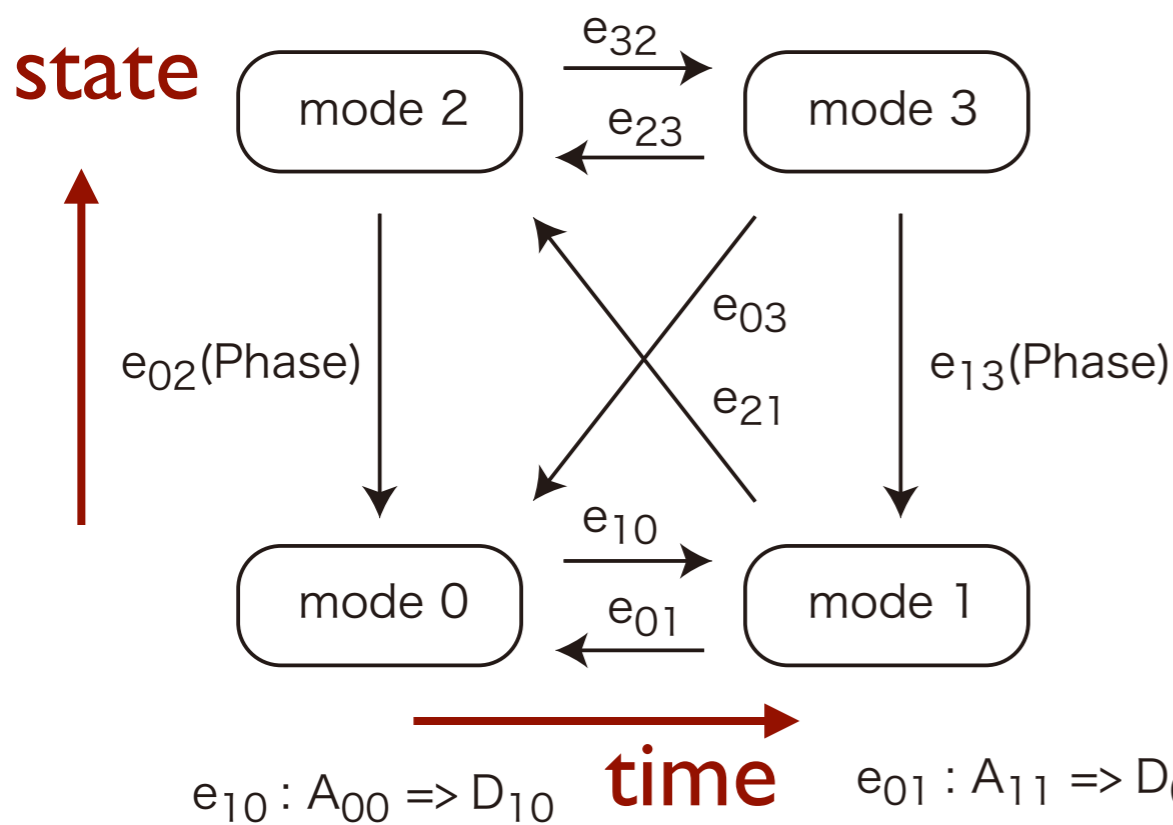
Forced KBC Circuit



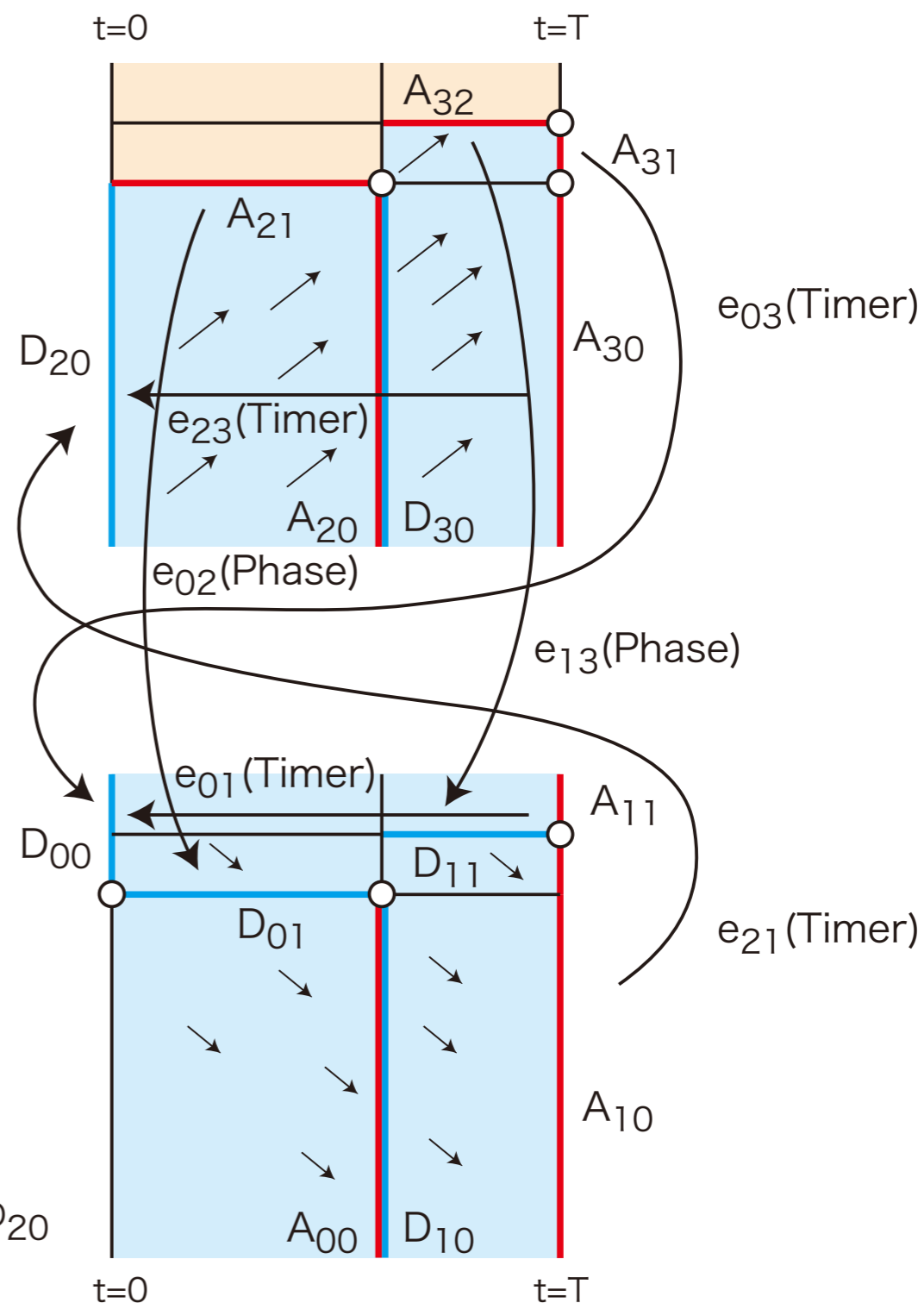


方形波入力でvrが変わる回路

$e_{02} : A_{21} \Rightarrow D_{01}$
 $e_{32} : A_{20} \Rightarrow D_{30}$
 $e_{23} : A_{30} \Rightarrow D_{20}$
 $e_{03} : A_{31} \Rightarrow D_{00}$
 $e_{13} : A_{32} \Rightarrow D_{11}$



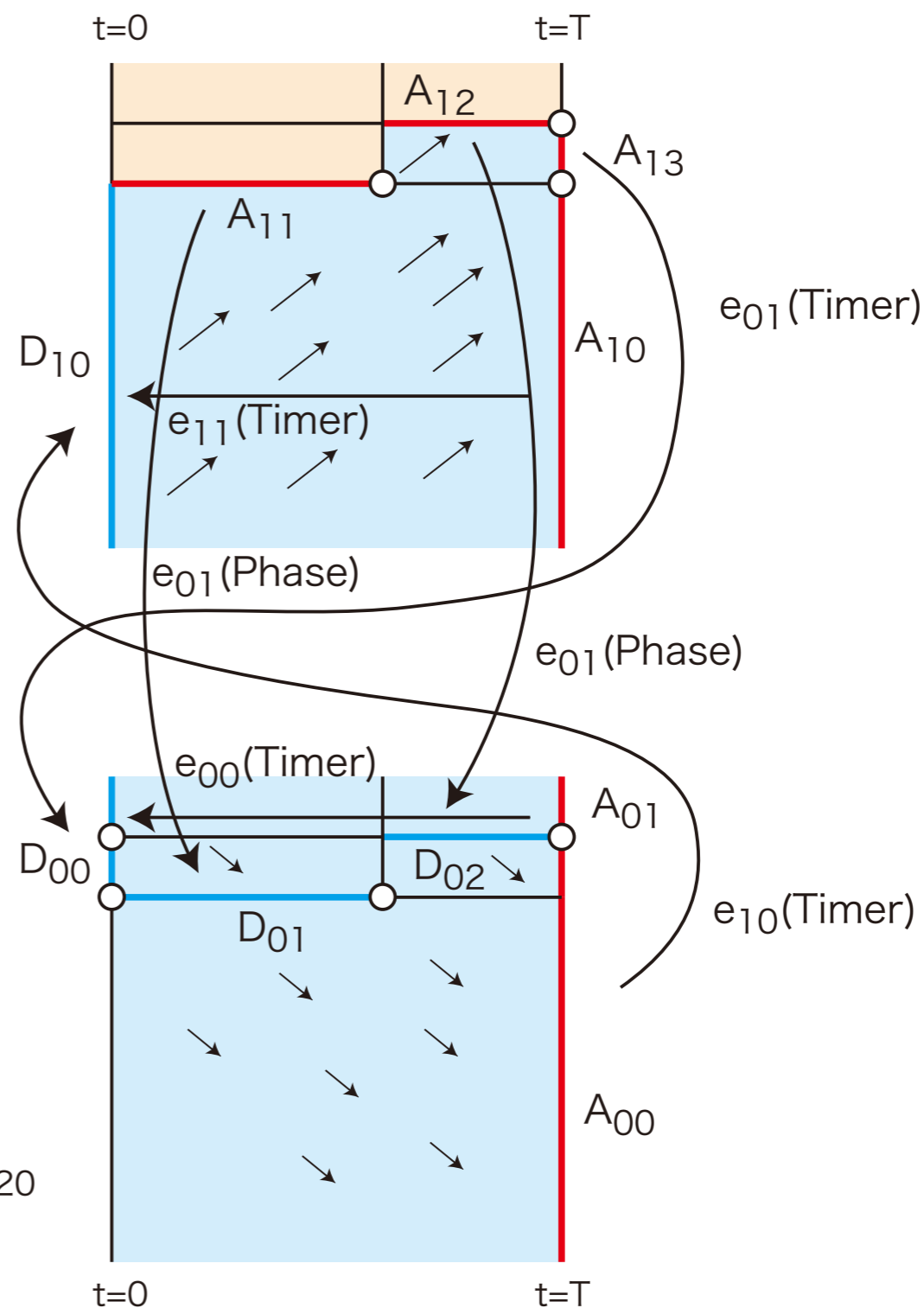
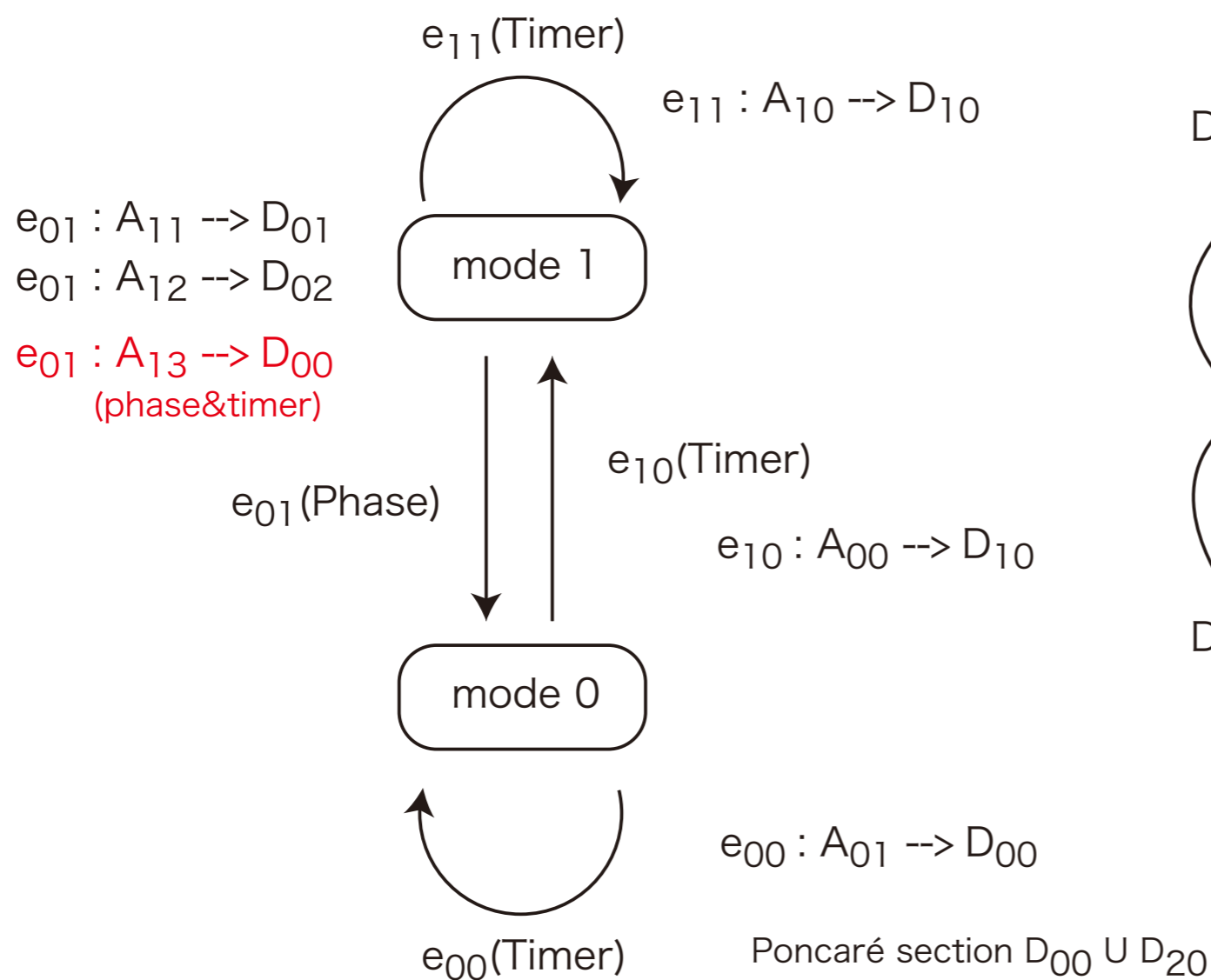
Poncare section $D_{00} \cup D_{20}$

