



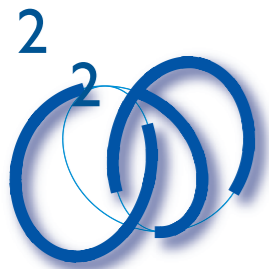
上田研 ゼミ

LEDホタルの定性的解析法

—相互結合系を例として—

川上 博

2014(H26).05.30



話の流れ

I 記法・用語・諸定義の相互確認

1. 方形波発振器

- ◎ コンパレータを使ったRC方形波発振器

2. 発振器の数学モデル

- ◎ FSMとODE混合系の定式化法

3. 状態の時間発展

- ◎ FSMと力学系の定性論

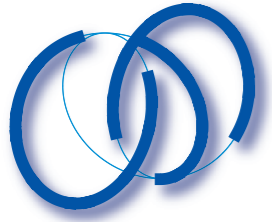
II 相互結合系の解析に関連した事項

4. 関川先生の Q&A

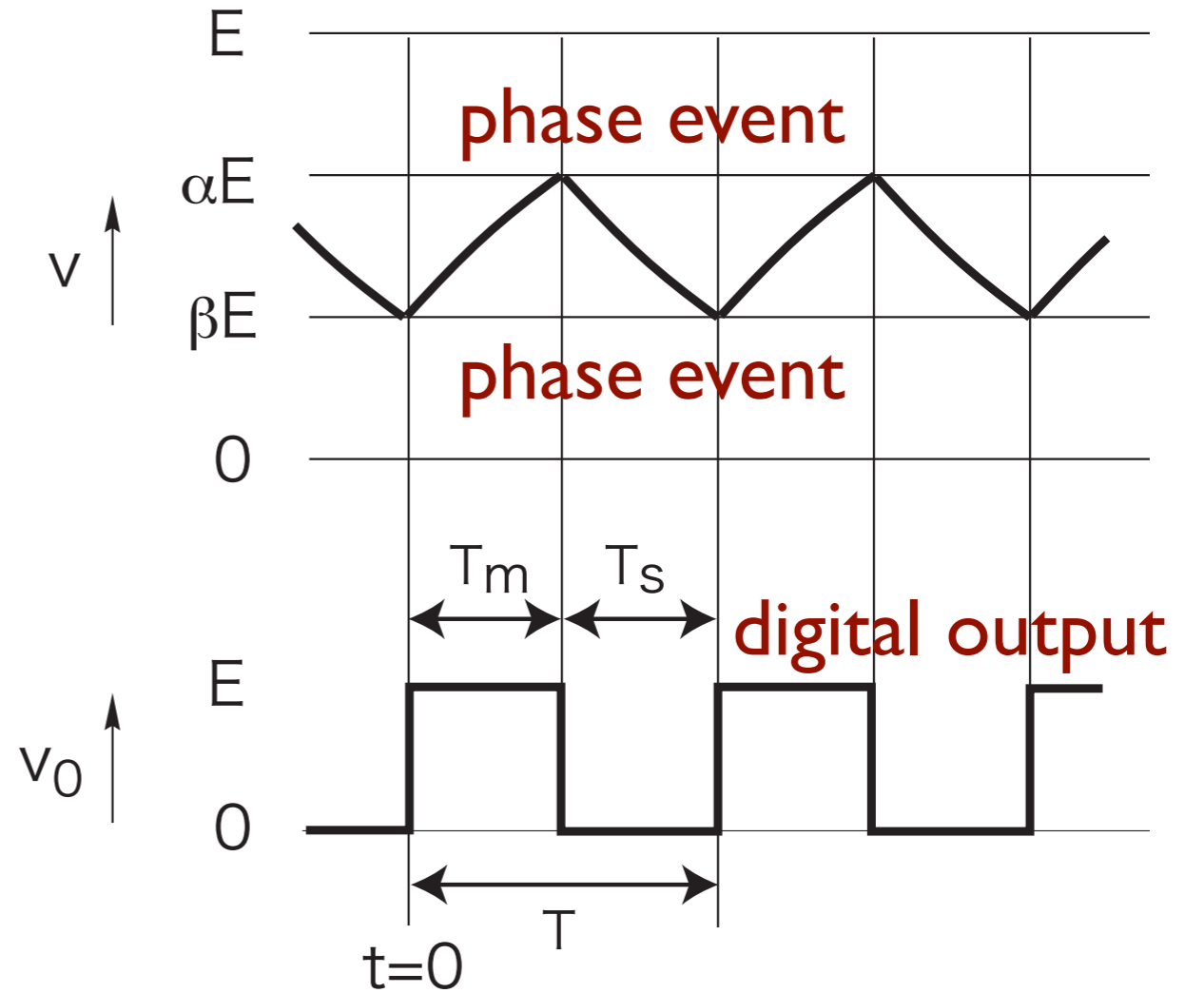
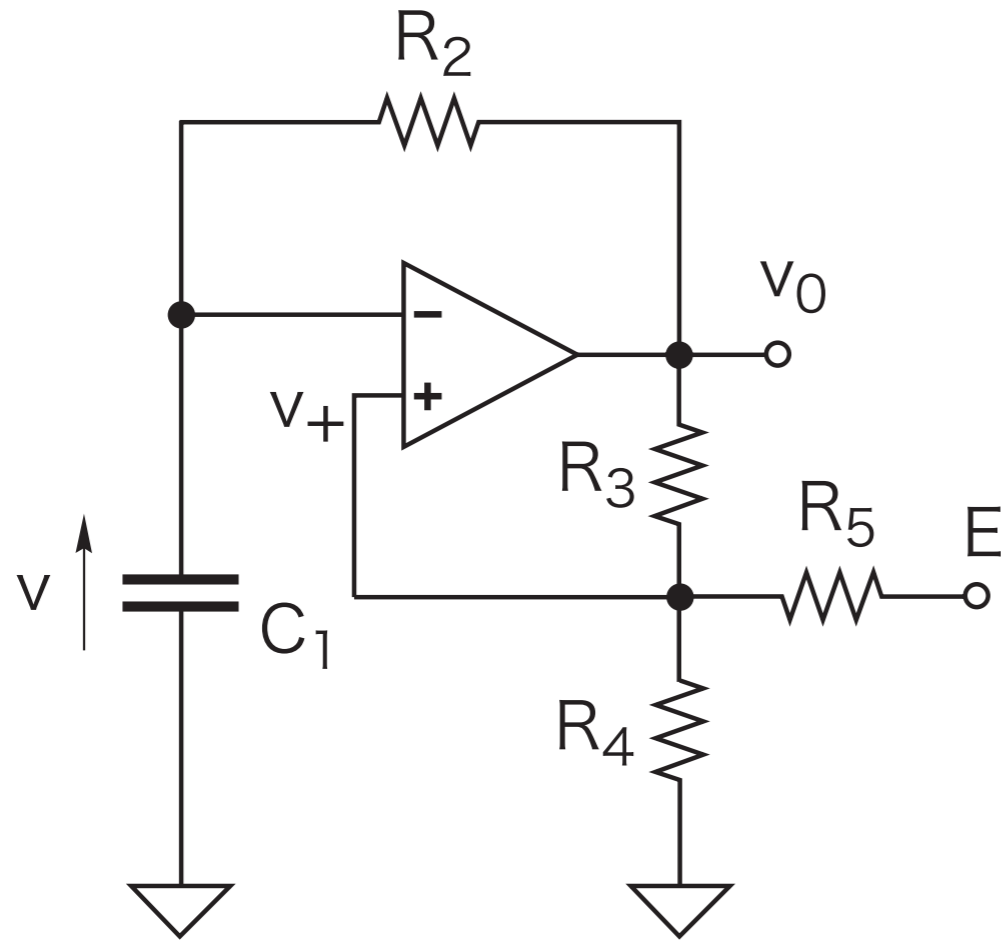
- ◎ 振動波形の不一致, oscillation death



方形波発振器



RC方形波発振器



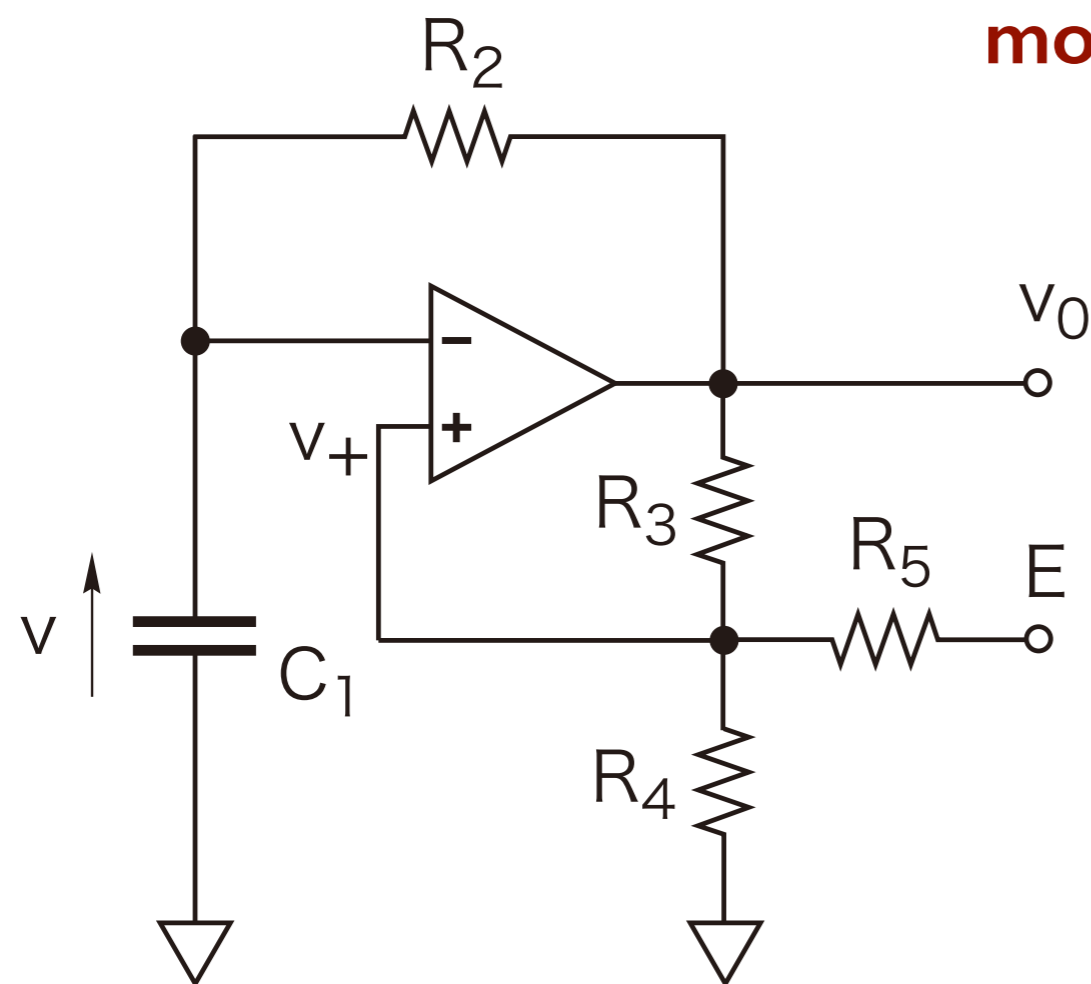
$$R_2 C_1 \frac{dv}{dt} + v = E$$

$$R_2 C_1 \frac{dv}{dt} + v = 0$$



ハイブリッド系としてみると

mode 変数: $v_0 \in F = \{0, E\}$



mode 遷移(mode transition):