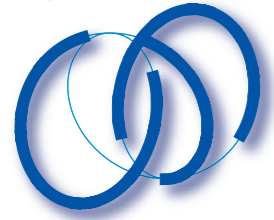




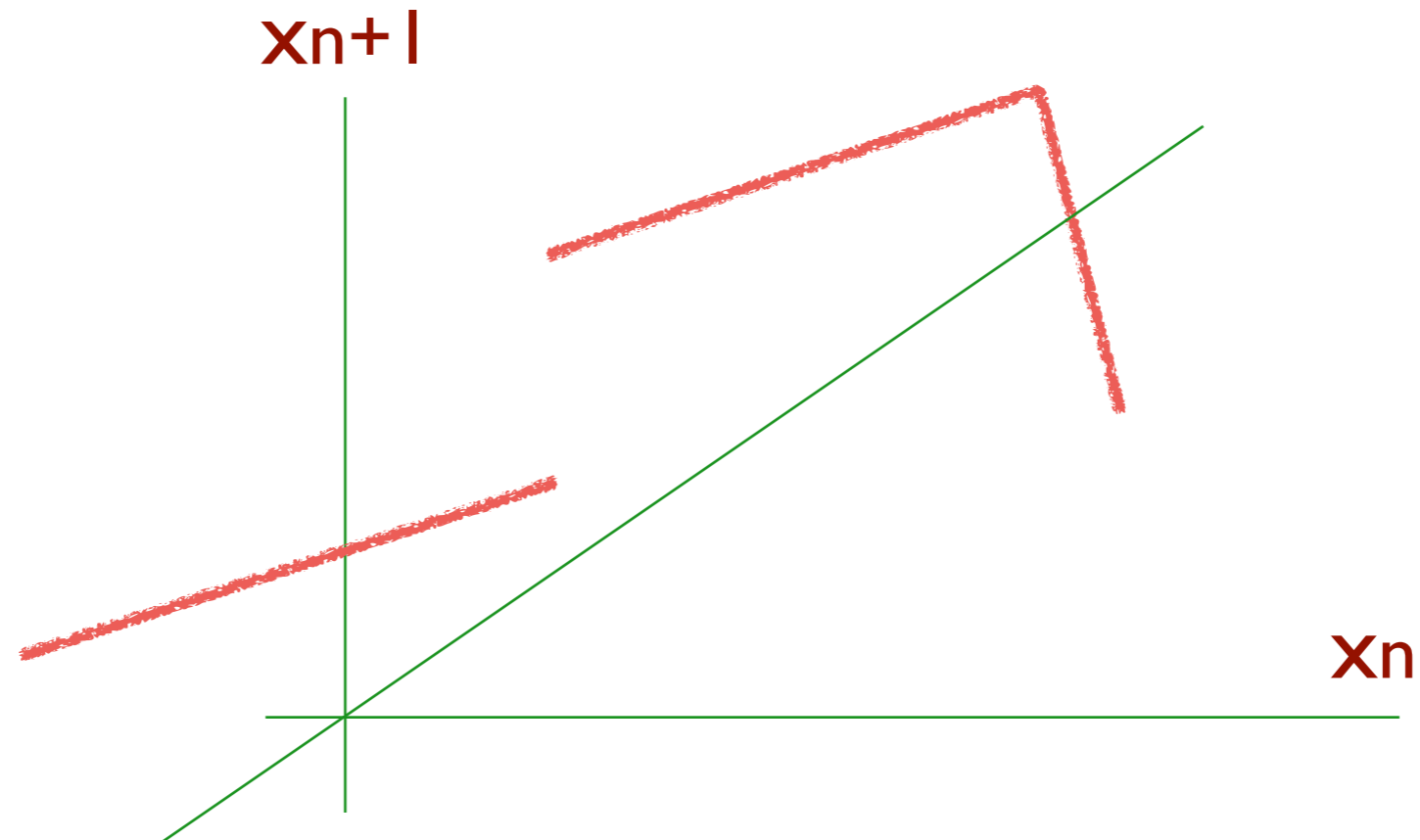
modeの簡略化

A set, D setは複雑になる





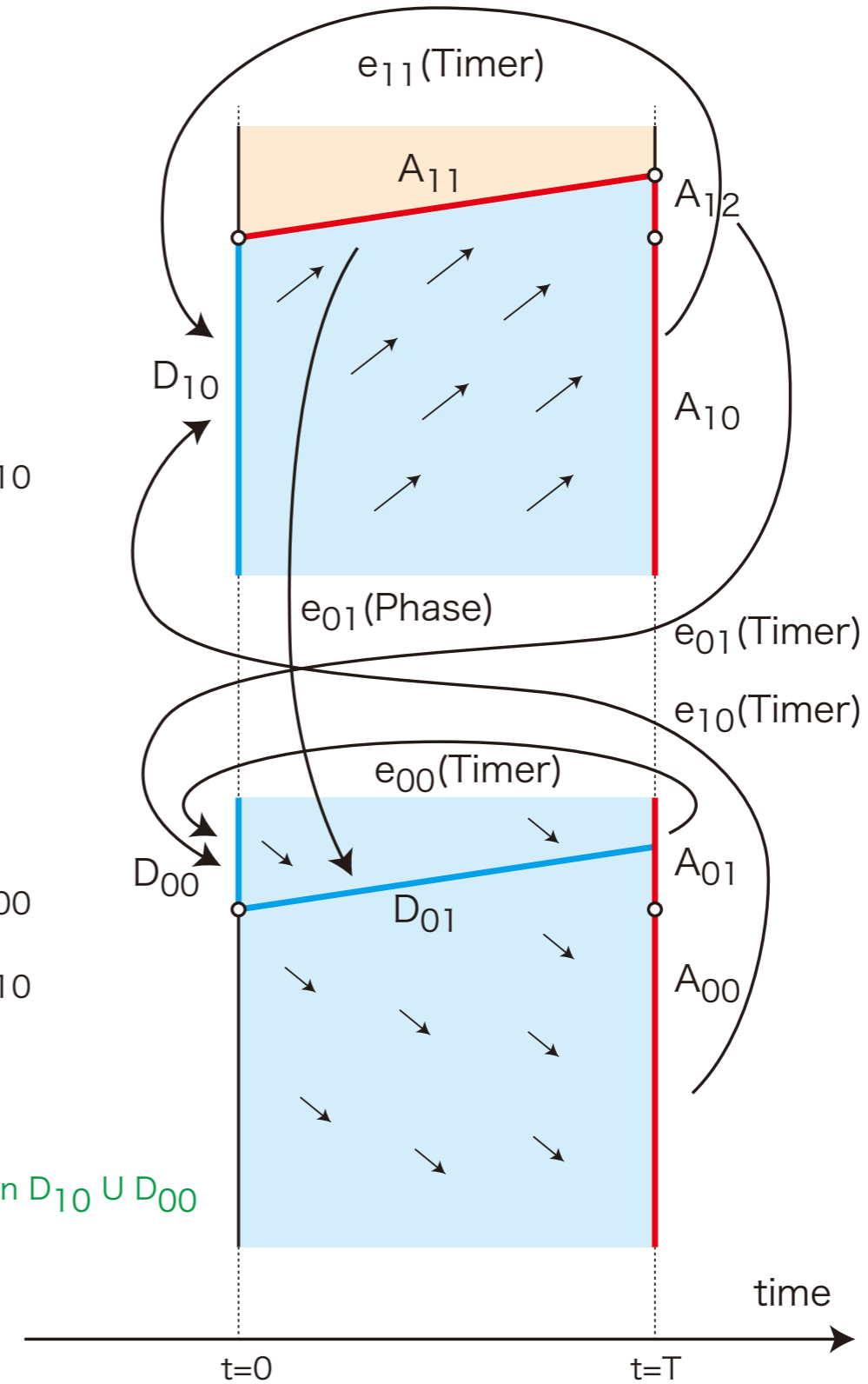
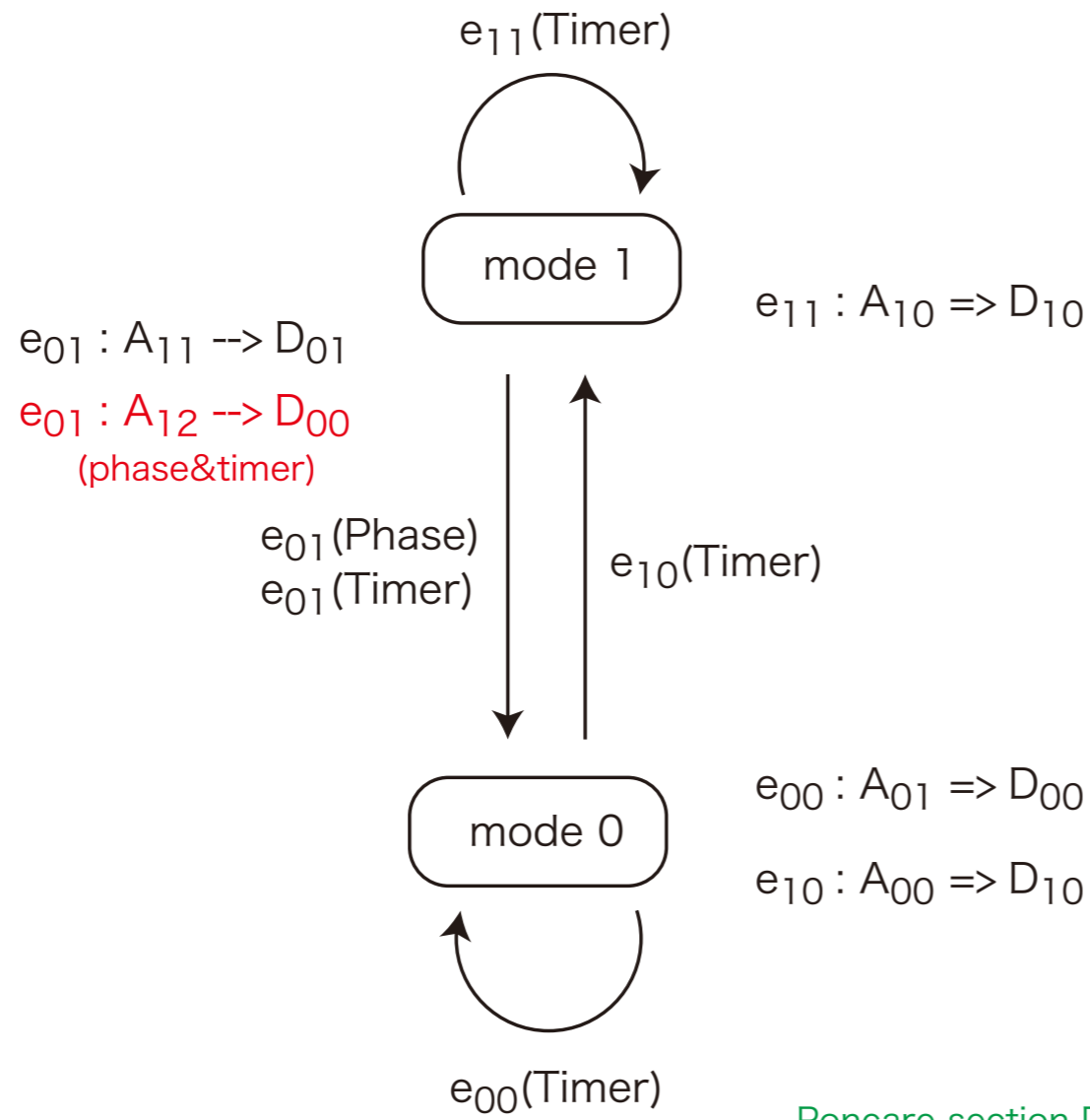
Poincaré写像に不連続点が見れる

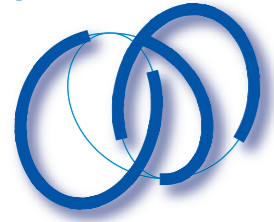


M. di Bernardo, C.J. Budd, A.R. Champneys and P. Kowalczyk, Piecewise-smooth Dynamical Systems, Springer Applied Mathematical Sciences 163, 2008; see Chap. 4

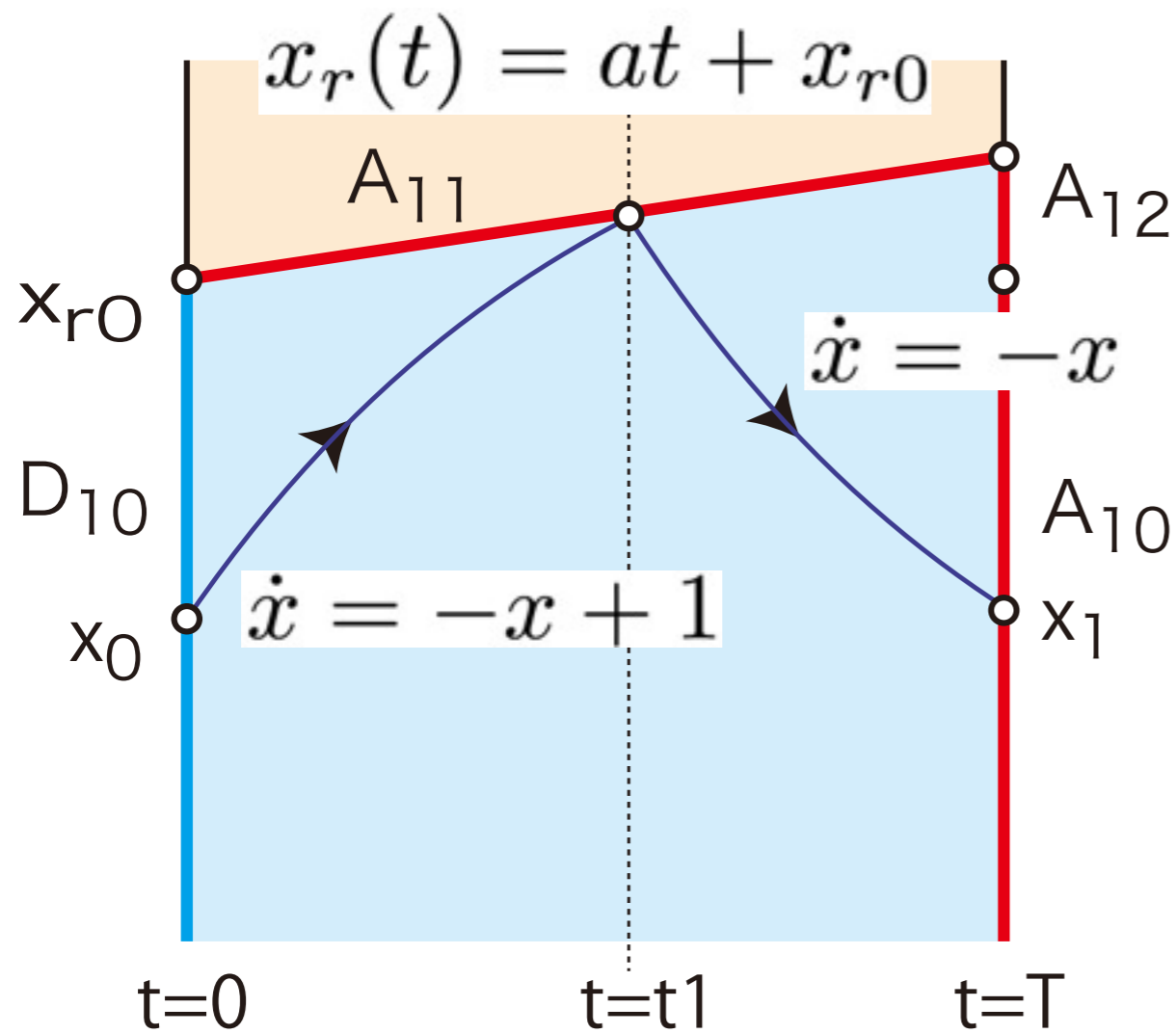


time variant border





circuit with moving border



$$x(\tau_1) = (x_0 - 1)e^{-\tau_1} + 1$$

$$x_r(\tau_1) = a\tau_1 + x_{r0}$$

$$x_1 = x(T - \tau_1) = x_r(\tau_1)e^{-(T - \tau_1)}$$

$$x_1 = P(x_0)$$

Poincaré map becomes nonlinear.

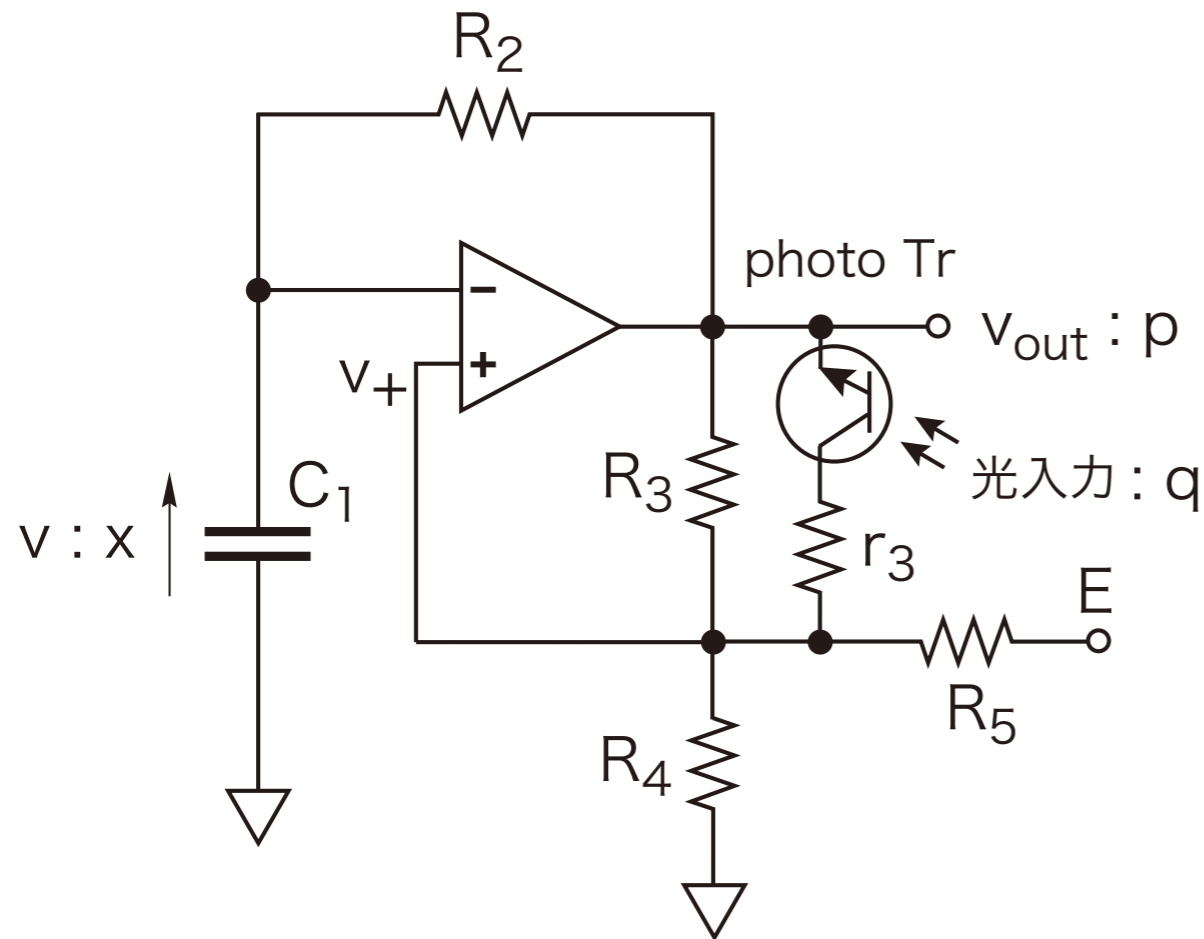
回路としてどんなmeritがあるのだろうか？



LEDホタルの強制振動



LED FF with SW forcing term



		comparator : p	
		on:1	off:0
光入力 : q	on:1	off:0	on:1
	off:0	off:0	off:0

mode0 : $(q, p) = (0, 0), dx/dt + x = 0$

mode1 : $(q, p) = (1, 0), dx/dt + x = 0, \beta_{on}$

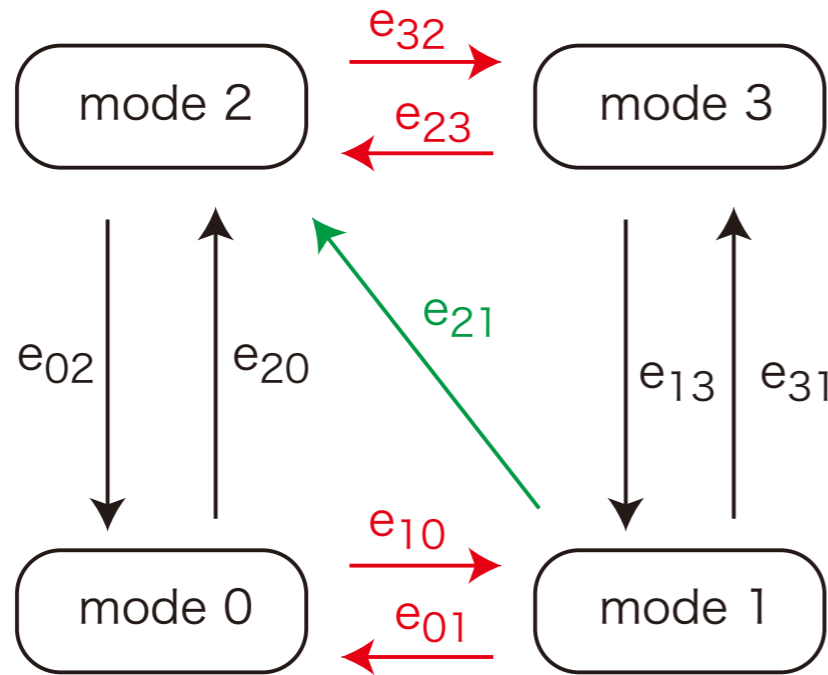
mode2 : $(q, p) = (0, 1), dx/dt + x = 1$

mode3 : $(q, p) = (1, 1), dx/dt + x = 1$



mode and phase diagrams

$e_{32} : A_{20} \Rightarrow D_{30}$ $e_{23} : A_{30} \Rightarrow D_{20}$
 $e_{02} : A_{21} \Rightarrow D_{01}$ $e_{23} : A_{31} \Rightarrow D_{21}$
 $e_{13} : A_{32} \Rightarrow D_{11}$

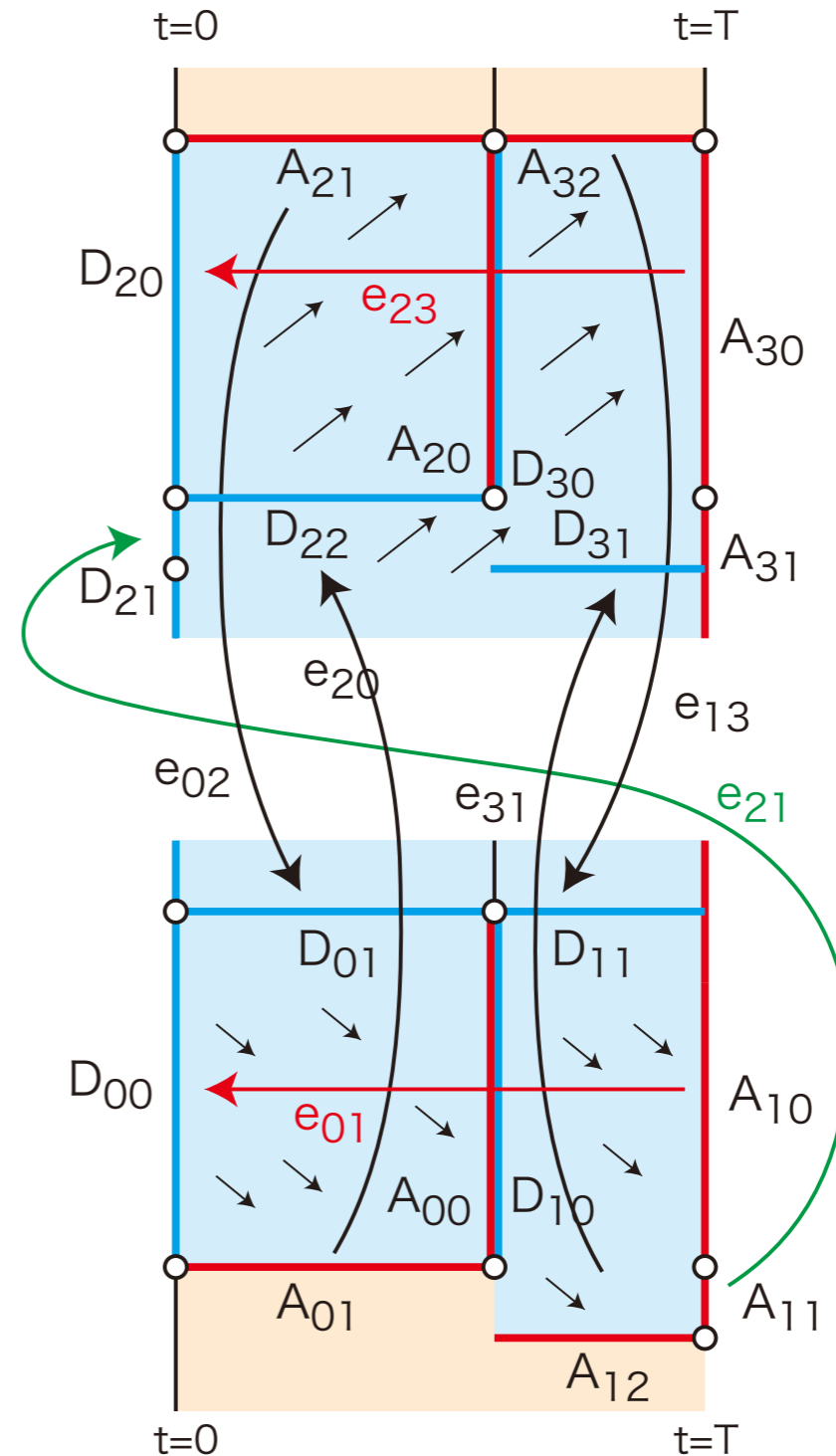


$e_{10} : A_{00} \Rightarrow D_{10}$ $e_{01} : A_{10} \Rightarrow D_{00}$
 $e_{20} : A_{01} \Rightarrow D_{22}$ $e_{21} : A_{11} \Rightarrow D_{21}$
 $e_{31} : A_{12} \Rightarrow D_{31}$

timer event

phase event

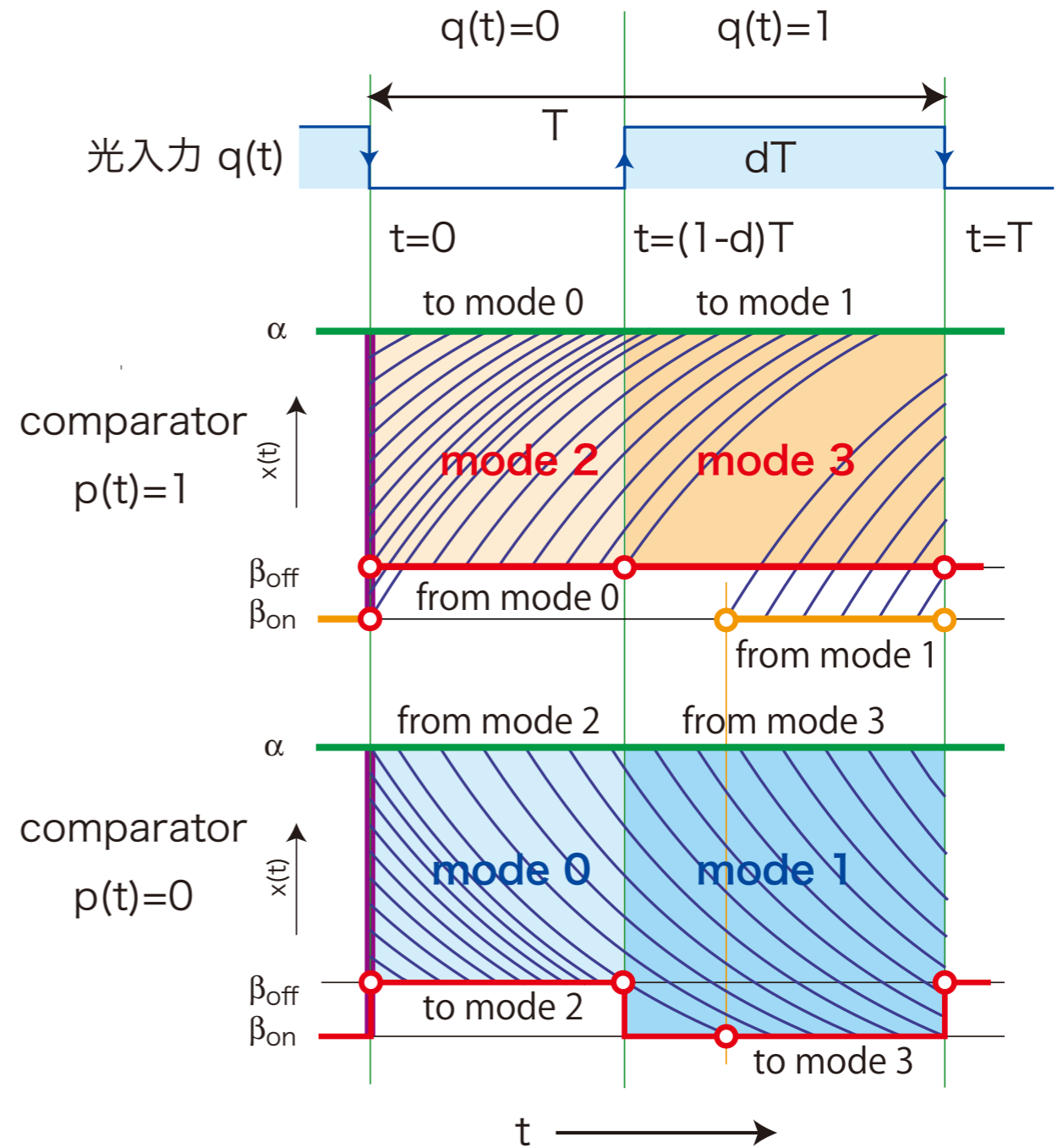
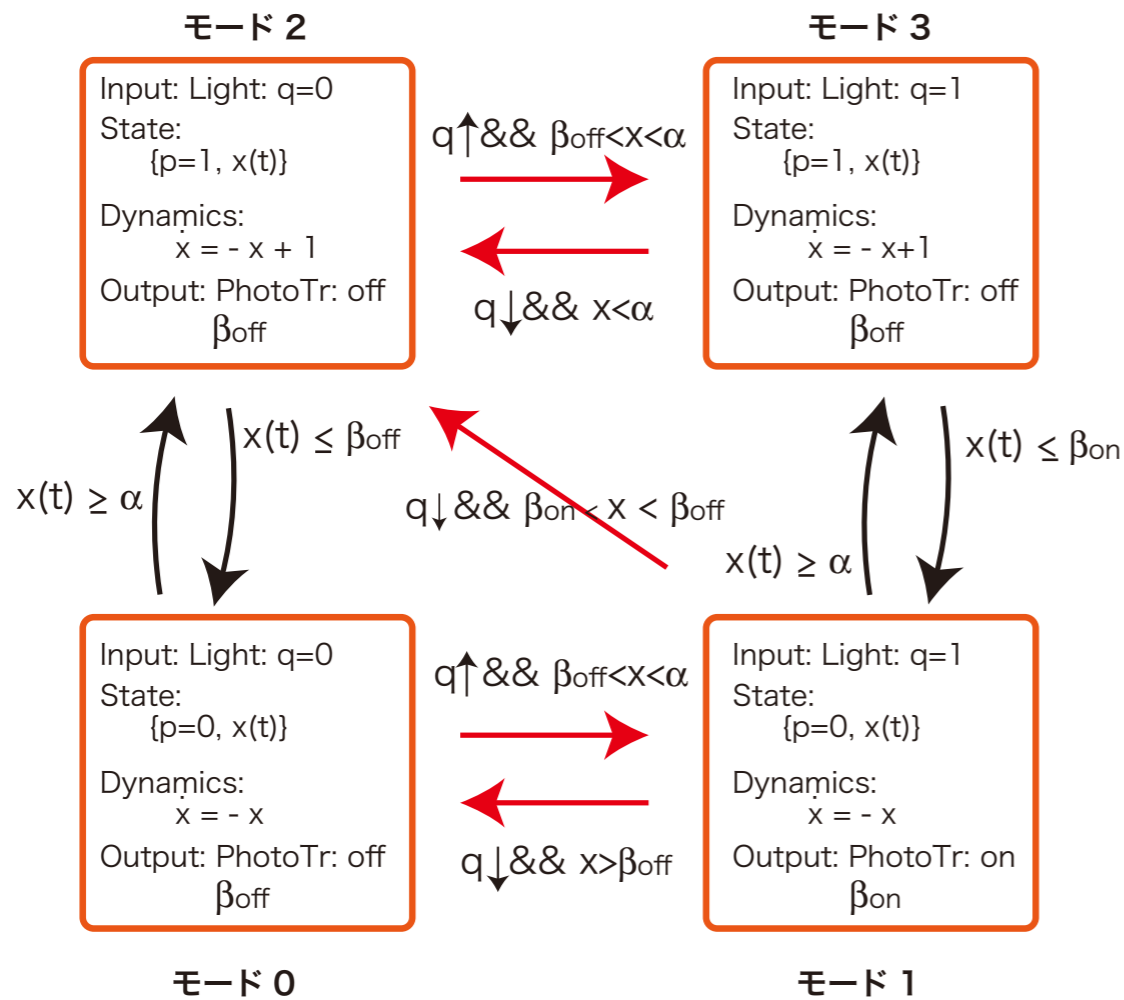
timer and phase event