mode I: $v_0 = E$

if($v \leq \beta E$) $v_0 = E$ if($v \geq \alpha E$) $v_0 = 0$

mode 0: $v_0 = 0$

phase 変数: $v(t)$

状態方程式(ode): $R_2 C_1 \frac{dv}{dt} + v = v_0$



集合 $F = \{0, 1\}$ 上のベクトル空間

F 上の加法 : $0 + 0 = 0, 0 + 1 = 1 + 0 = 1, 1 + 1 = 0$

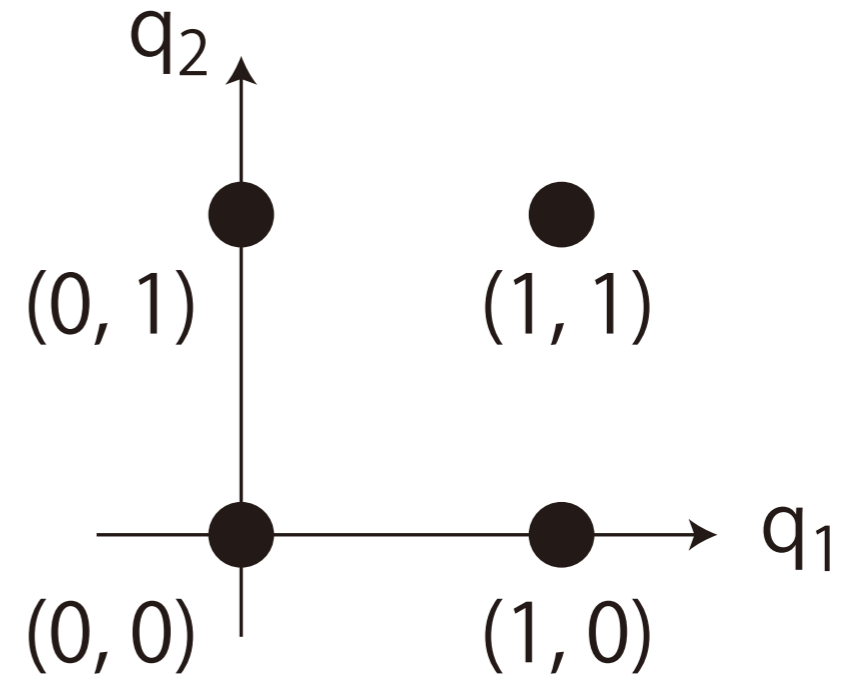
F 上の乗法 : $0 \cdot 0 = 0 \cdot 1 = 1 \cdot 0 = 0, 1 \cdot 1 = 1$

このとき, F は有限体(finite field)となる

F^m は F 上のベクトル空間(vector space)

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \in F^2$$

$$q = \underset{\text{binary}}{0bq_2q_1} = \underset{\text{decimal}}{2q_2 + q_1}$$





ハイブリッド系は2種類の状態をもつ

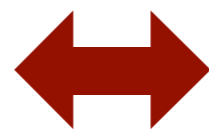
phase変数とmode変数: $\tau = \frac{1}{R_2 C_1} t$, $x = \frac{v}{E}$, $x_+ = \frac{v_+}{E}$, $q = \frac{v_{out}}{E}$

vector field:

$$\frac{dx}{dt} + x = q, \quad x \in R, \quad q \in F = \{0, 1\}$$

mode 遷移(mode transition):

$$q(n+1) = f(q(n), x(t))$$



if($x \leq \beta$)

$q = 1$



if($x \geq \alpha$)

$q = 0$



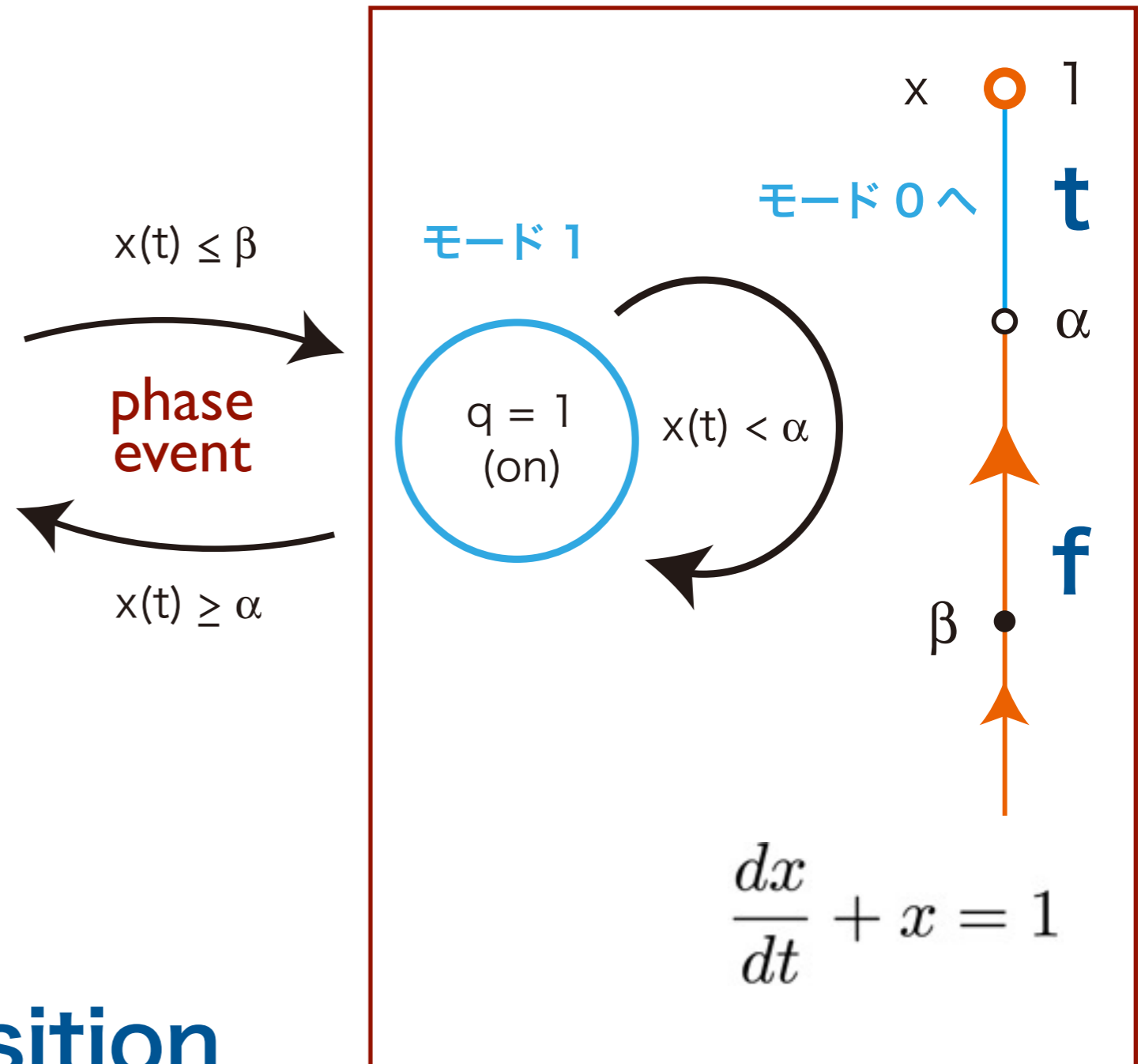
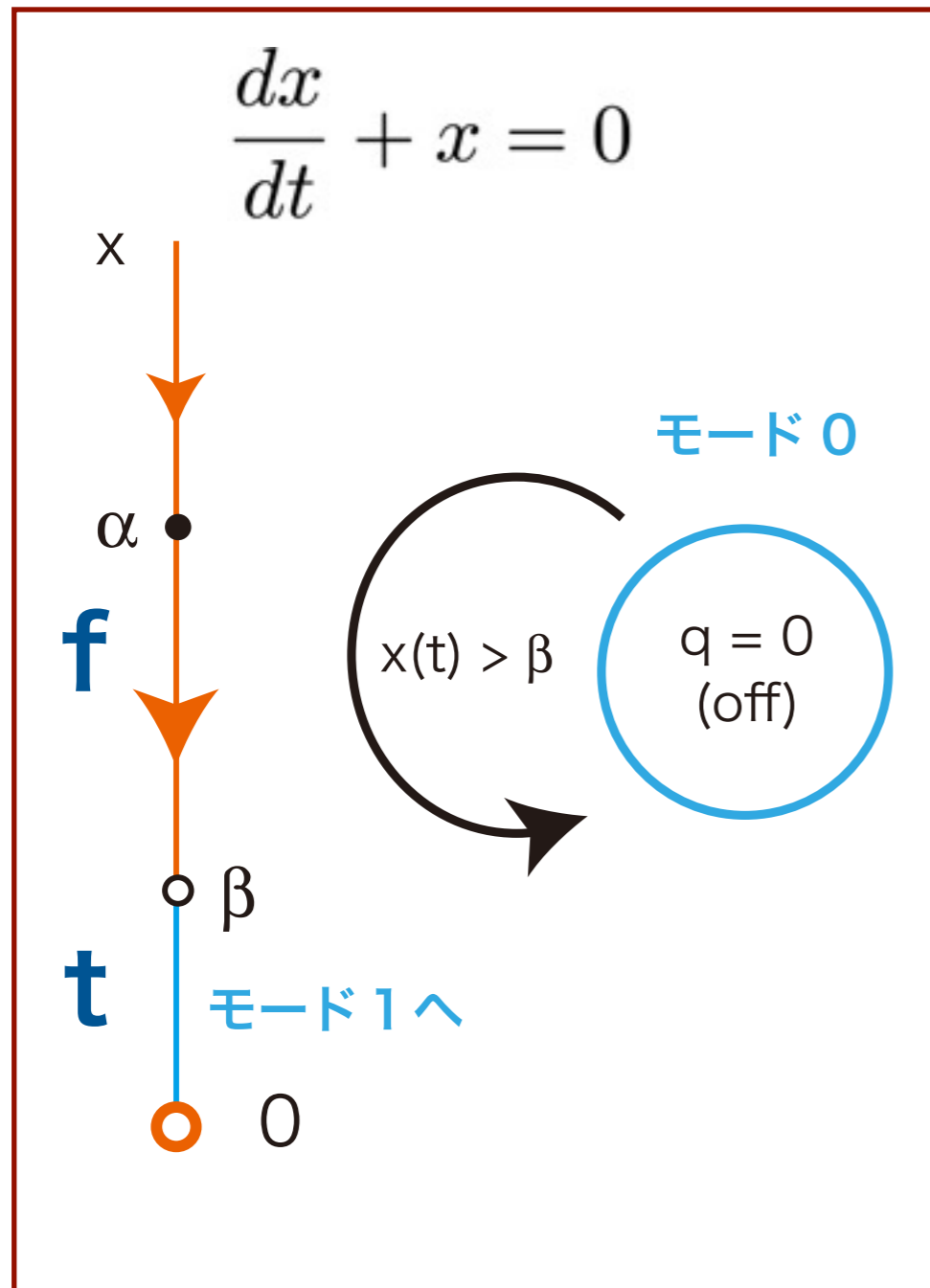
mode 1: $q=1$