Poincaré section $D_{00} \cup D_{20} \cup D_{21}$

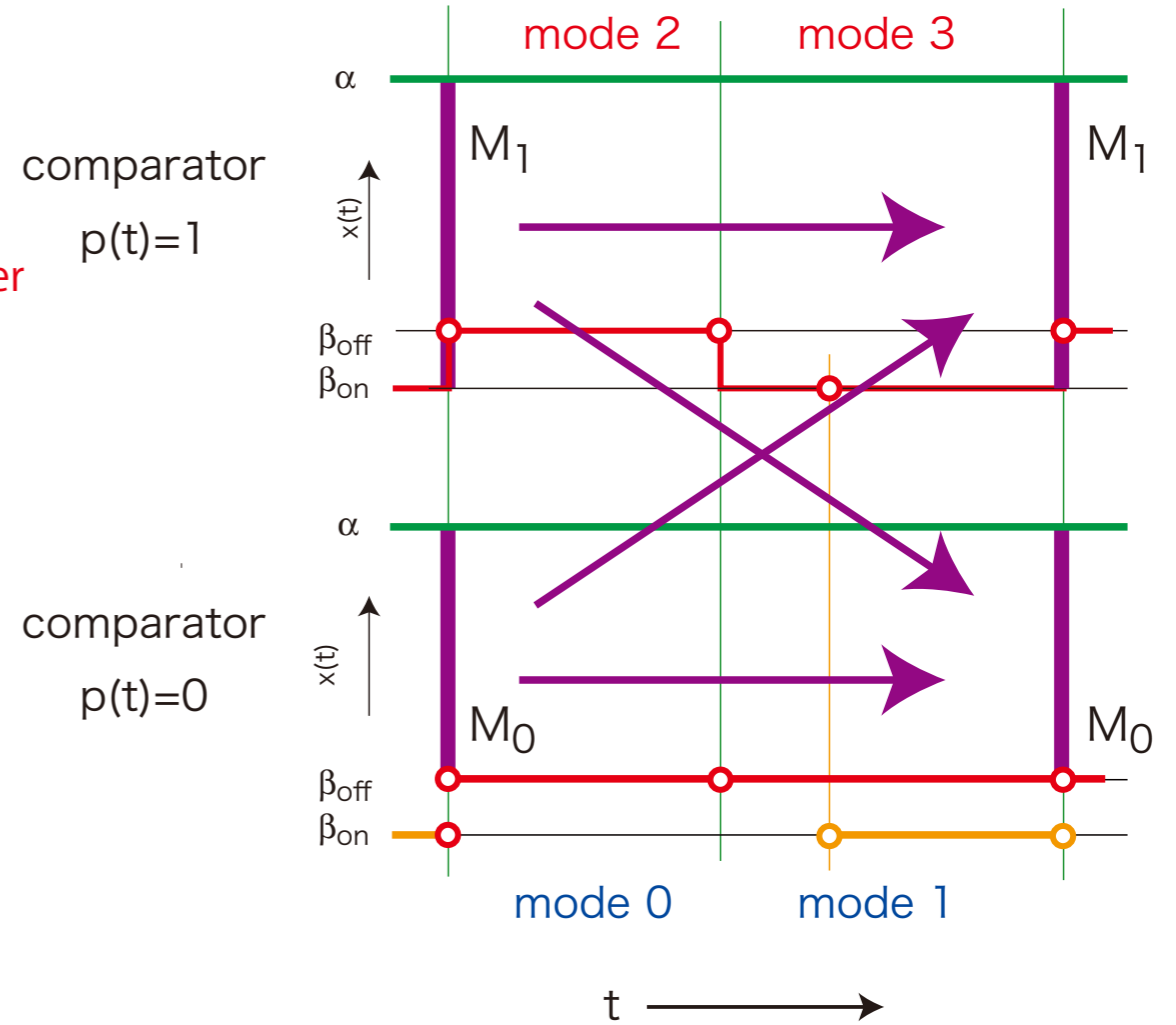
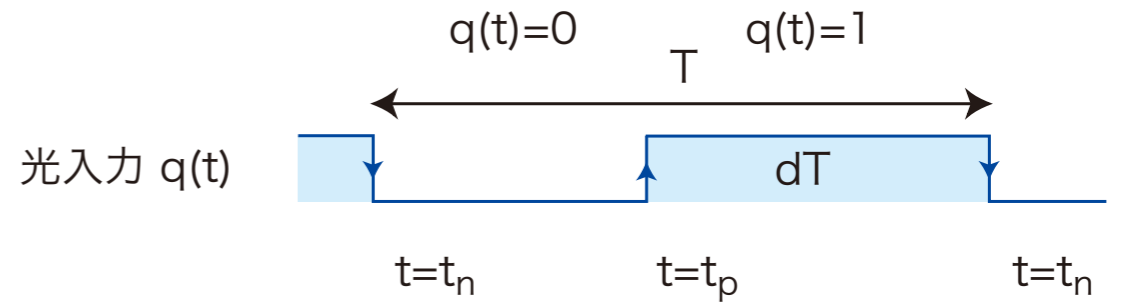
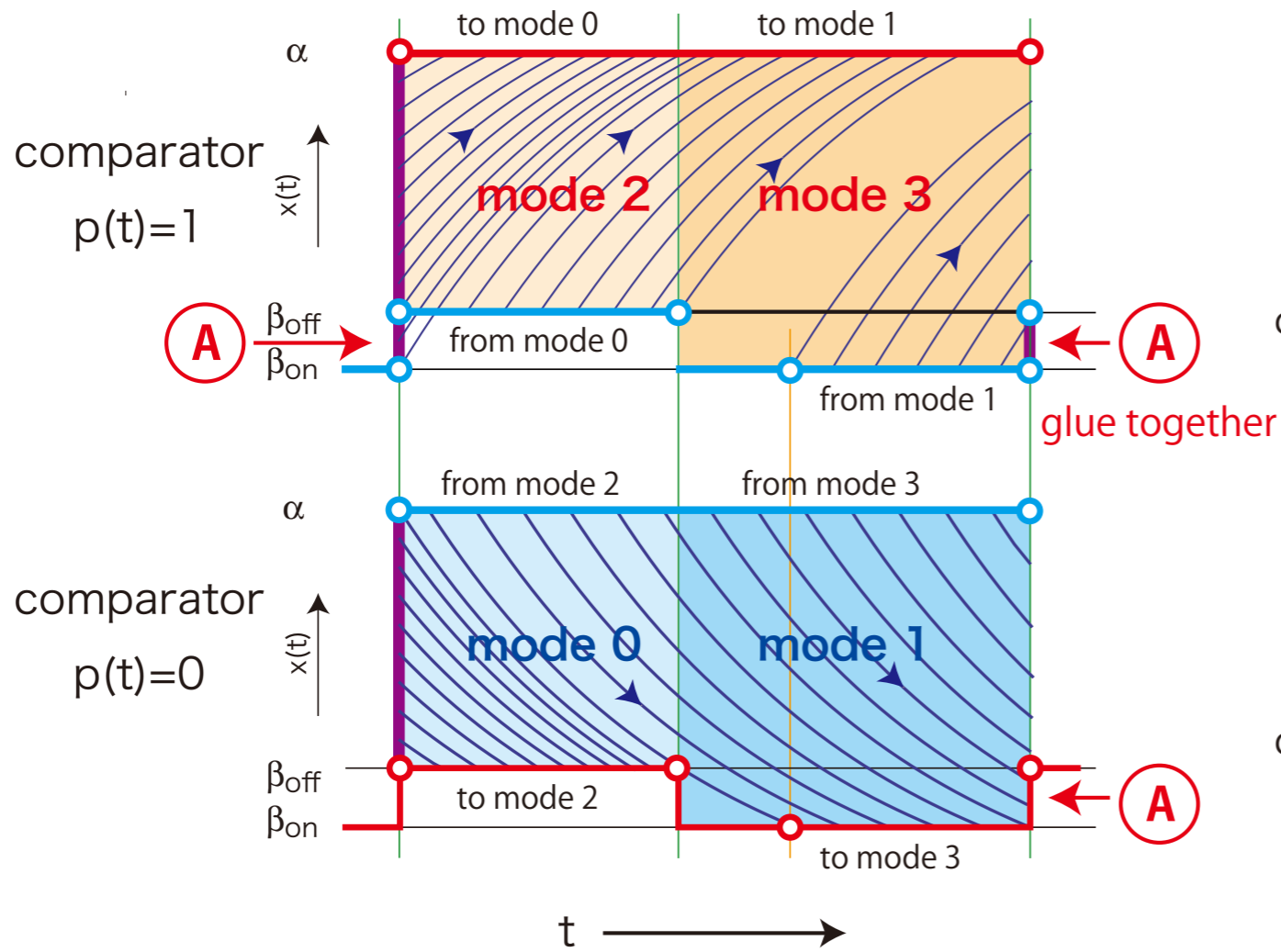


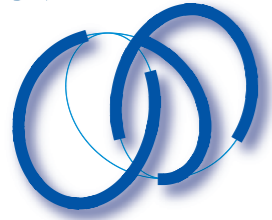
各モードのflowの様子



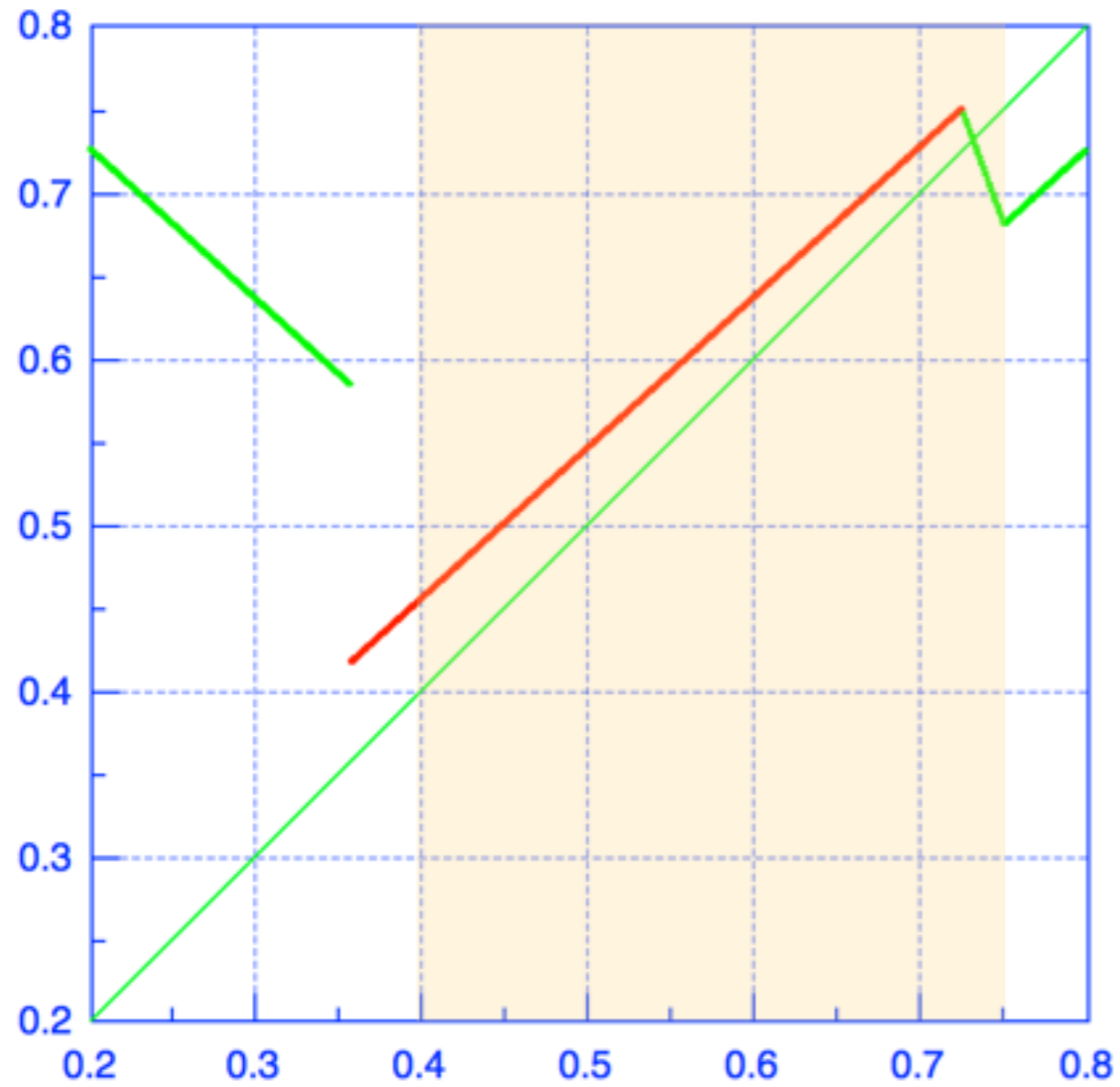


貼合せ多様体は？

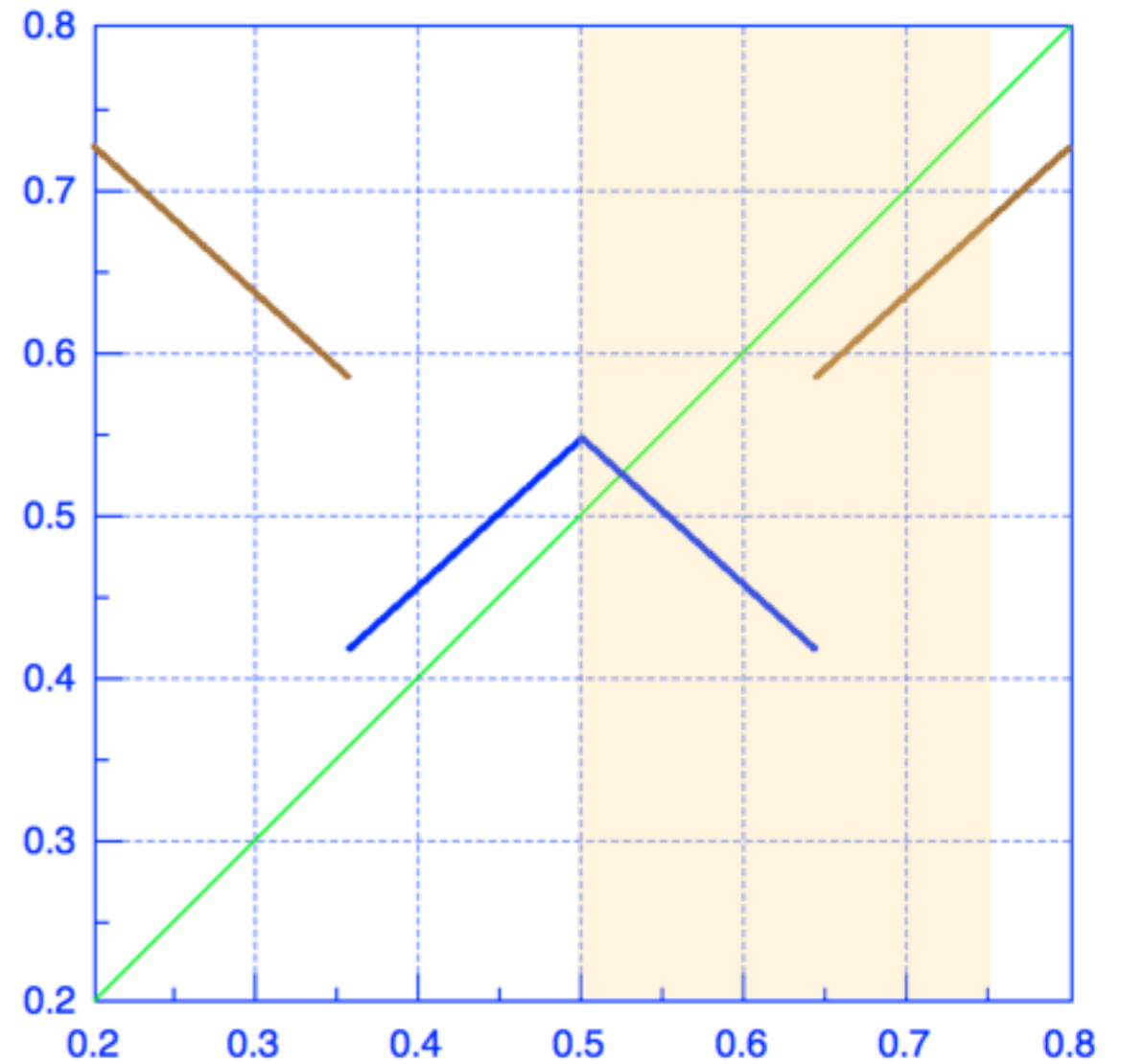




Example of Poincaré map



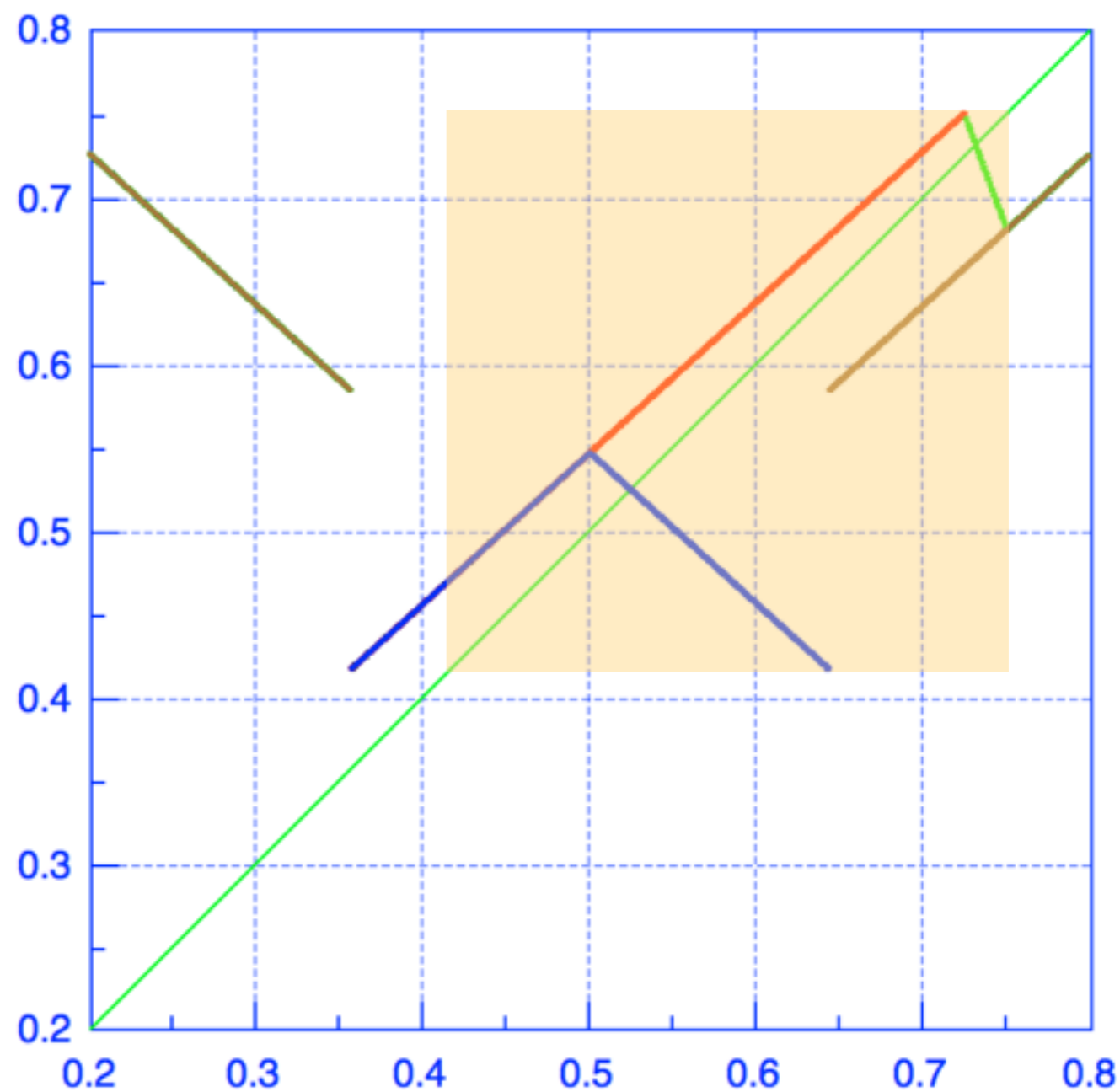
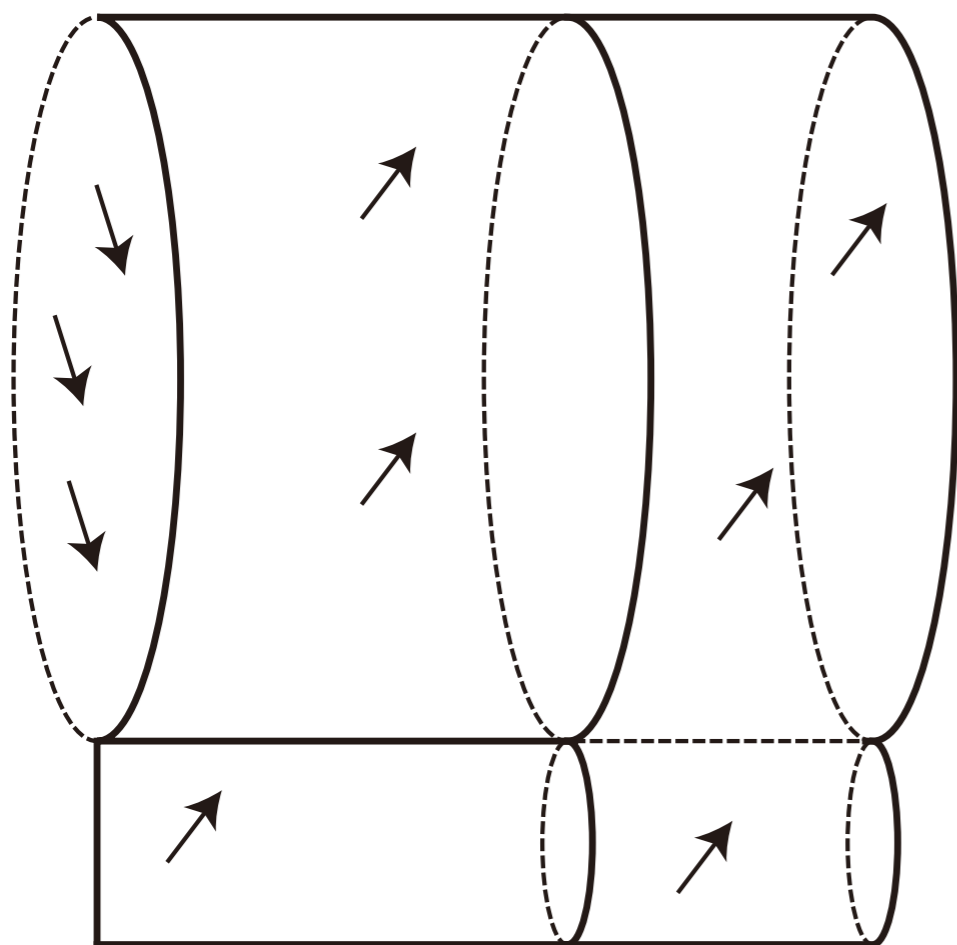
red: $M_I \rightarrow M_I$,
green: $M_I \rightarrow M_0$



brown: $M_0 \rightarrow M_0$,
blue: $M_0 \rightarrow M_I$

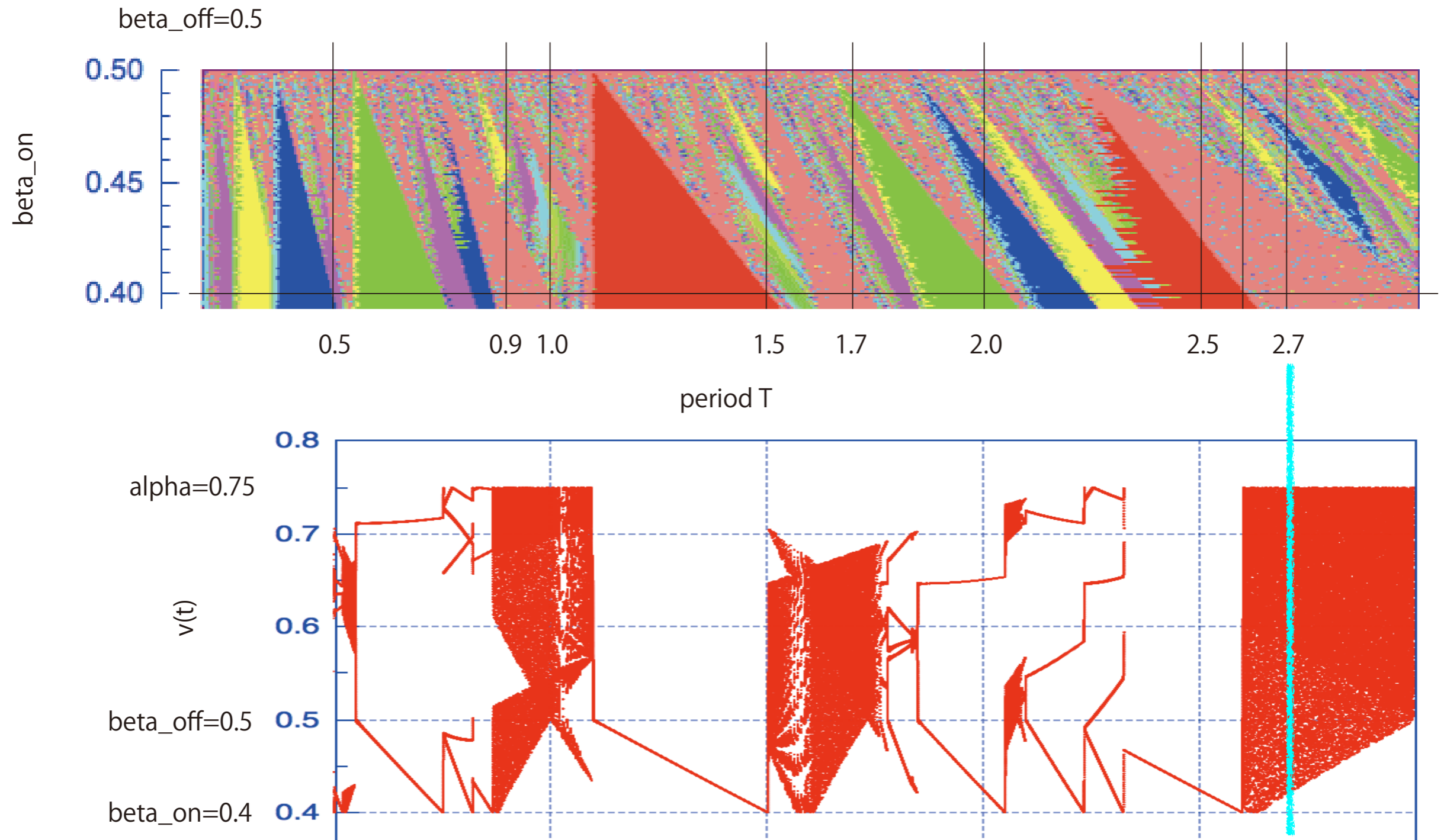


Poincaré map on torus-like surface



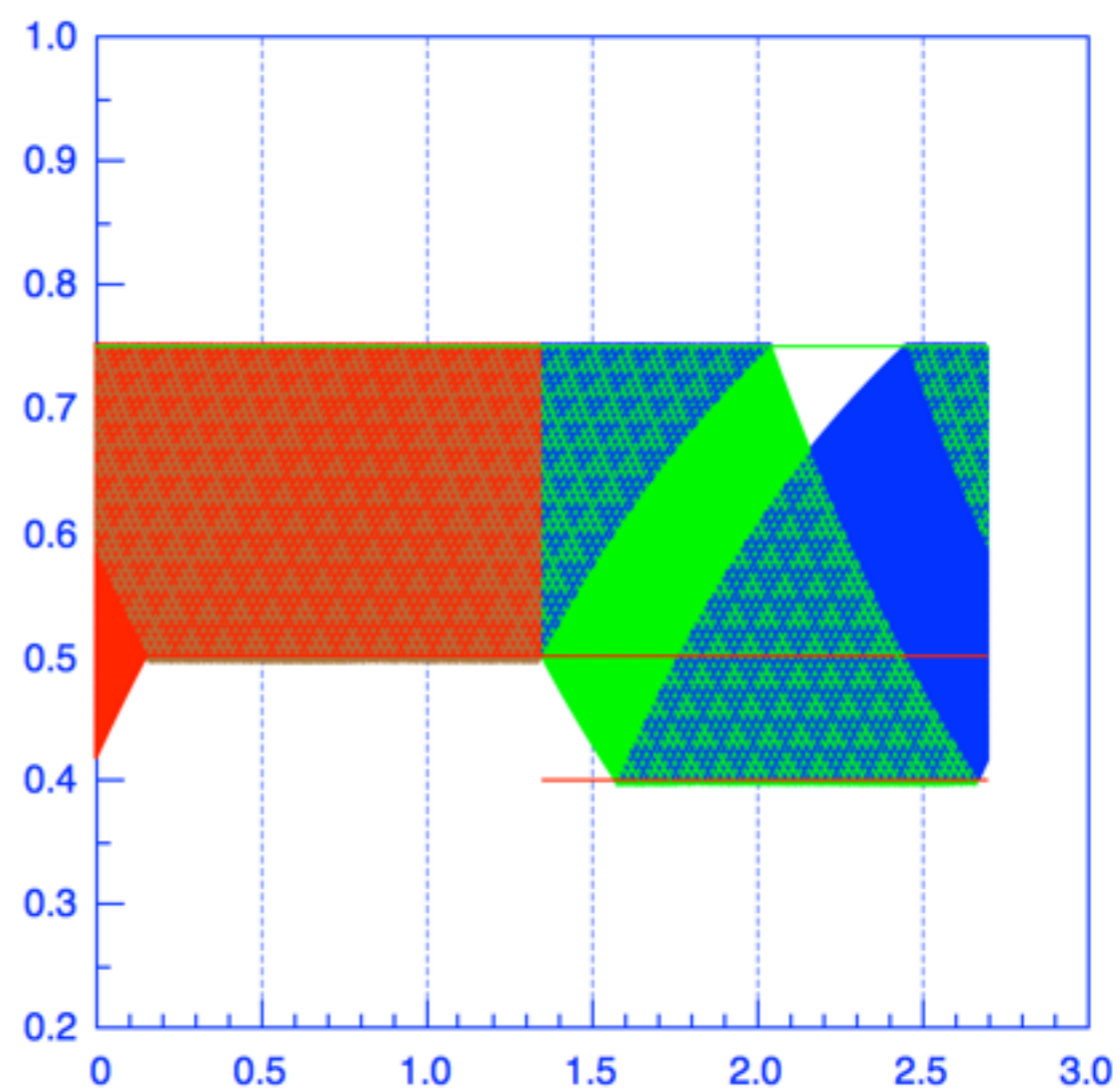
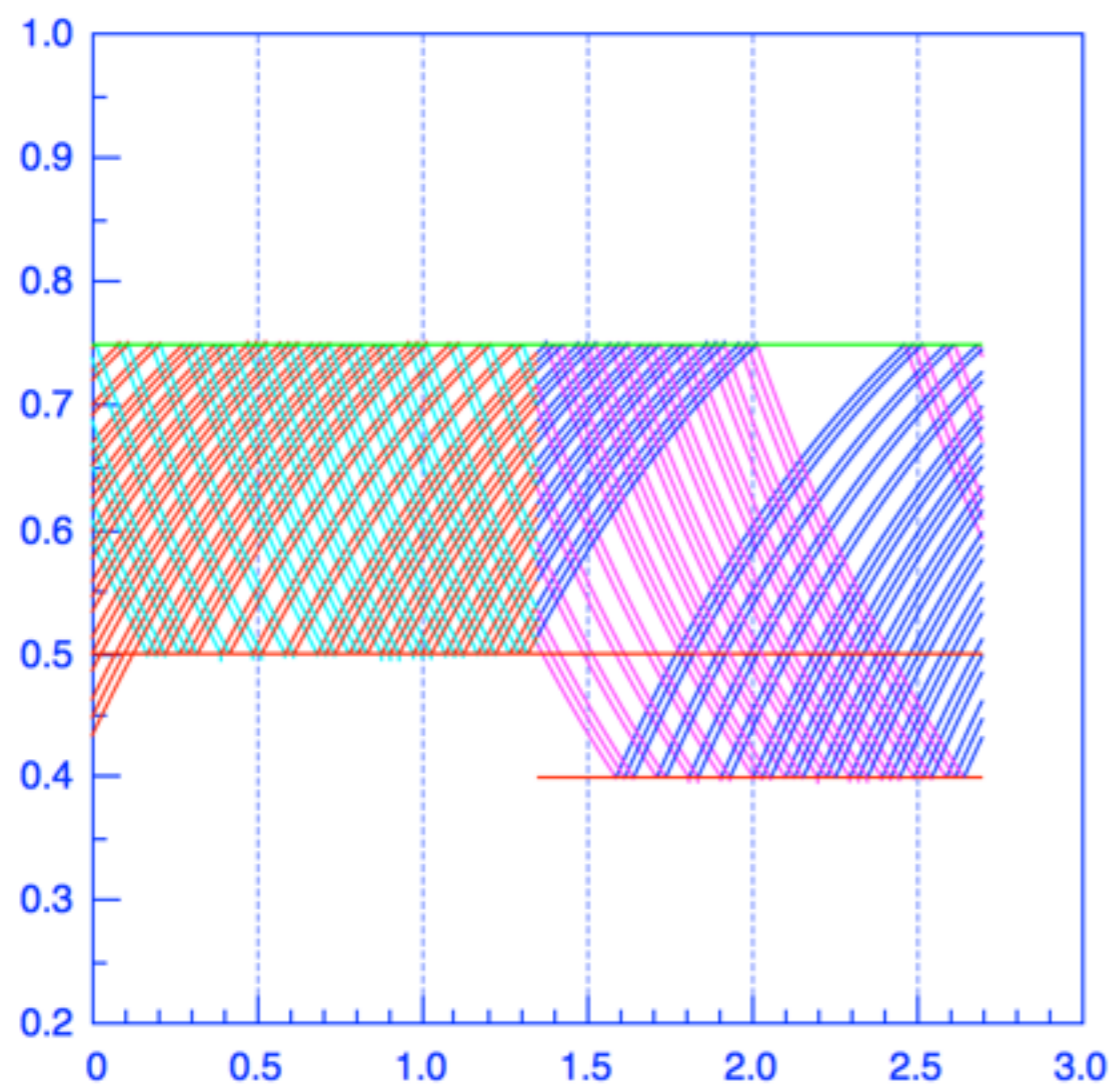