mode 0: $q=0$

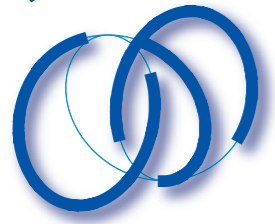


ハイブリッド系の運動

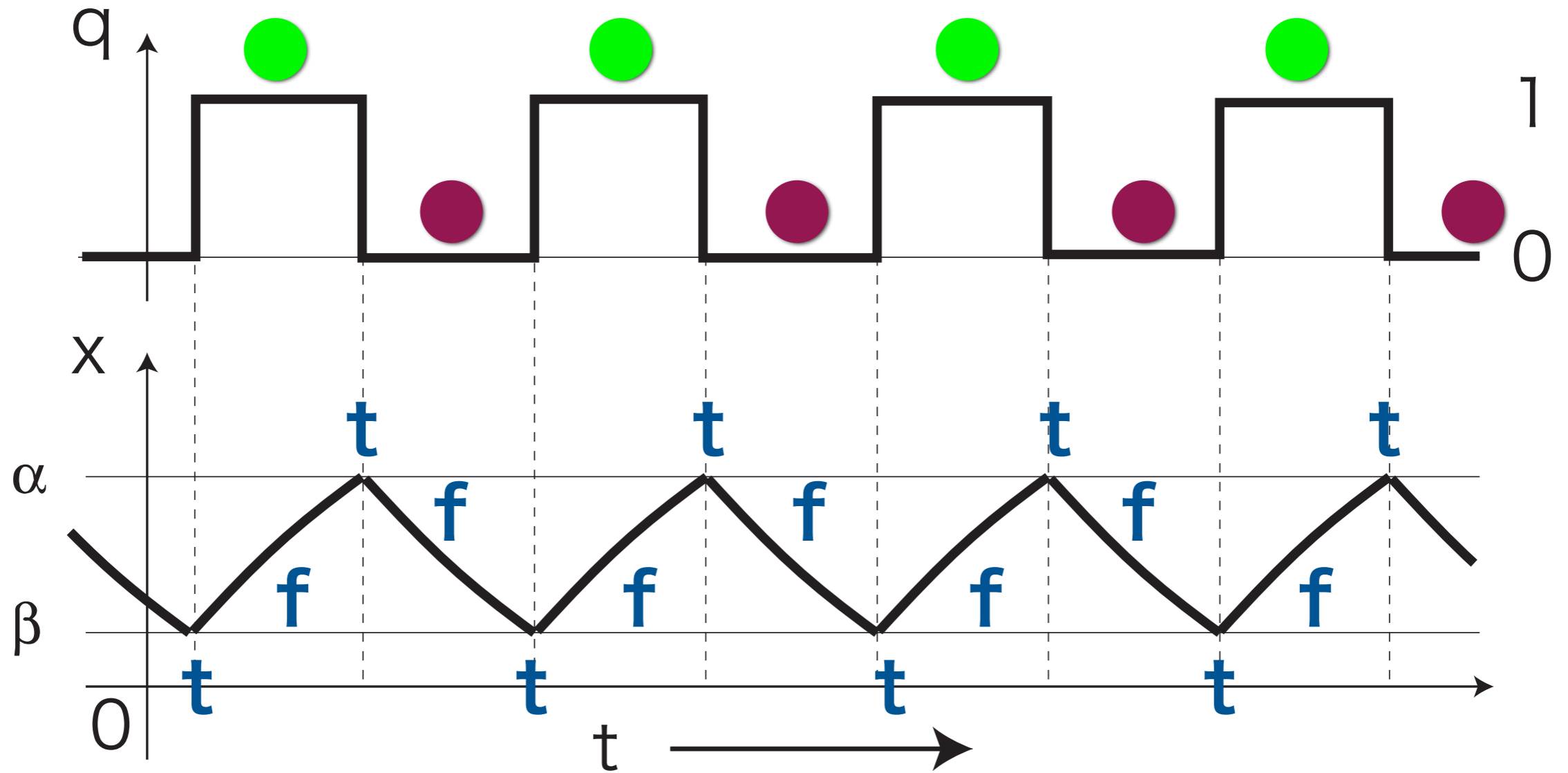
event: modeを遷移させる条件



f : flow, t : transition



状態の時間発展

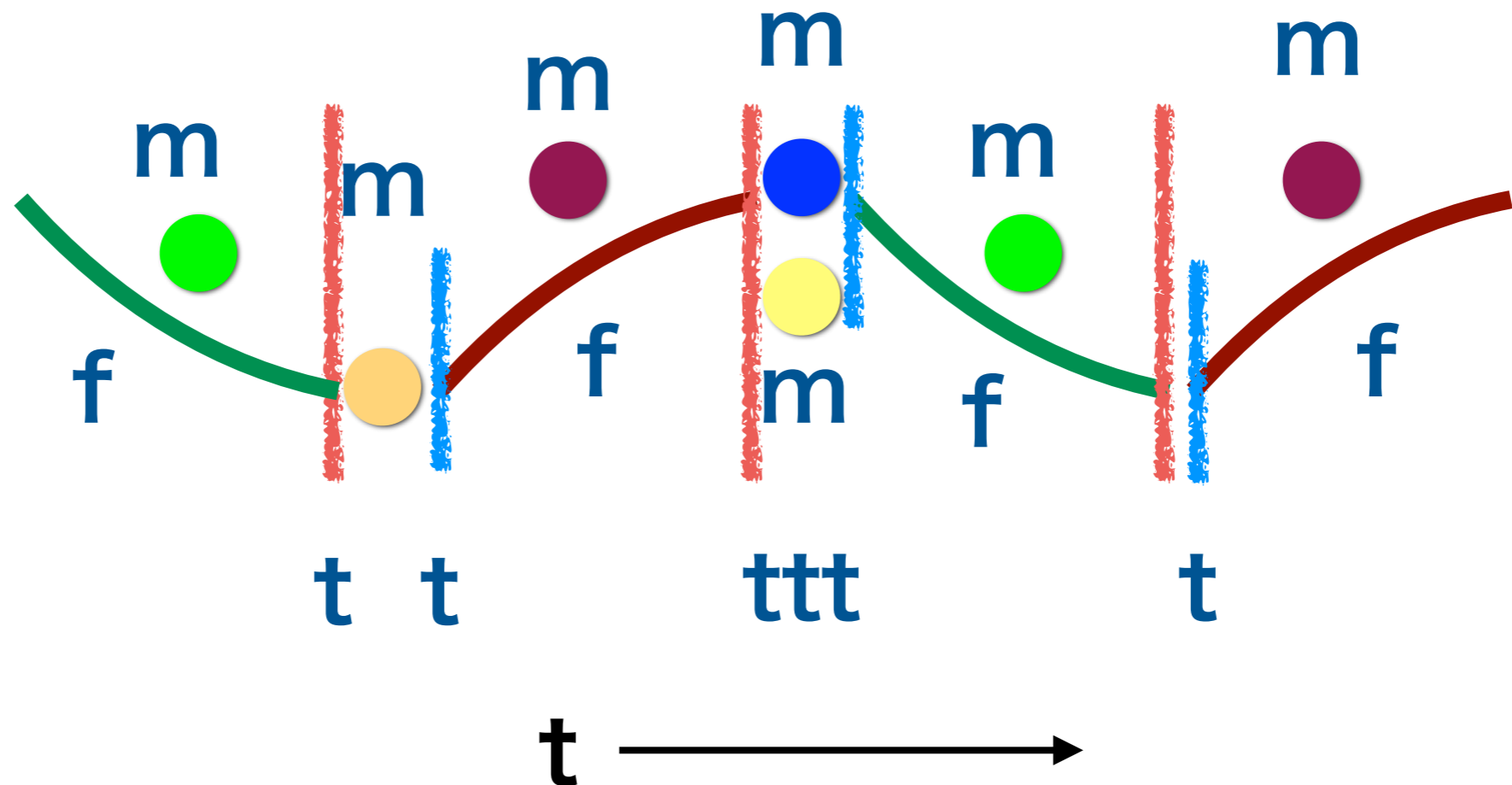




Hybrid系：状態の進展と遷移

flowが進展している時は、modeは固定されている
 modeが遷移している時は、flowは固定されている
meta-stable state

ftttftttft...



11

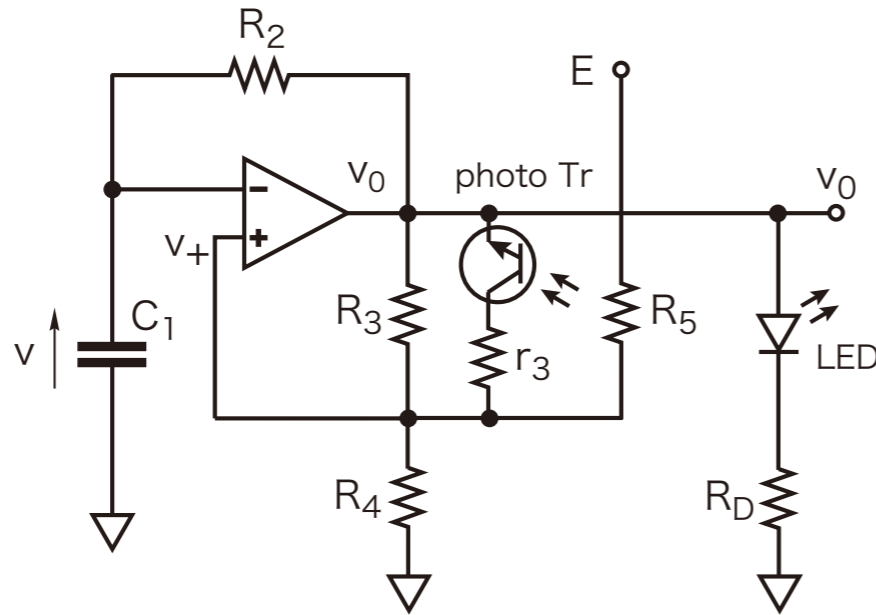


LEDホタルとその結合系



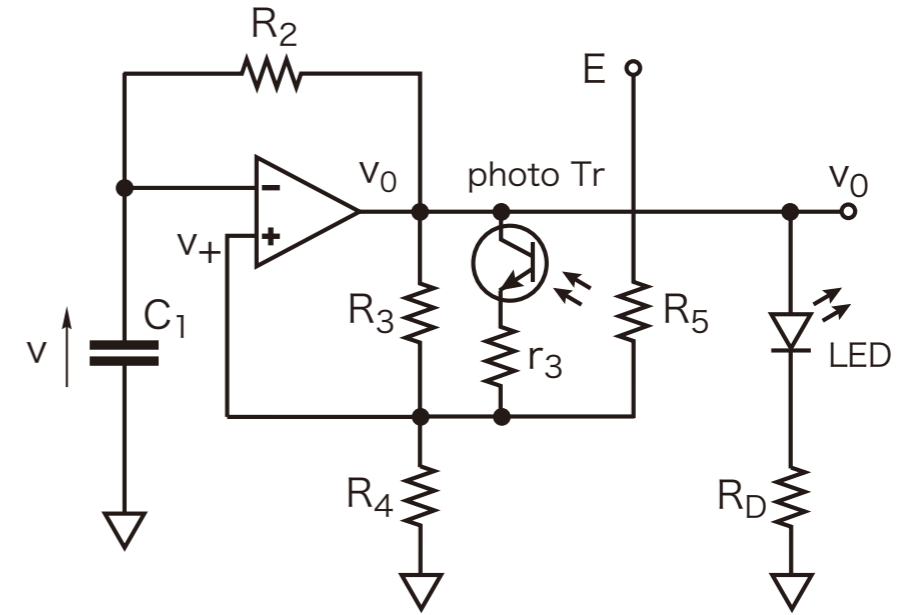
LEDホタルの回路例

A I



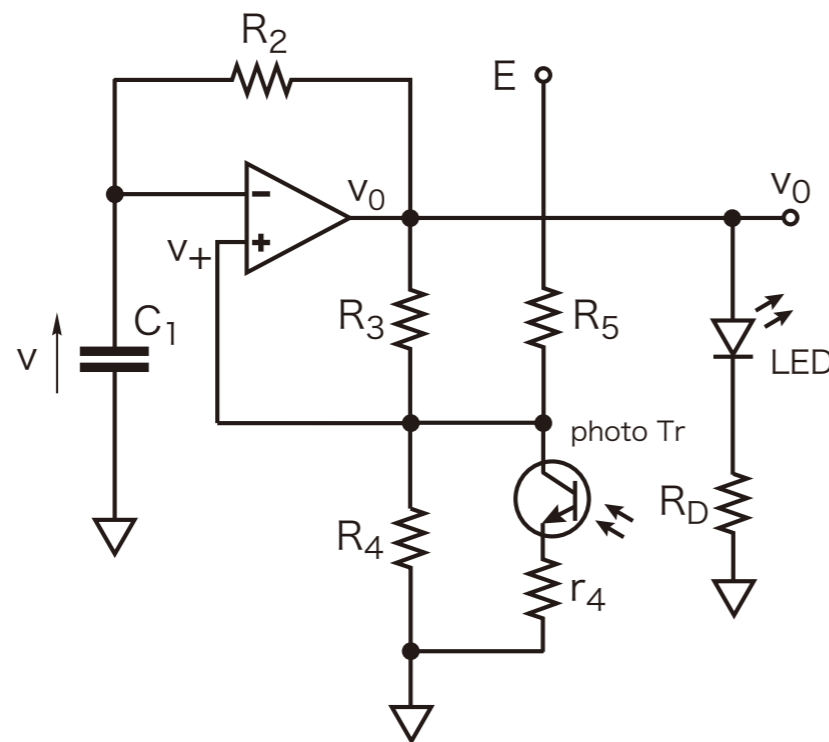
(1) 回路 a1: $v_0=0[v]$ && 光あり : $\beta_{on} < \beta_{off}$

B I



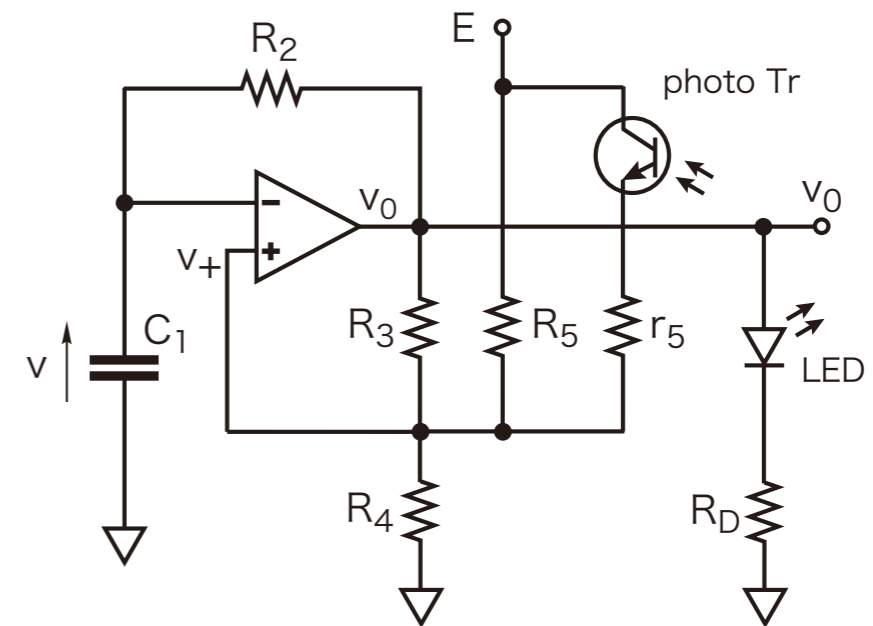
(2) 回路 b1: $v_0=V_{cc}[v]$ && 光あり : $\alpha_{off} < \alpha_{on}$

C I



(3) 回路 c1: 光あり : $\beta_{on} < \beta_{off}, \alpha_{on} < \alpha_{off}$

D I



(4) 回路 d1: 光あり : $\beta_{off} < \beta_{on}, \alpha_{off} < \alpha_{on}$



LEDホタルの回路 : type A1

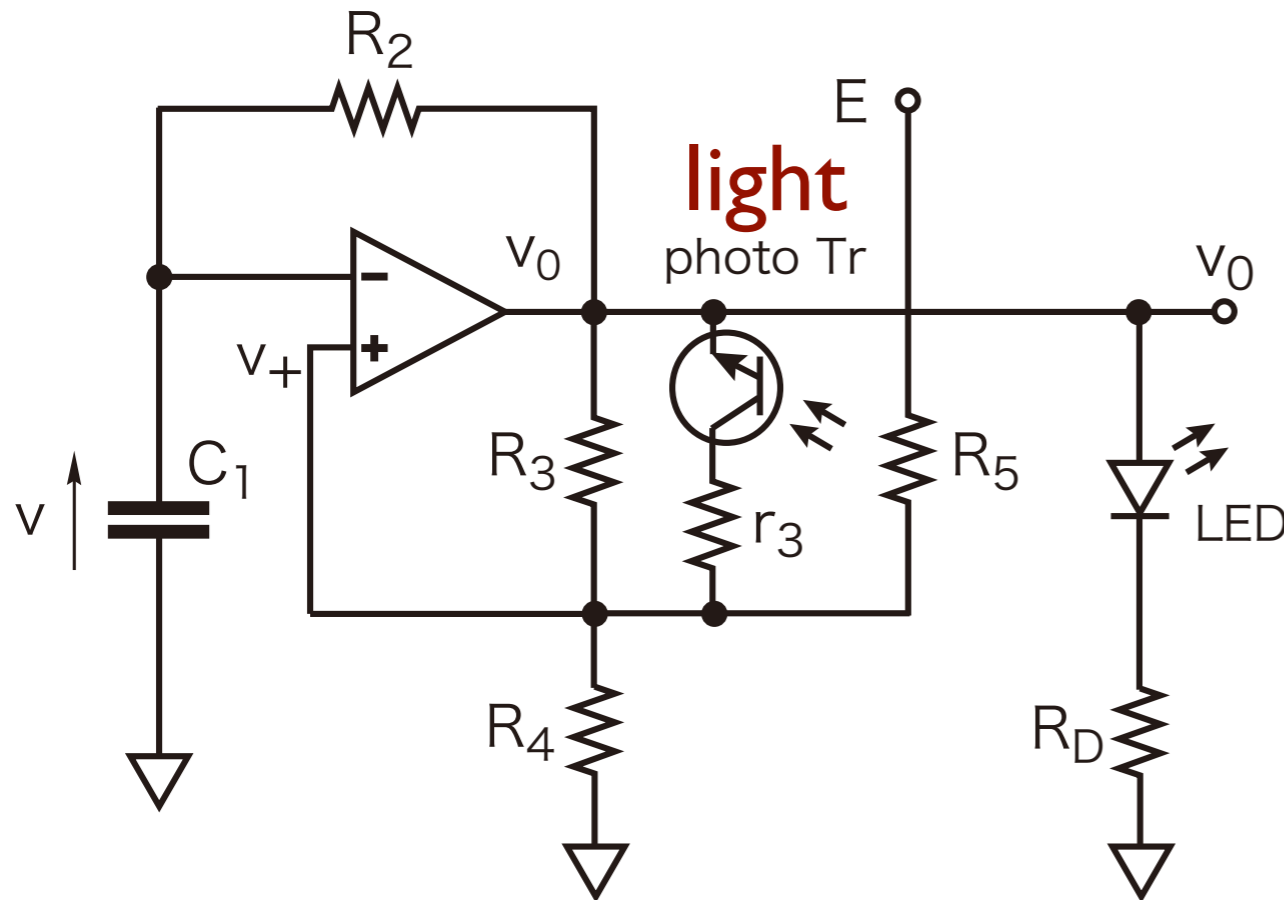


photo Tr	light on	light off
$v_0 = 0$	on	off
$v_0 = E$	off	off

光センサー部
(Photo Tr)

発光部 (LED)