bifurcation diagrams: duty cycle=0.5



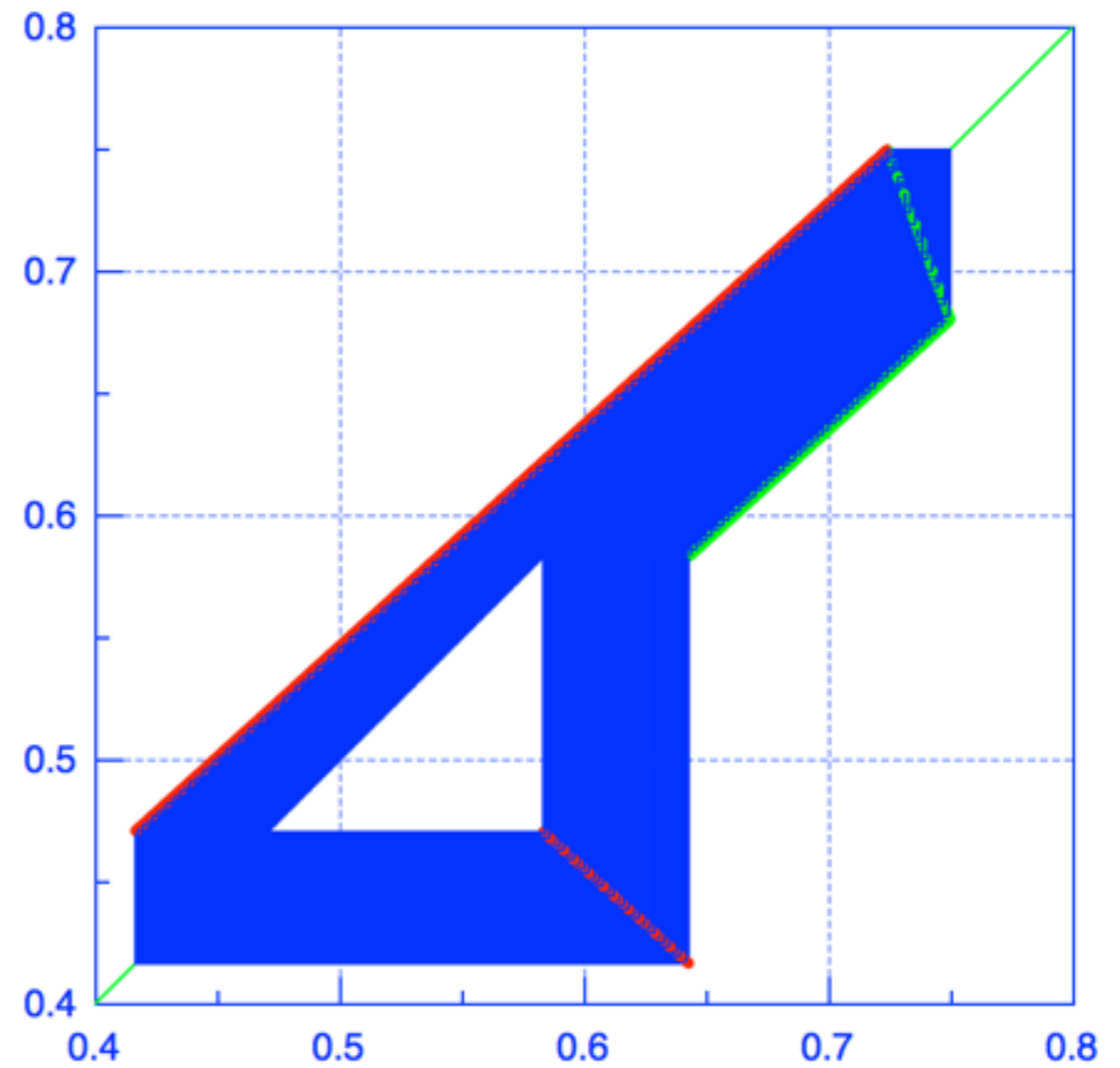
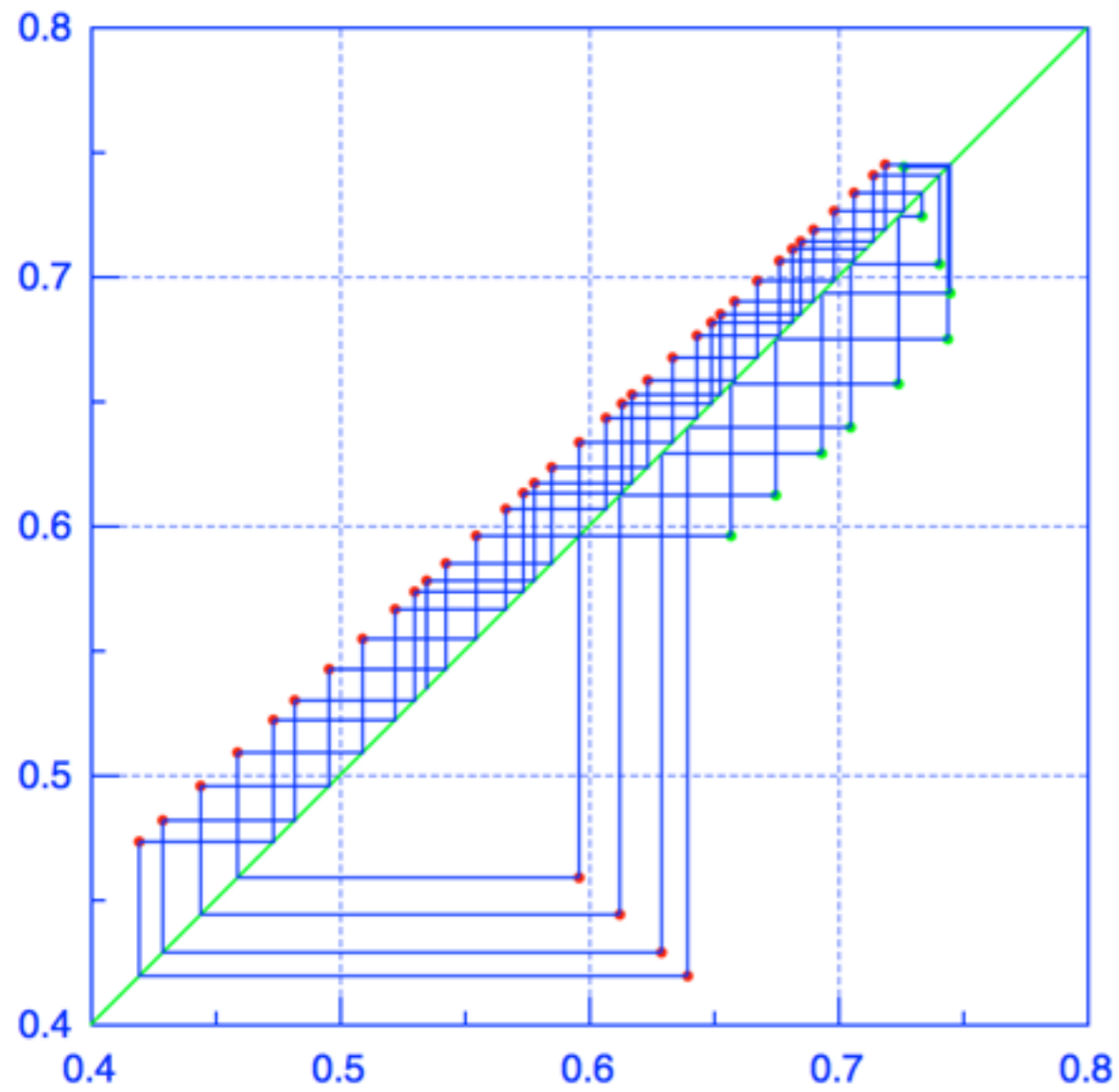


waveform for $T=2.7$





mapping trajectories for $T=2.7$





2つのLEDホタルの同期



回路例 5 : OCO type A [4: 1]

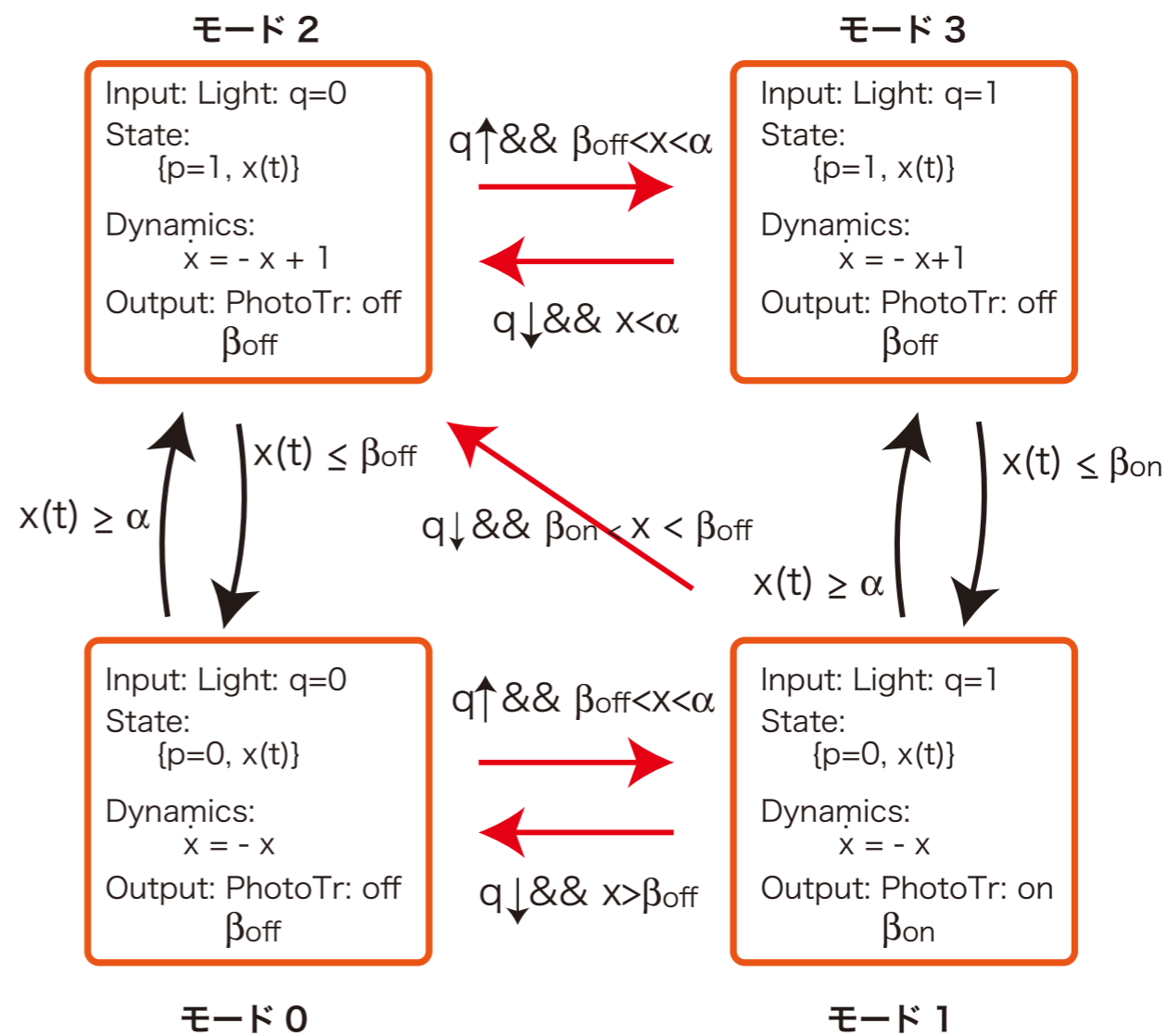
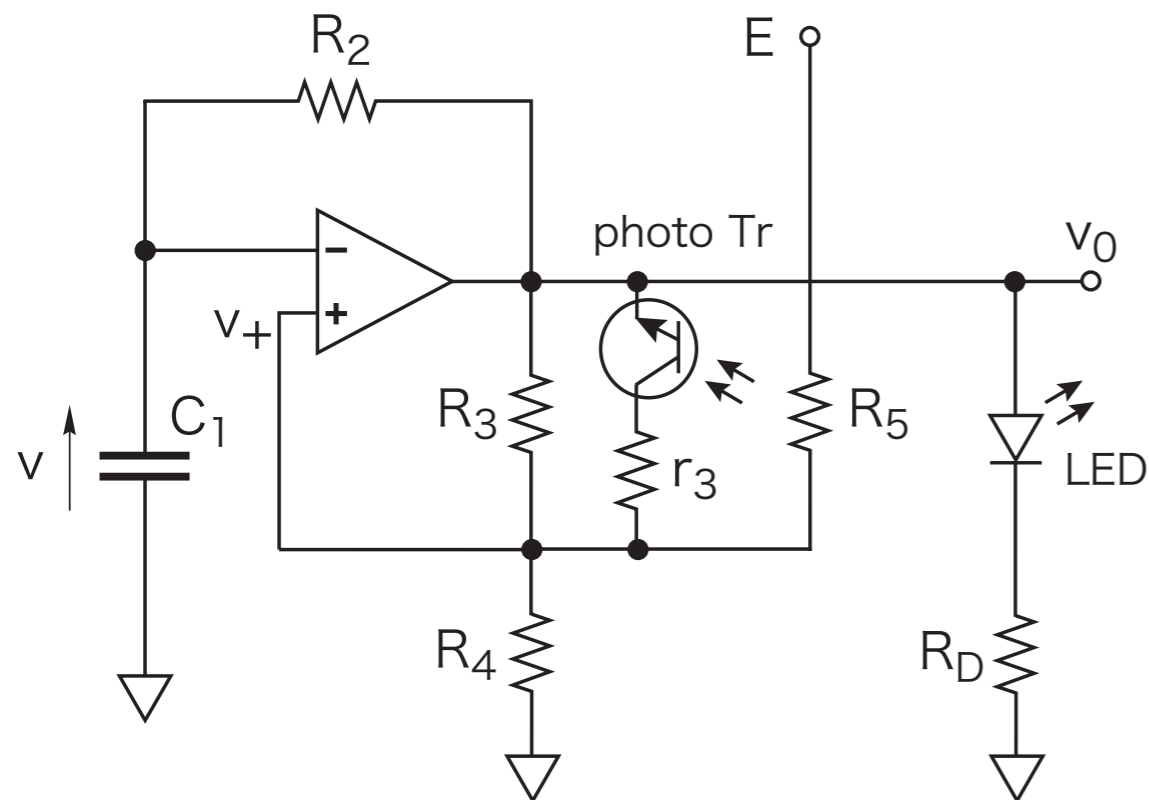
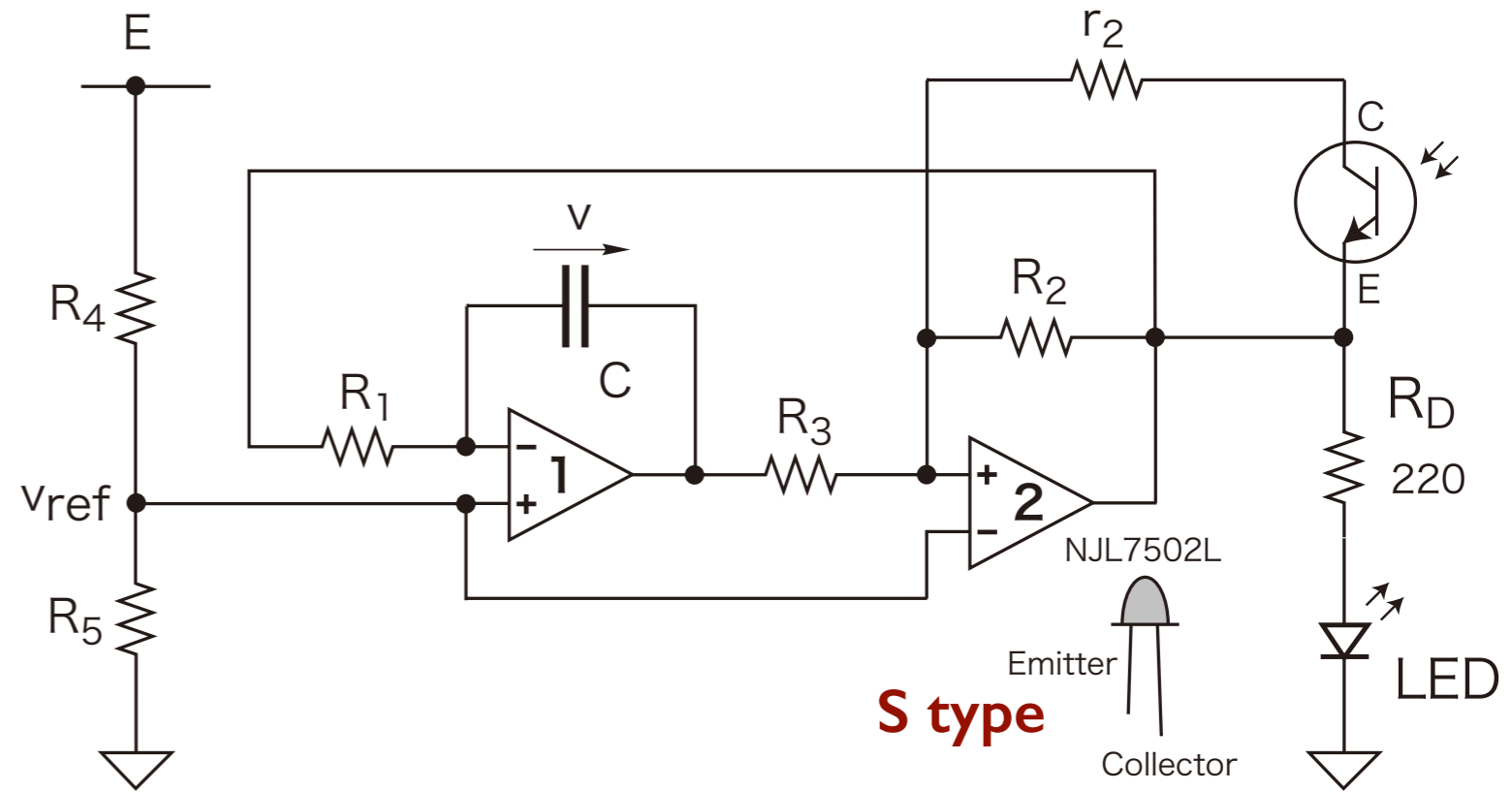
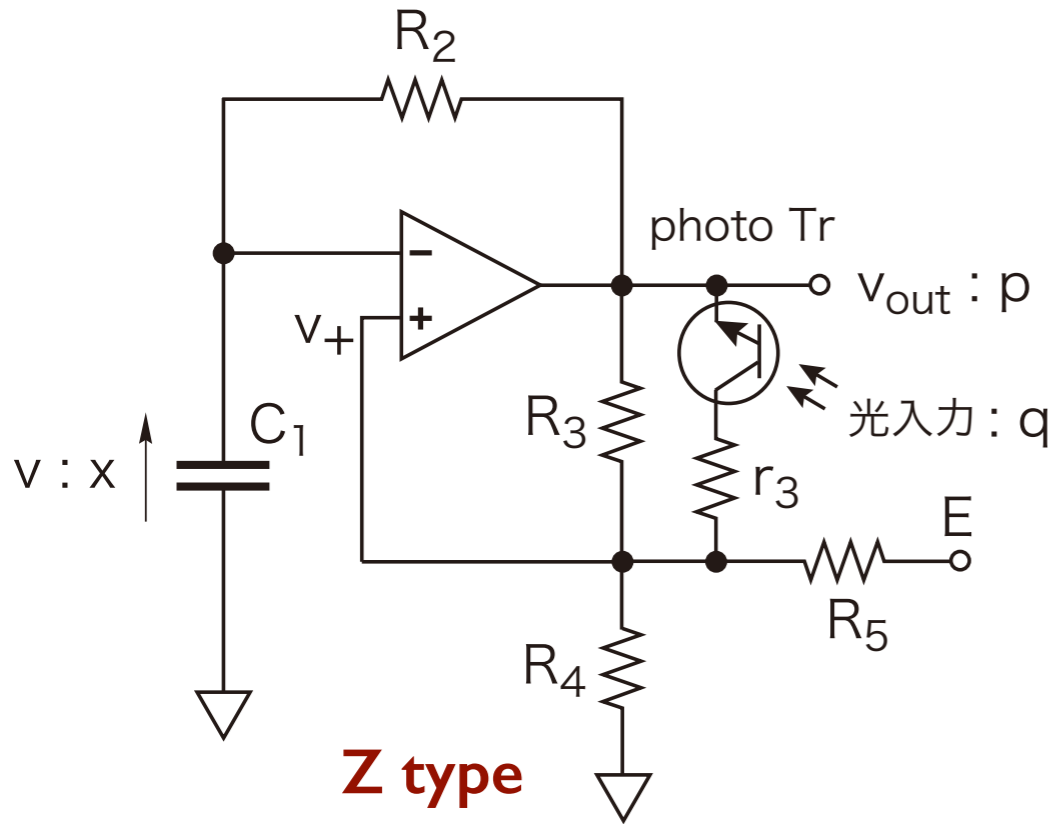


photo Tr		comparator : p	
		on:1	off:0
光入力 : q	on:1	off:0	on:1
	off:0	off:0	off:0

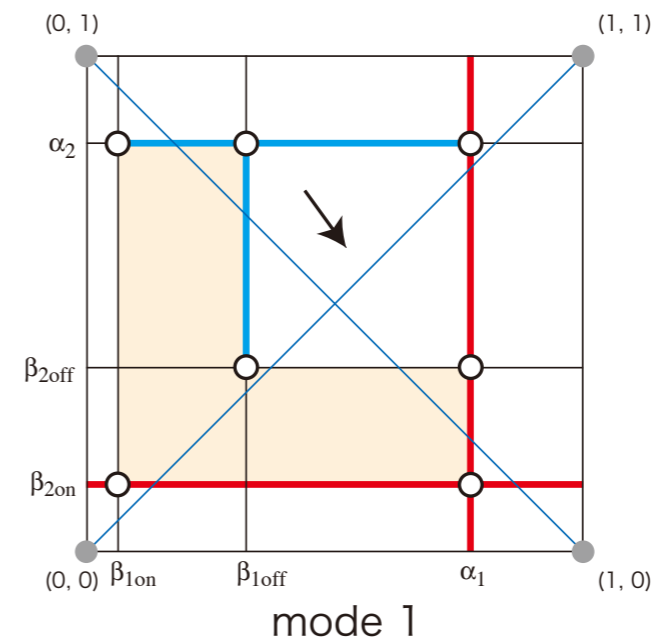
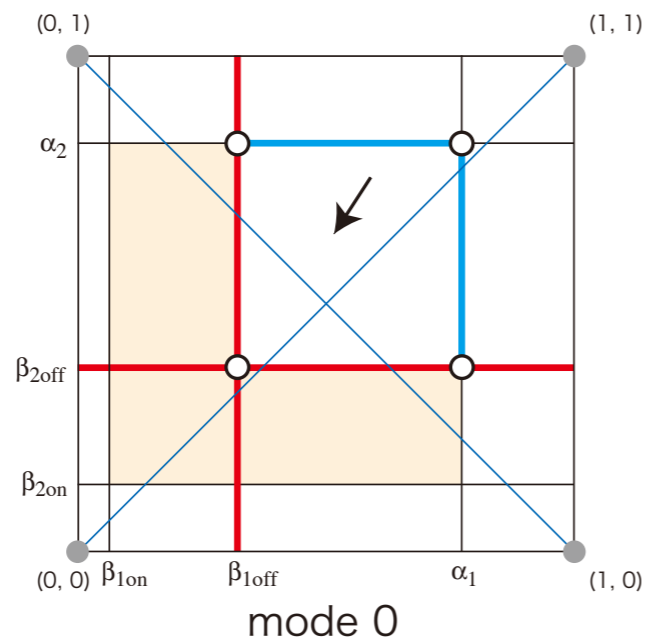
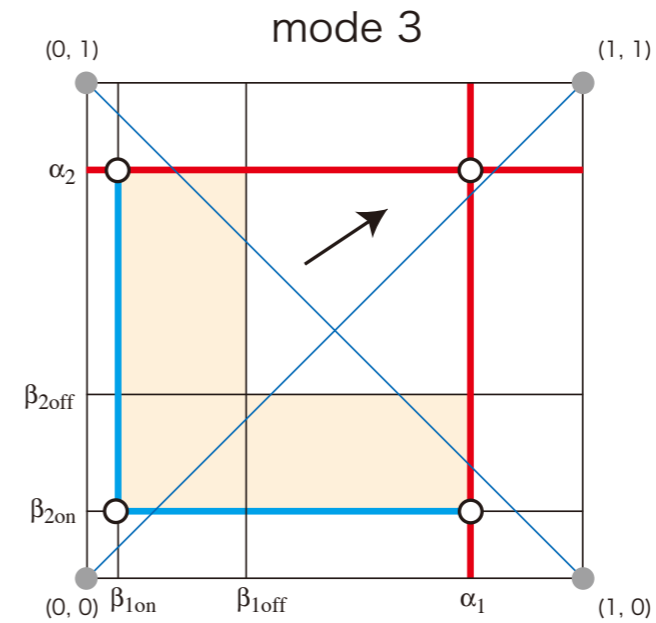
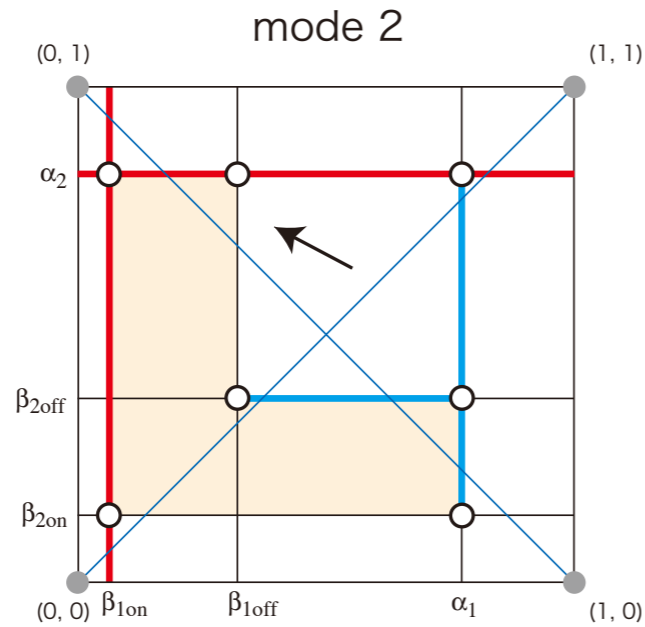


LED firefly oscillators



RC Square Wave Oscillator

Triangular Wave Oscillator



mode0 : $(q, p) = (0, 0)$, $dx/dt+x = 0$, $dy/dt+y=0$

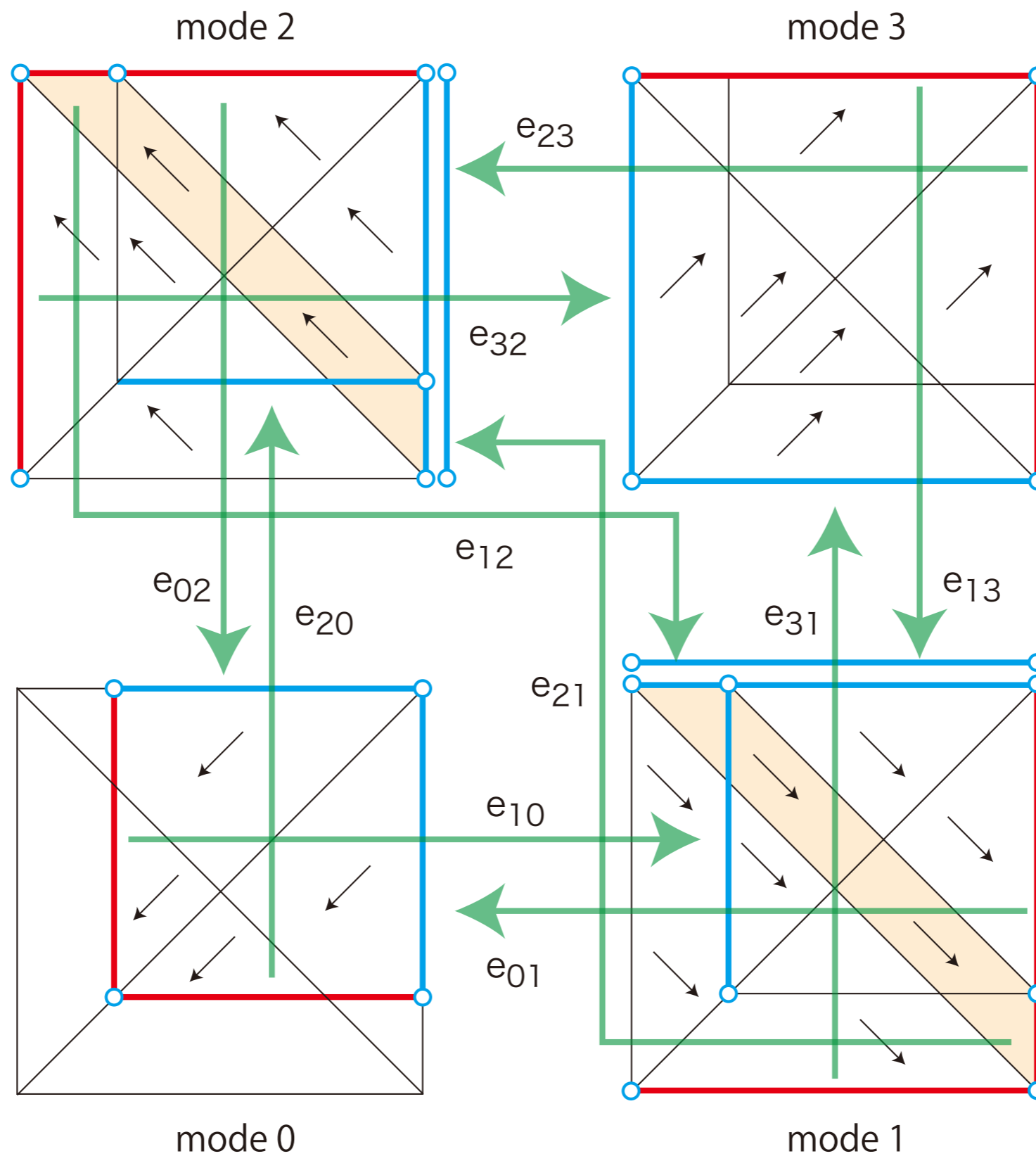
mode1 : $(q, p) = (1, 0)$, $dx/dt+x = 1$, $dy/dt+y=0$, β_{2_on}