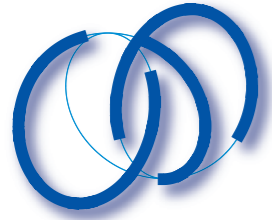
$$C_1 R_2 \frac{dv}{dt} + v = E$$

α

β_{on} β_{off}

$$C_1 R_2 \frac{dv}{dt} + v = 0$$

v



Hybrid回路の数学モデル

- ◎ 1つのFSM (Finite State Machine: 有限状態機械)と
このFSMのモード数と同じ数の力学系からなる複合系

mode —— FSM の状態 phase —— 力学系の状態

- ◎ 系の運動 —— FSMのtransitionと力学系のflowで時間発展する

event —— phase event, timer event

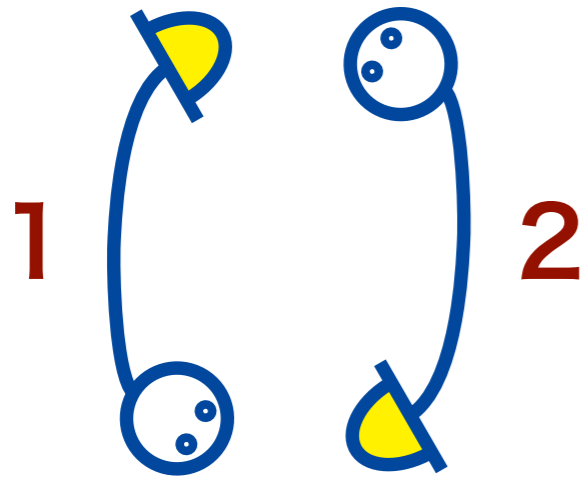
mode遷移図(graph)

vector場(ODE)

mode遷移図+phase portrait(vector場+transition場)



LEDホタルの回路：2個結合 A1-A1



$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2 \quad q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \in \mathbb{F}^2$$

$$\frac{dx}{dt} + x = q$$

photo Tr 1	q2=0	q2=1
q1=0	off	on
q1=1	off	off

photo Tr 2	q2=0	q2=1
q1=0	off	off
q1=1	on	off

if($q_1 == 0 \ \&\& \ q_2 == 1$) β_{1on}

if($q_1 == 1 \ \&\& \ q_2 == 0$) β_{2on}

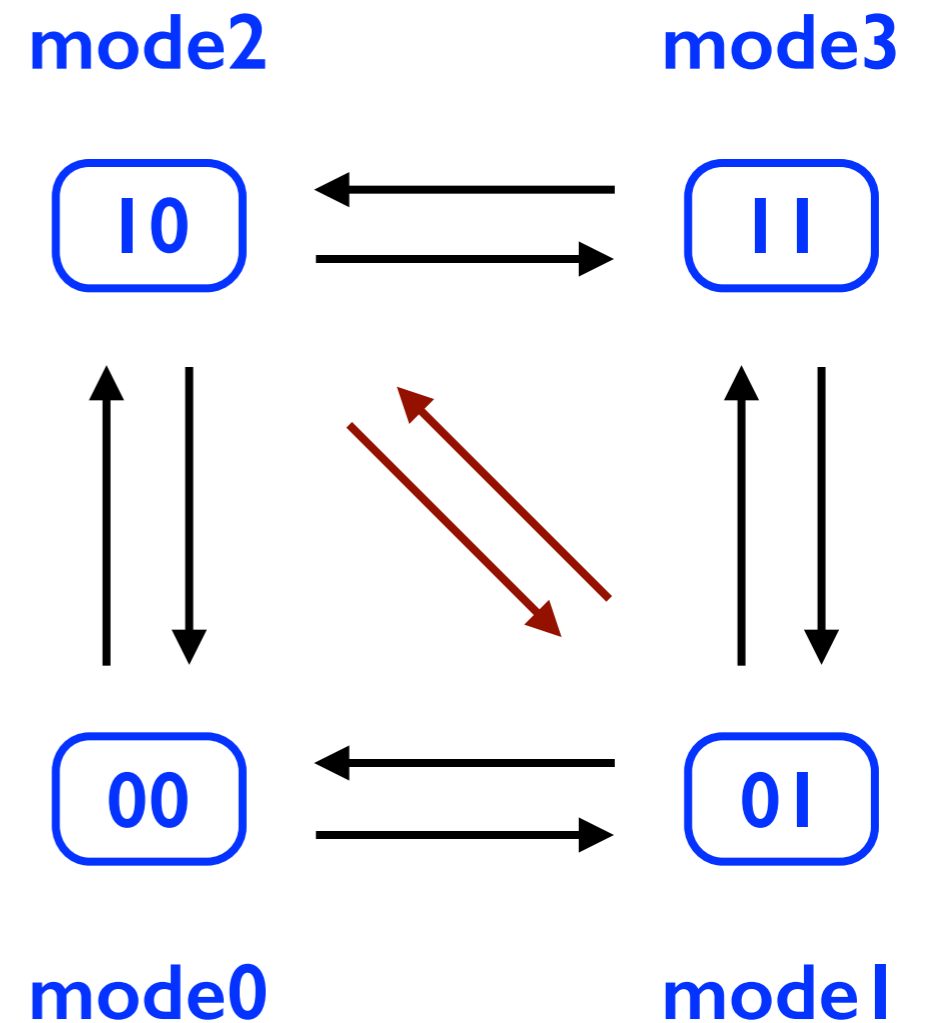


LEDホタルの回路：4つのモード

FSM 部の4つのモード

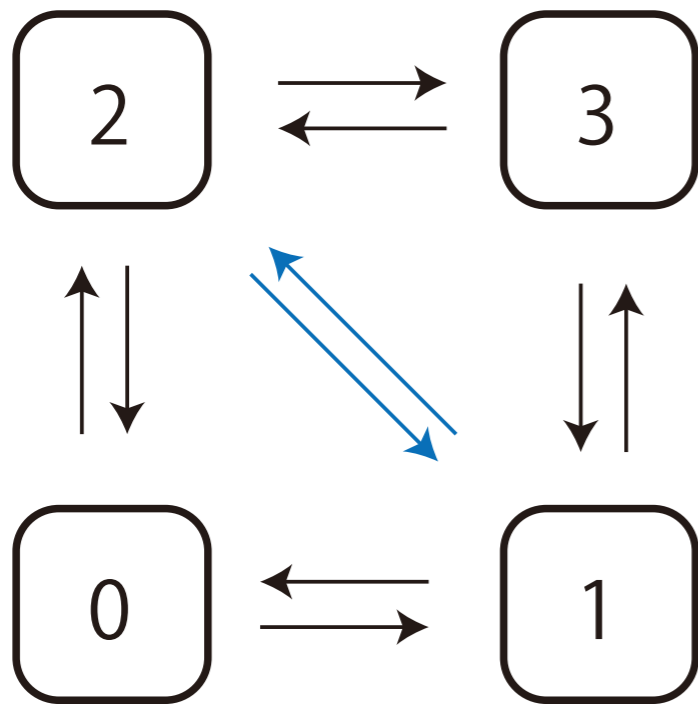
mode 0	$q_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$	0b00
mode 1	$q_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$	0b01
mode 2	$q_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$	0b10
mode 3	$q_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$	0b11

モード遷移図

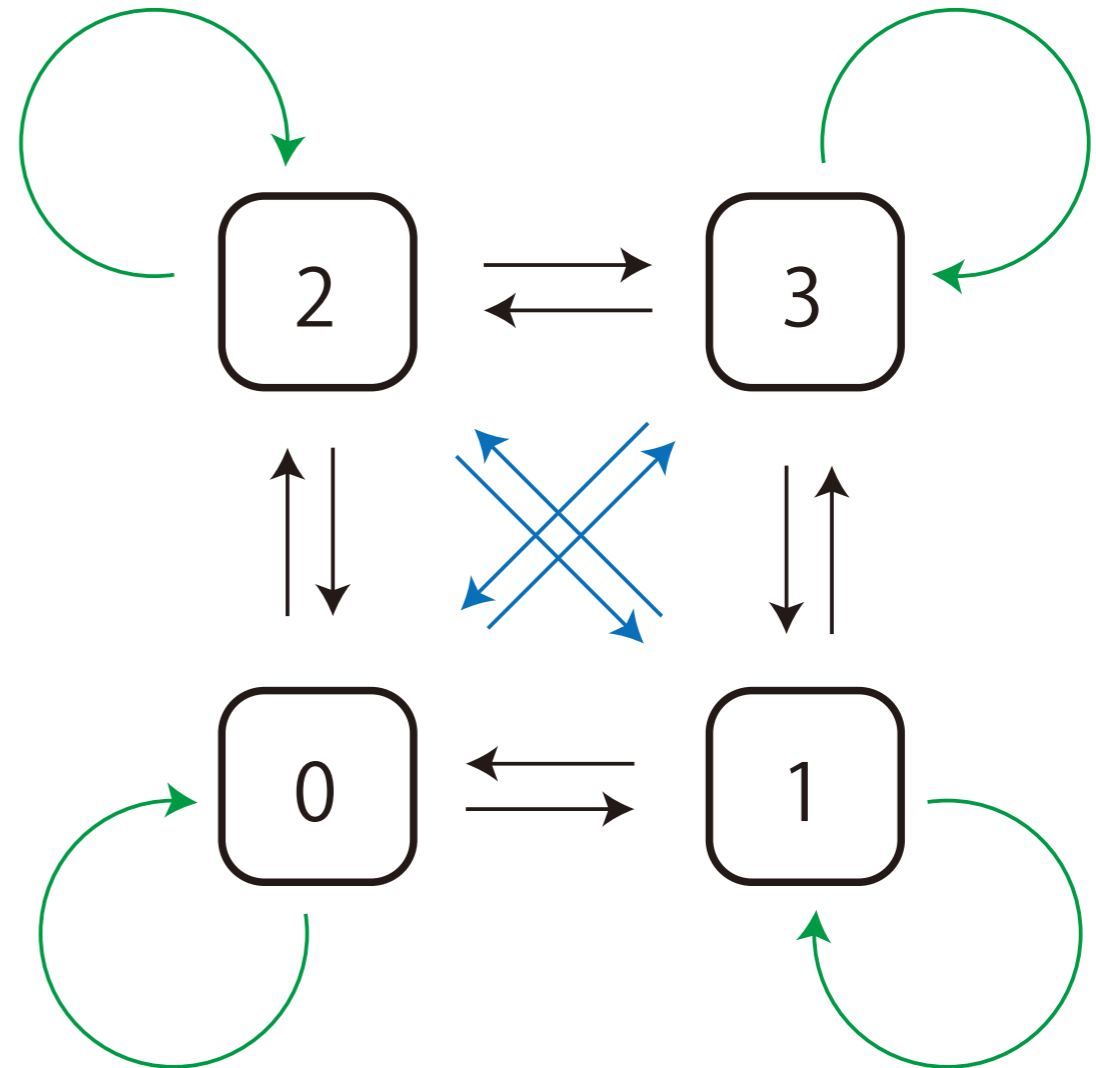




LEDホタル相互結合系 : A1



(a) 遷移図 (従来のモデル)



(b) 遷移図 (今回のモデル)