mode2 : $(q, p) = (0, 1)$, $dx/dt+x = 0$, $dy/dt+y=1$, β_{1_on}

mode3 : $(q, p) = (1, 1)$, $dx/dt+x = 1$, $dy/dt+y=1$

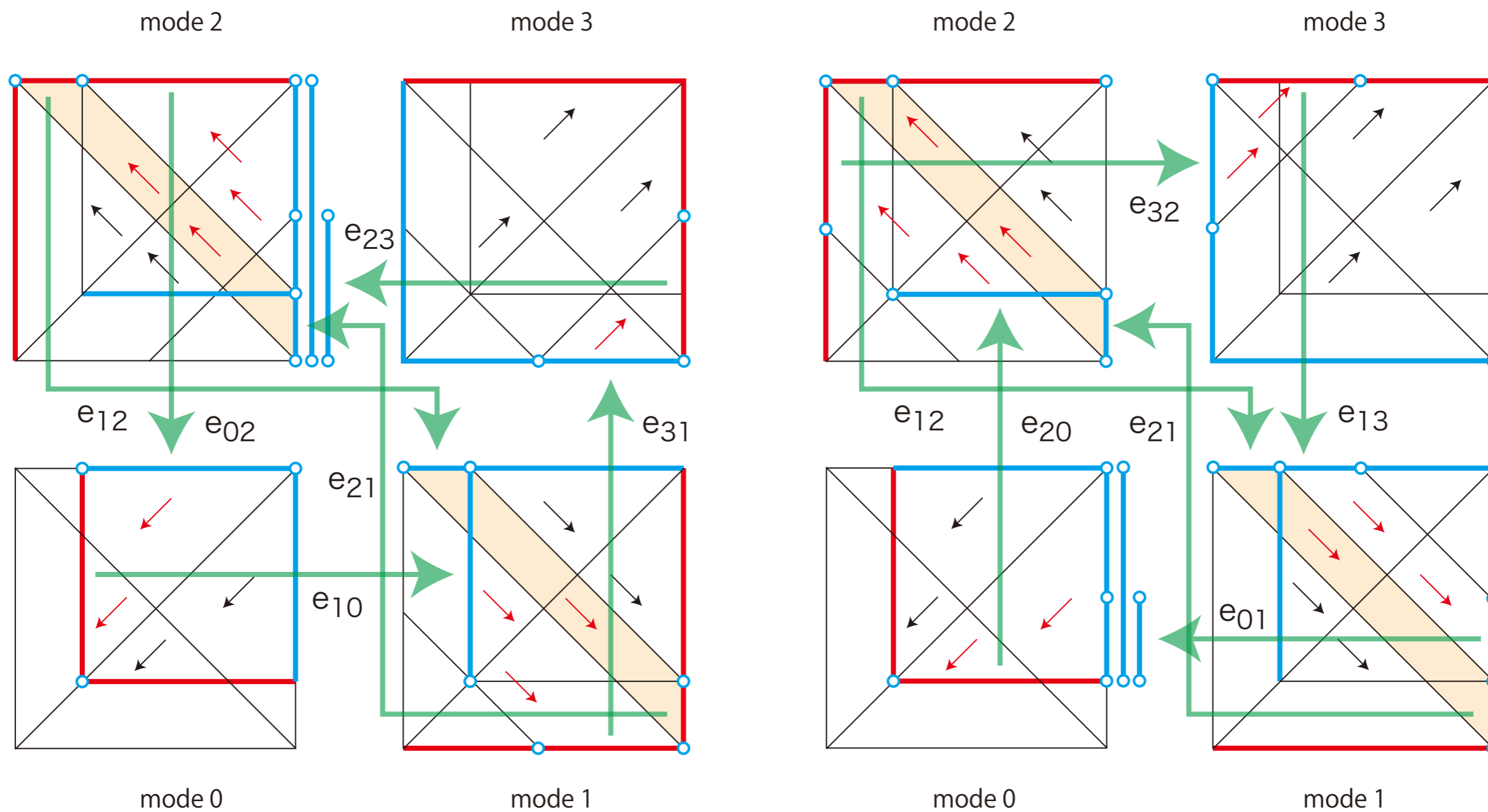


area preserving map



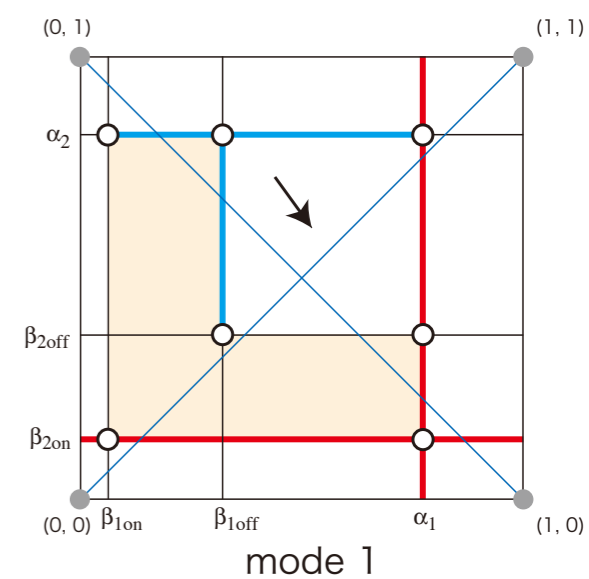
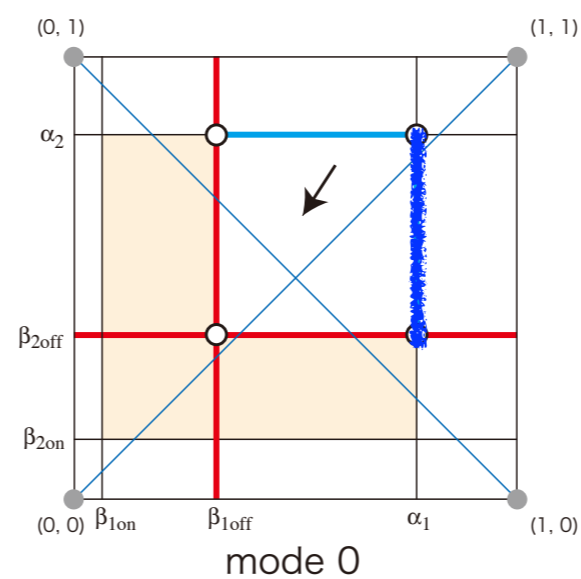
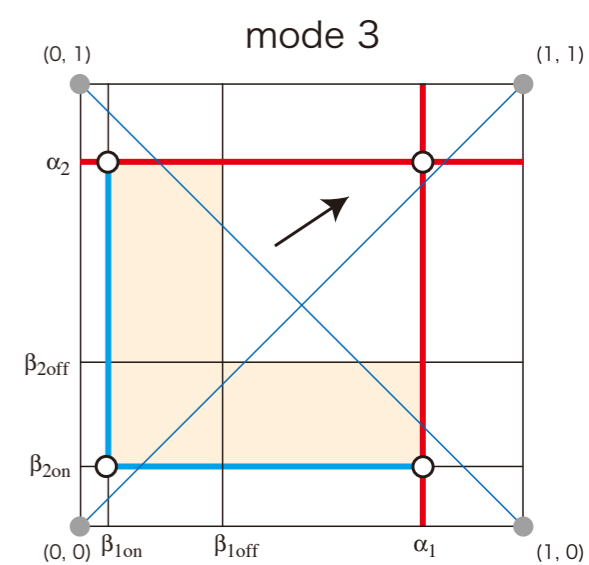
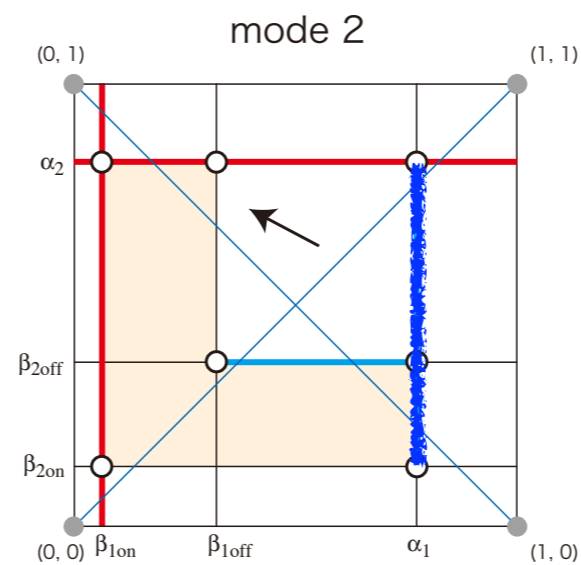
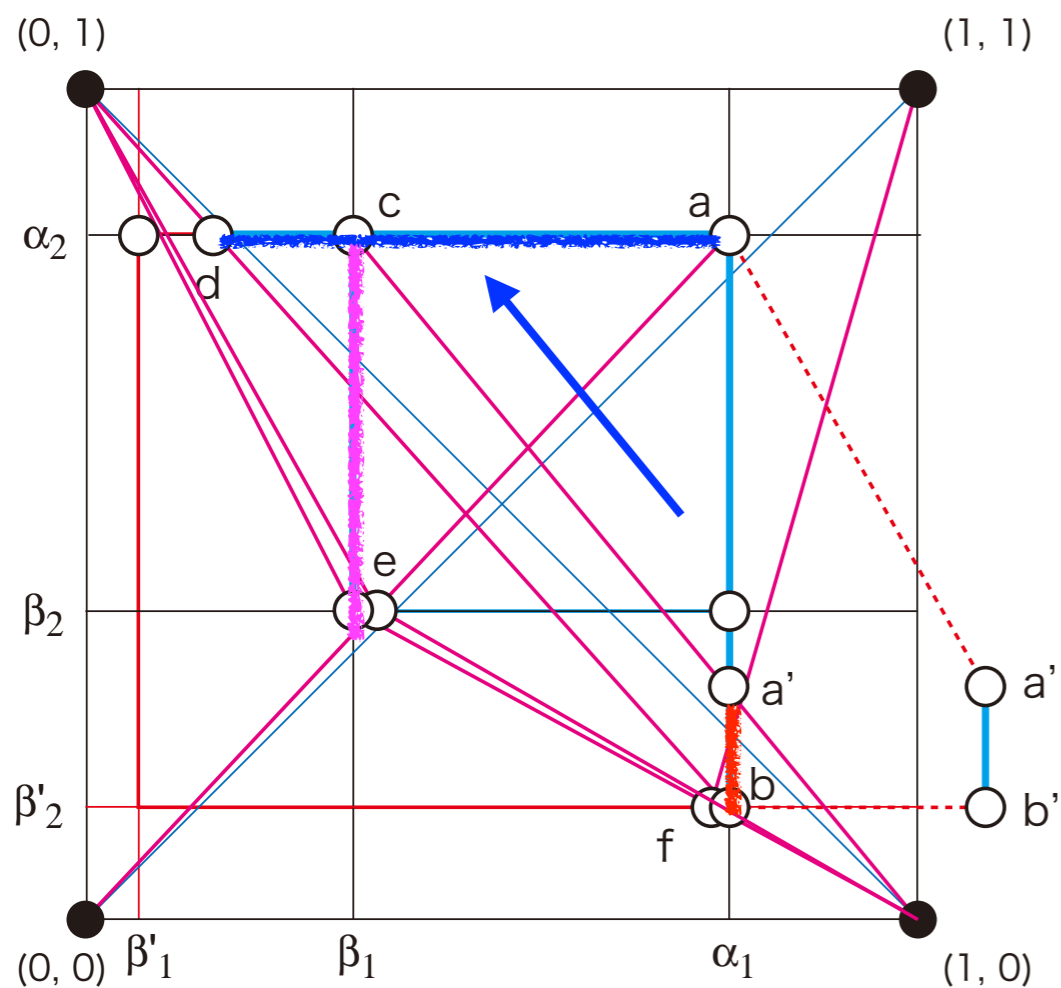


Poincaré section and mapping



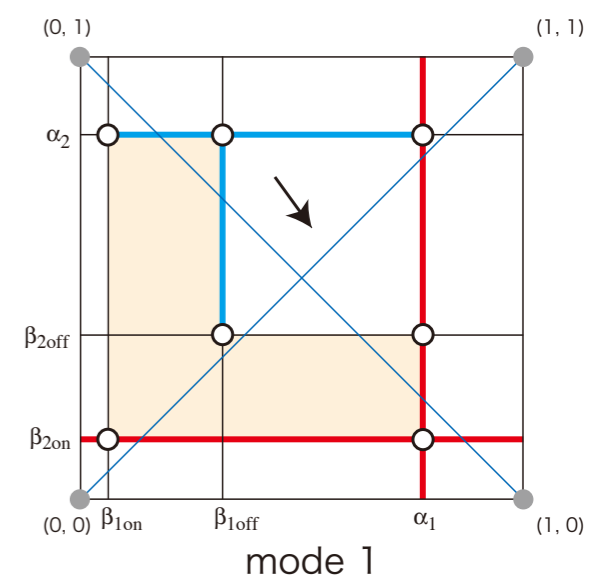
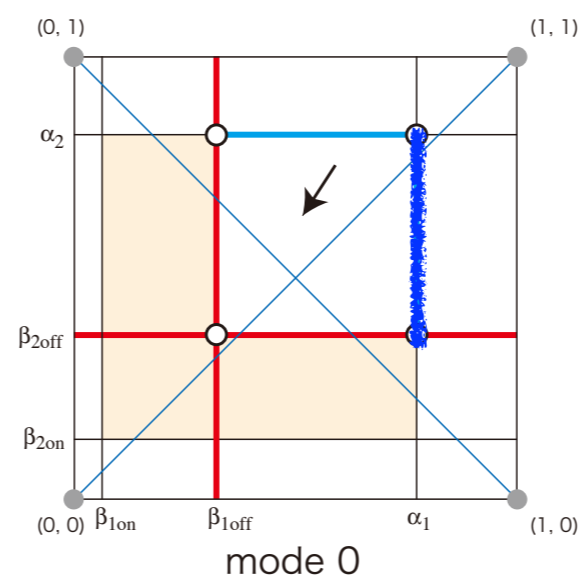
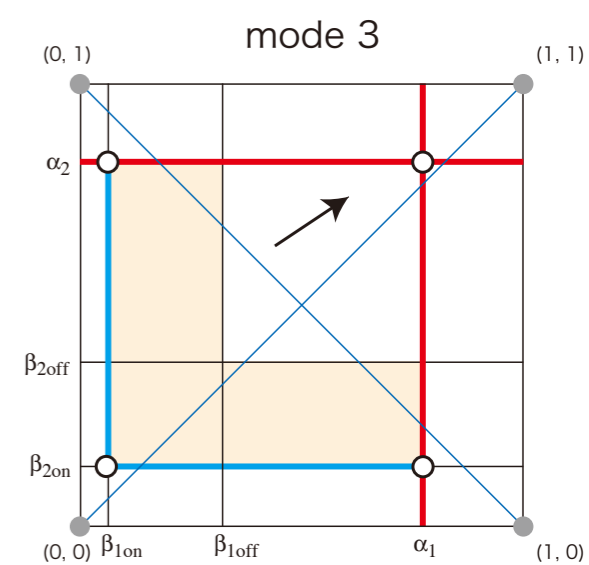
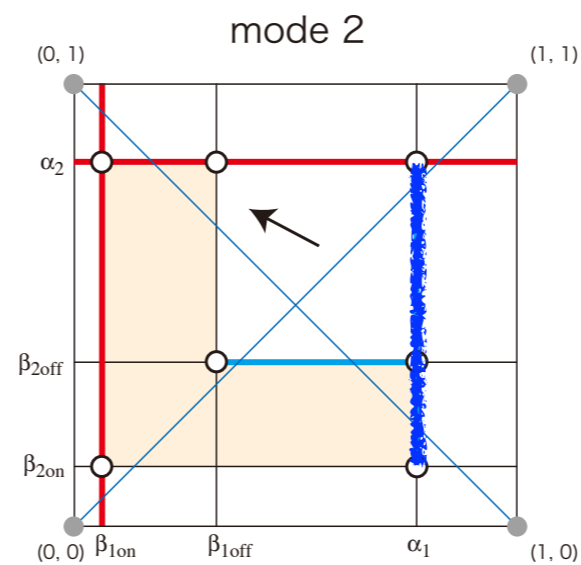
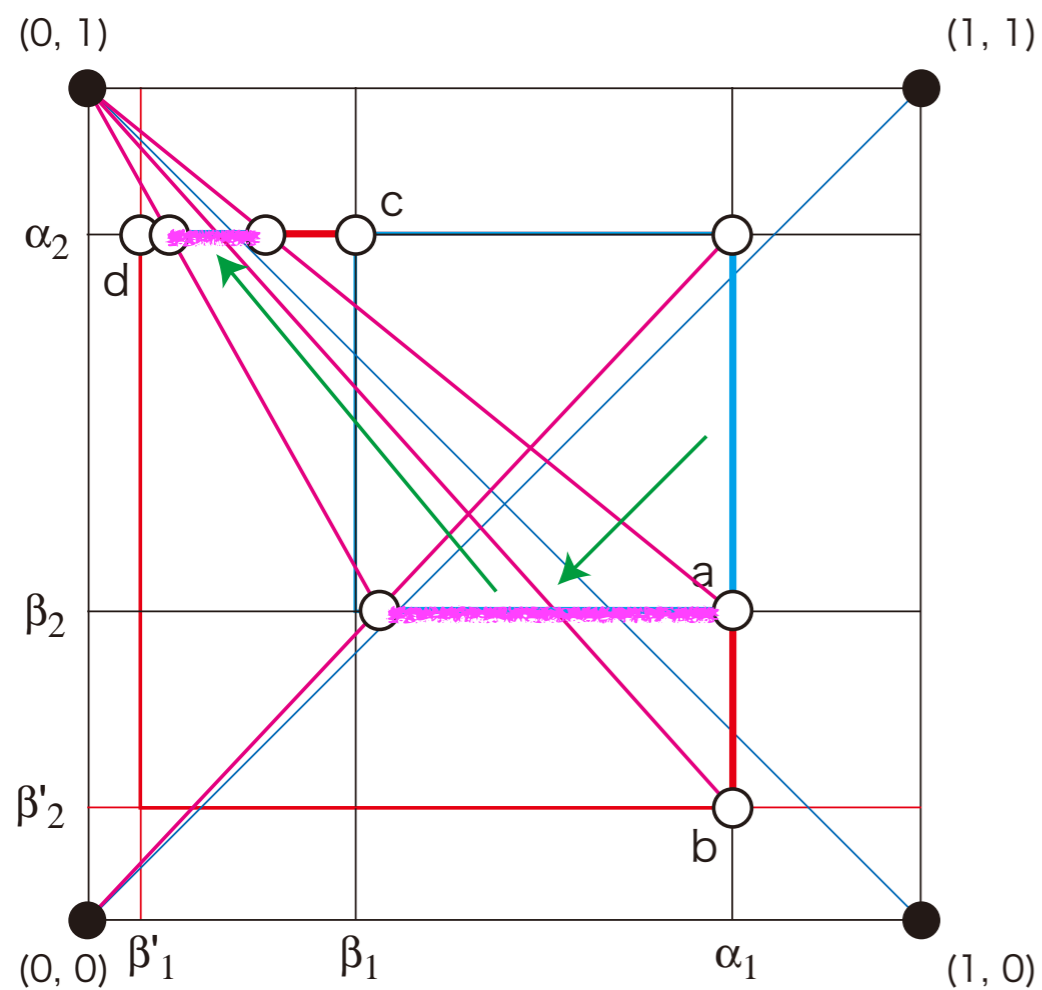


Poincaré断面は同期領域に沈み込む





Poincaré断面は同期領域に沈み込む





ハイブリッド回路の定性的解析法

1. 数学モデルの定式化

a) digital part: mode数の決定, mode遷移の導出

b) analog part: vector field, 回路方程式の導出

c) ADC part: mode数の決定, mode遷移の導出

c-1) border mfd, phase event, flowの定義域

c-2) flowの到着集合, 出発集合を求めmode遷移を再確認

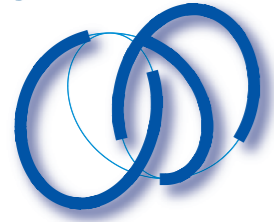
c-3) timer eventを求めmode遷移を再確認

2. 貼合せ多様体をつくる

Poincaré断面を定義し, Poincaré写像をつくる

3. 運動の解析

各種不変集合を求め, 安定性や分岐を考察する



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[4] N.N. Leonov; Discontinuous map of the straight line, *Dokl. Akad. Nauk. SSSR*, 143(5) 1962, pp. 1038-1041.

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[2] Н. Н. Леонов; О разрывном кусочно-линейном точечном преобразовании прямой в прямую, *Радиофизика*, Том. 3, *№*. 3, 1960, pp.496-510.

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