LEDホテルの回路：4つの相平面

mode 0

$$q_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

mode2

10

mode3

11

$$\begin{array}{l} x_1 > \beta_1 \\ x_2 < \beta_2 \end{array}$$

$$\begin{array}{l} x_1 < \beta_1 \\ x_2 < \beta_2 \end{array}$$

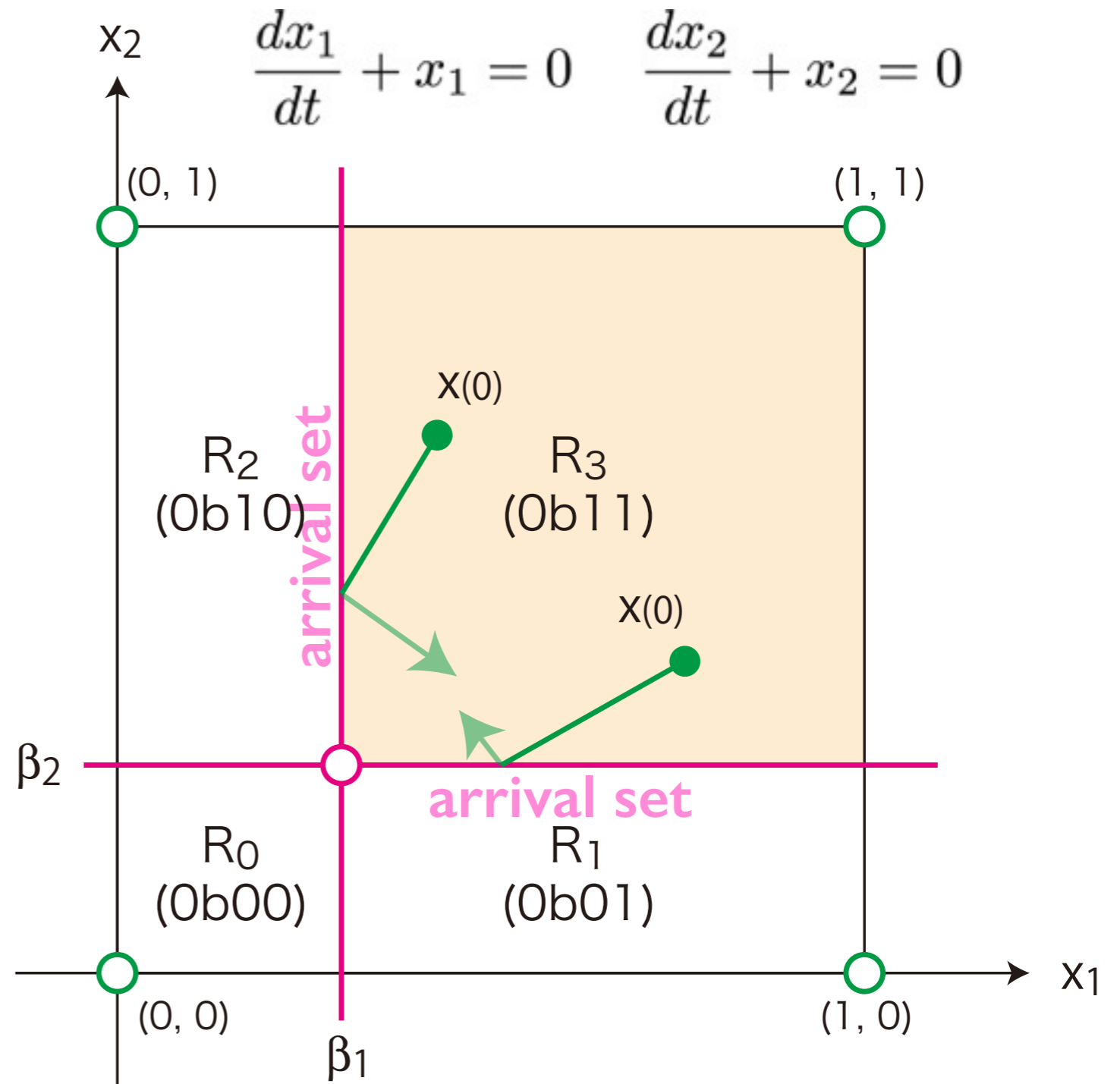
00

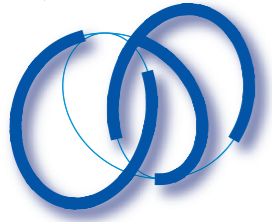
$$\begin{array}{l} x_1 < \beta_1 \\ x_2 > \beta_2 \end{array}$$

01

mode0

mode1





LEDホテルの回路：4つの相平面

mode 1

$$q_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

mode 2

10

mode 3

11

$$\begin{array}{l} x_1 > \alpha_1 \\ x_2 < \beta_{2on} \end{array}$$

$$\begin{array}{l} x_1 > \alpha_1 \\ x_2 < \beta_{2on} \end{array}$$

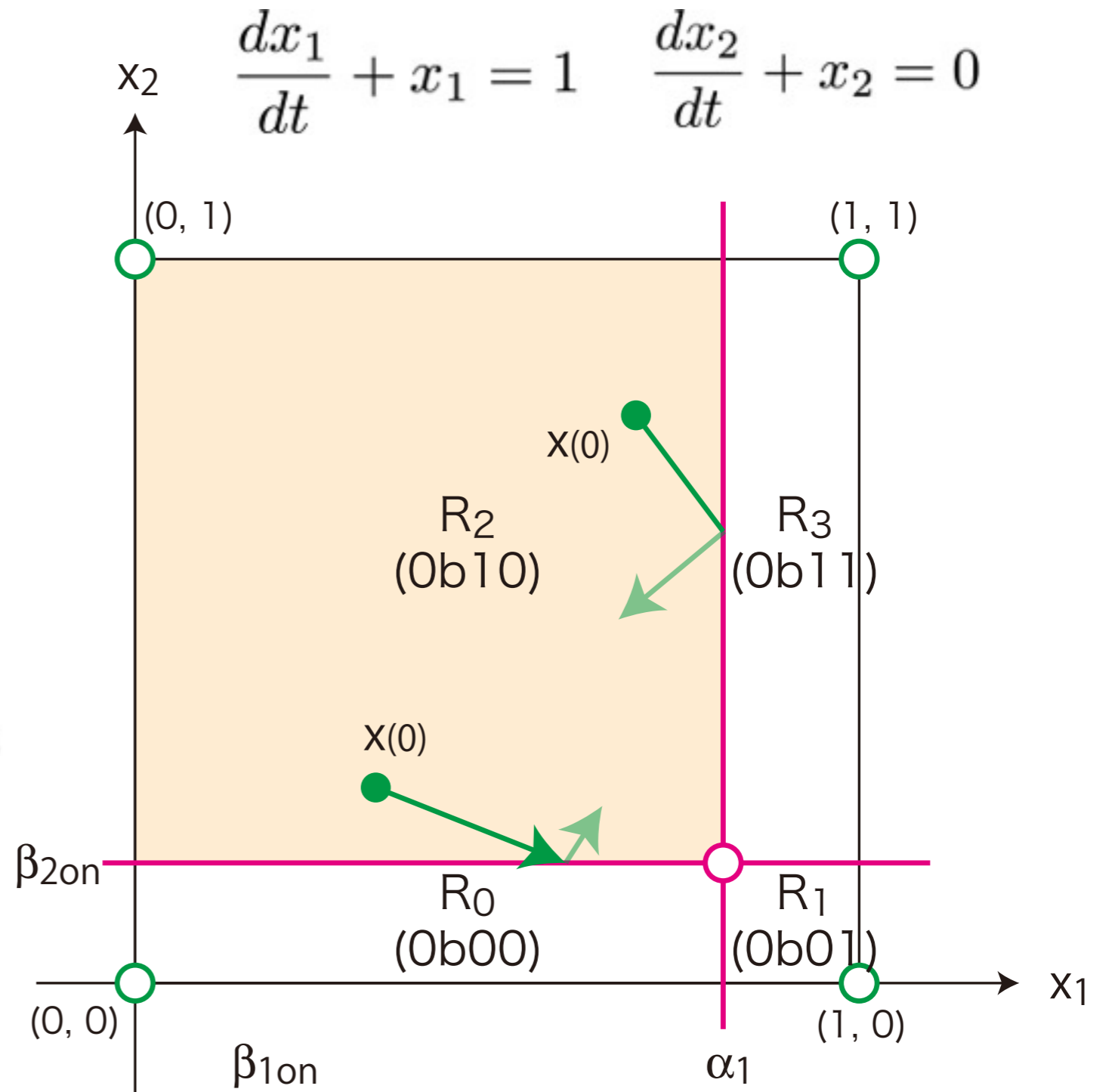
00

mode 0

$$\begin{array}{l} x_1 > \alpha_1 \\ x_2 > \beta_{2on} \end{array}$$

01

mode 1





LEDホテルの回路：4つの相平面

mode 2

$$q_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

mode2

10

$$\begin{array}{c} x_1 < \beta_{1on} \\ \xrightarrow{x_2 < \alpha_2} \end{array}$$

mode3

11

$$\begin{array}{c} x_1 > \beta_{1on} \\ \downarrow x_2 > \alpha_2 \end{array}$$

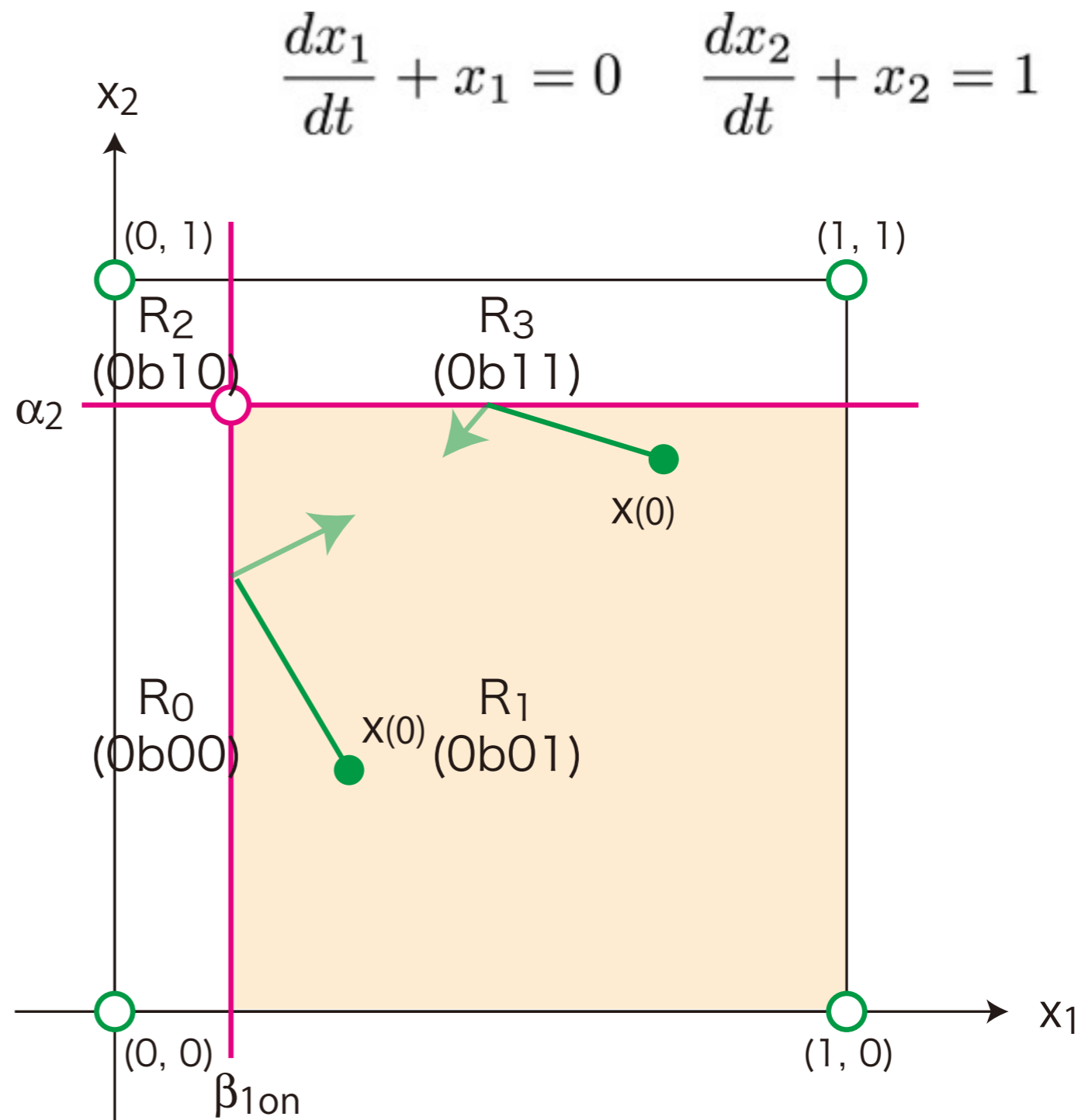
00

mode0

$$\begin{array}{c} x_2 > \alpha_2 \\ \swarrow x_1 < \beta_{1on} \end{array}$$

01

mode1





LEDホテルの回路：4つの相平面

mode 3

$$q_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

mode 2

10

$$\begin{array}{l} x_1 > \alpha_1 \\ \leftarrow \\ x_2 < \alpha_2 \end{array}$$

mode 3

11

$$\begin{array}{l} x_2 > \alpha_2 \\ \swarrow \\ x_1 > \alpha_1 \end{array}$$

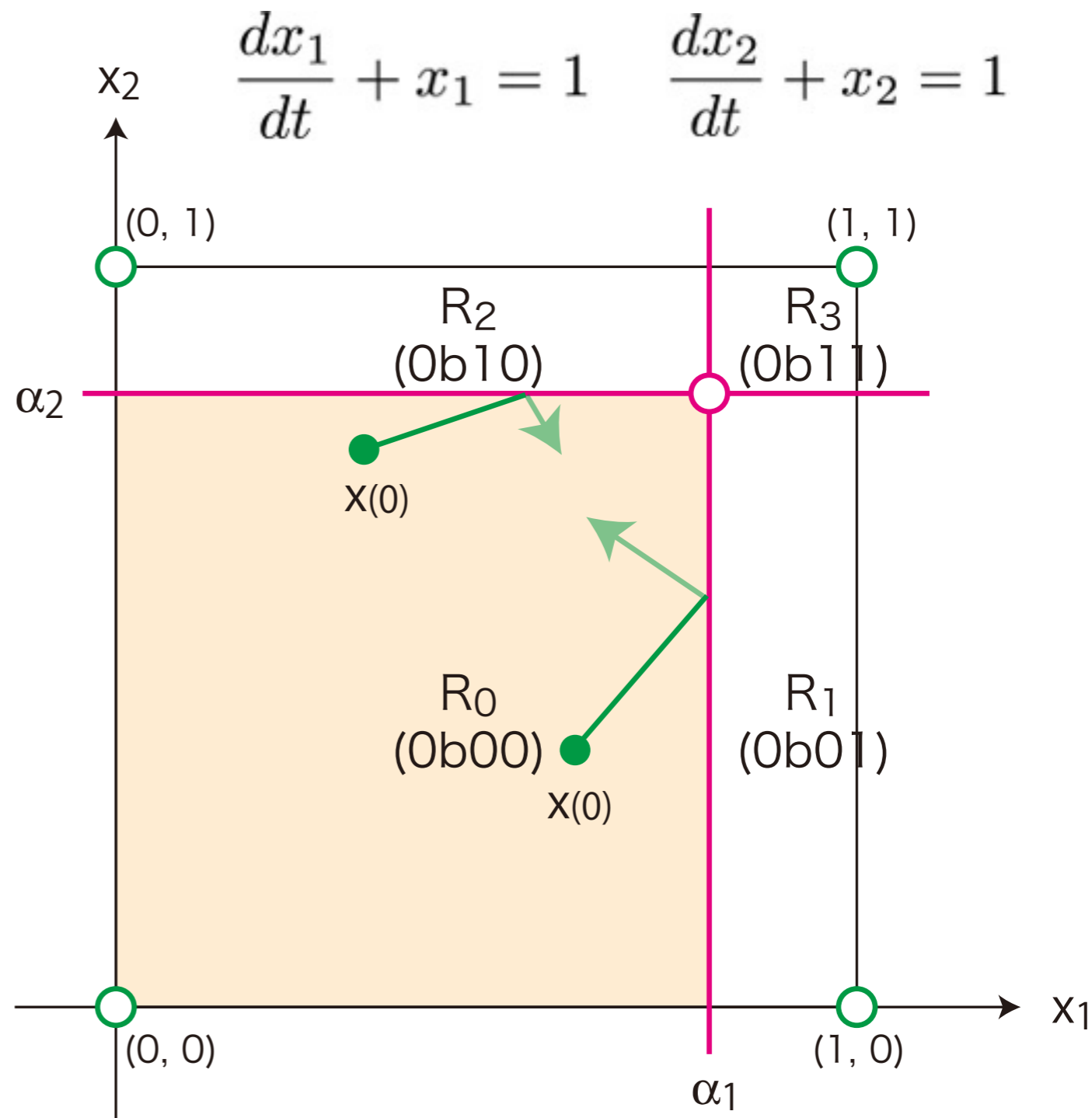
$$\begin{array}{l} x_1 < \alpha_1 \\ \downarrow \\ x_2 > \alpha_2 \end{array}$$

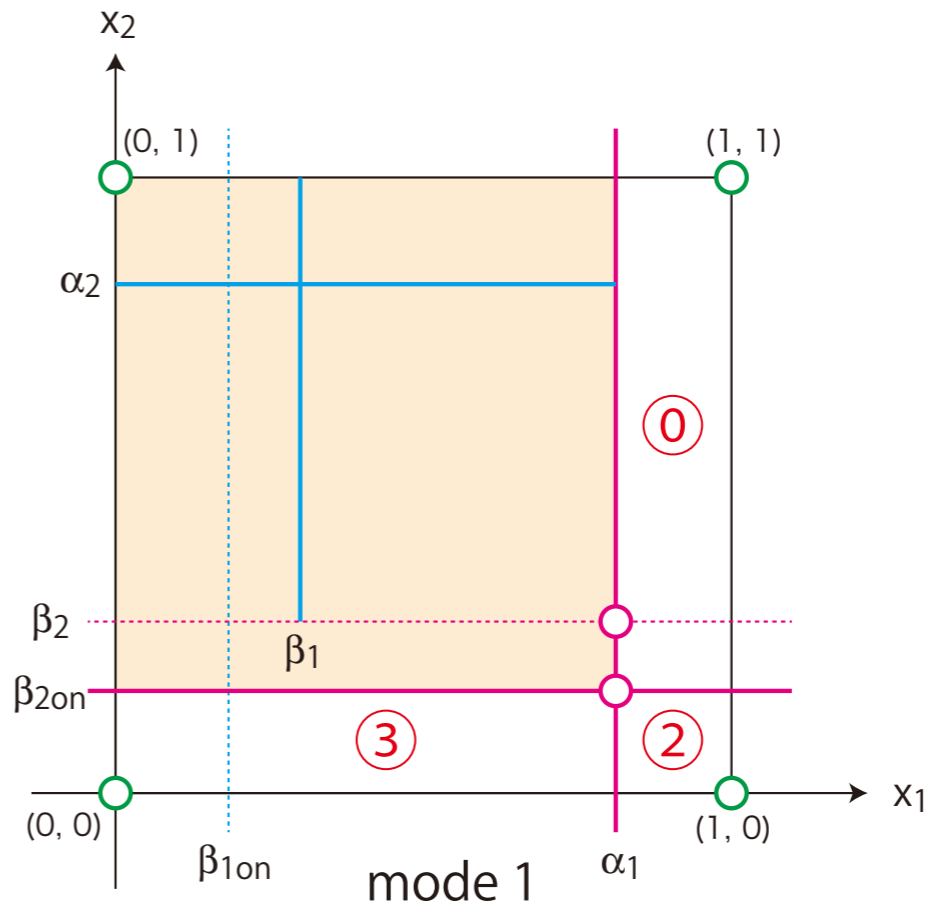
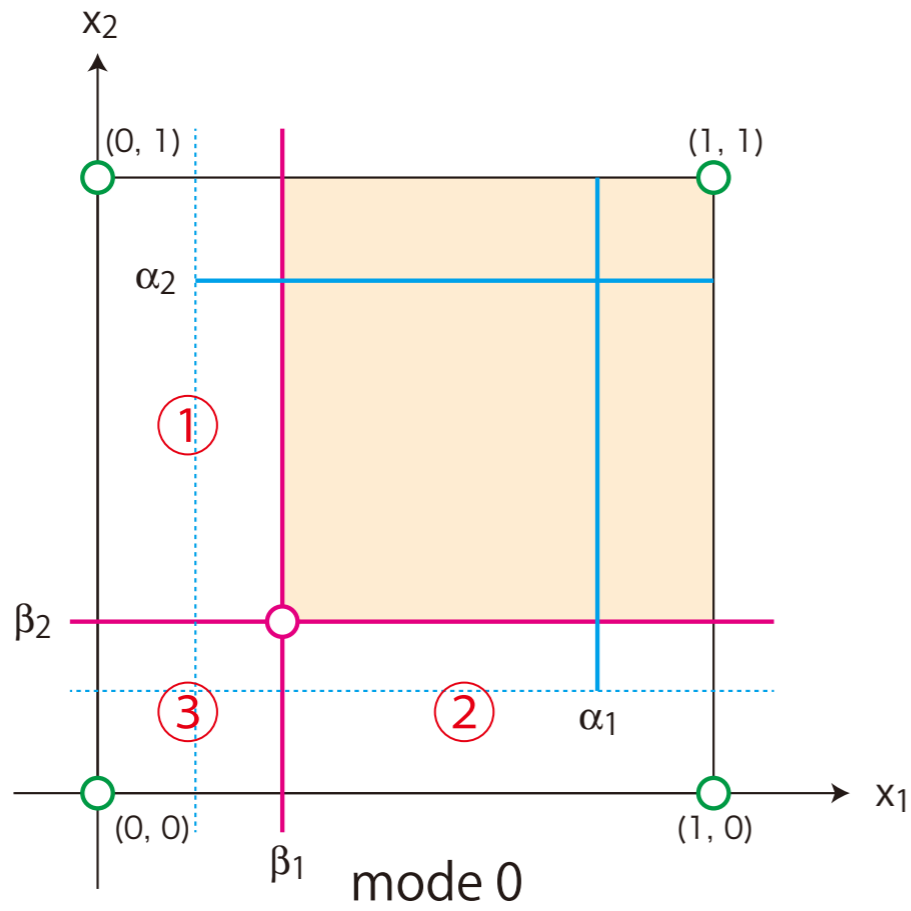
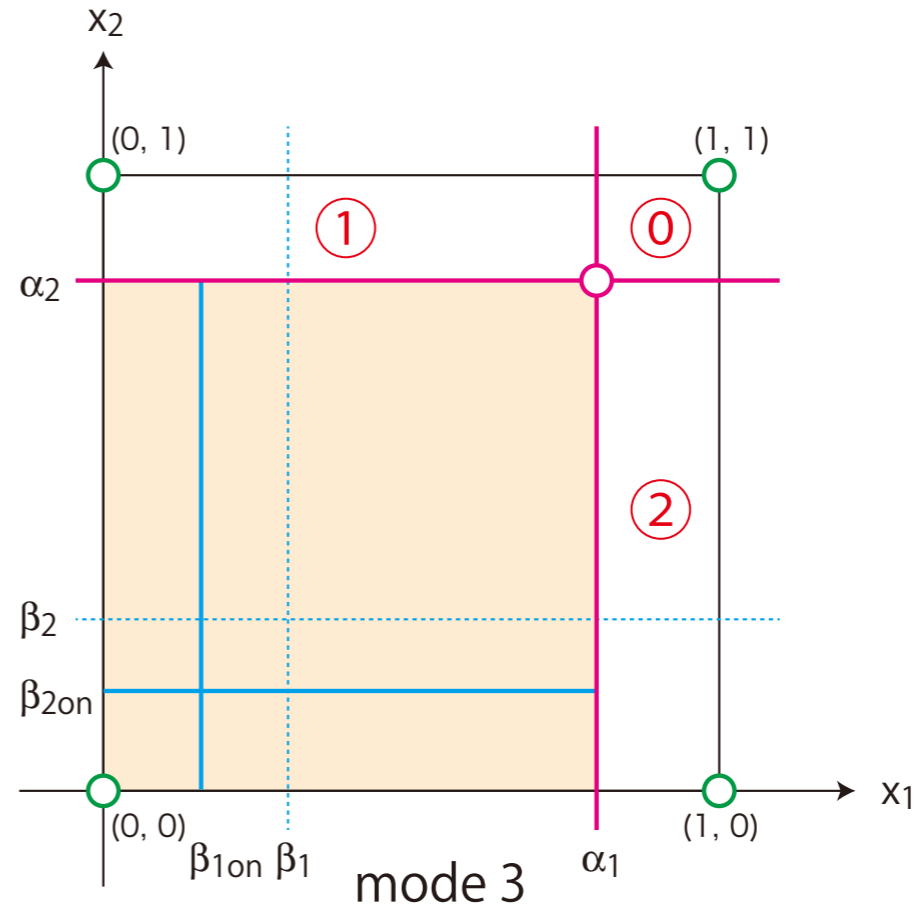
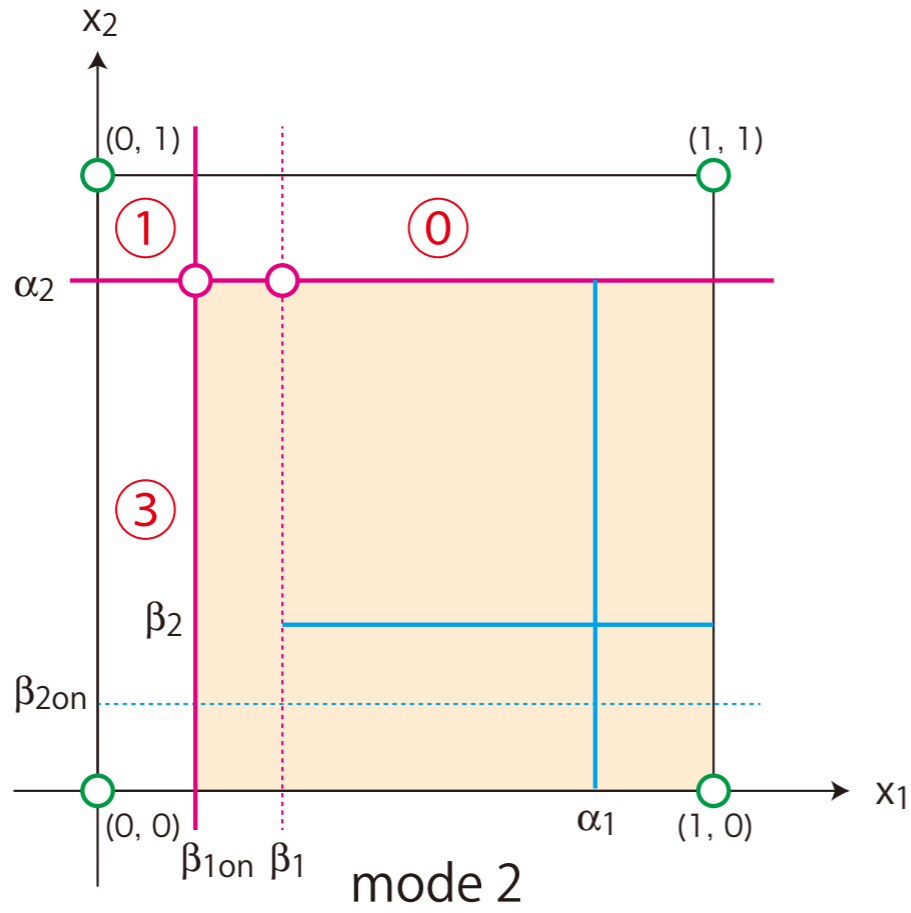
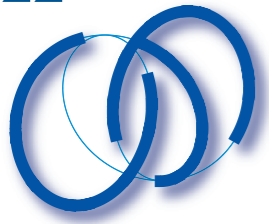
00

mode 0

01

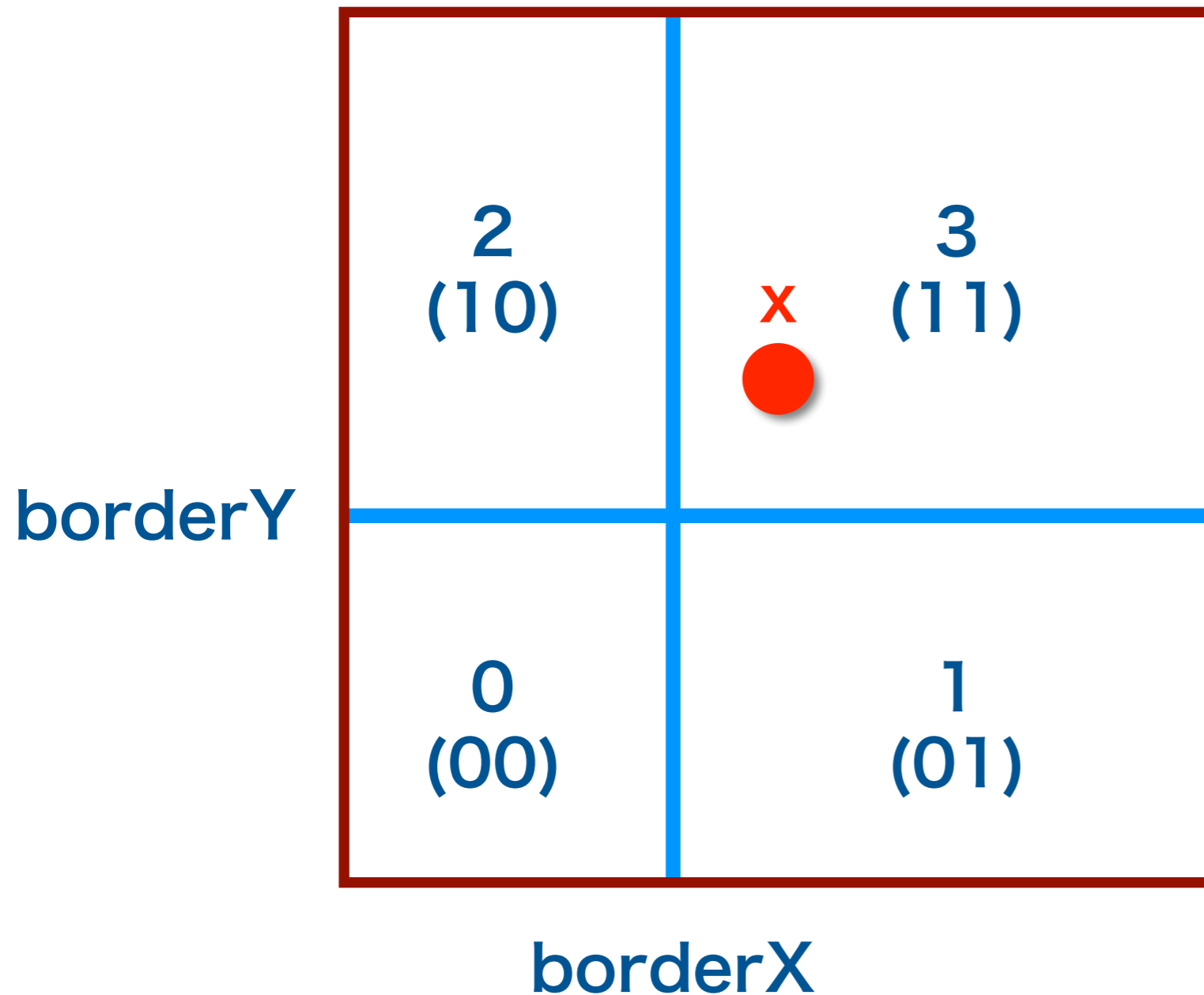
mode 1





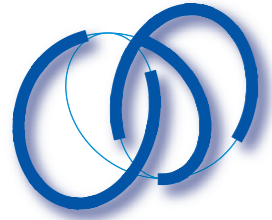


phase plane: region number



borderMfd(q, bdr)

mode qを与えると
borderが決まる



next transition number

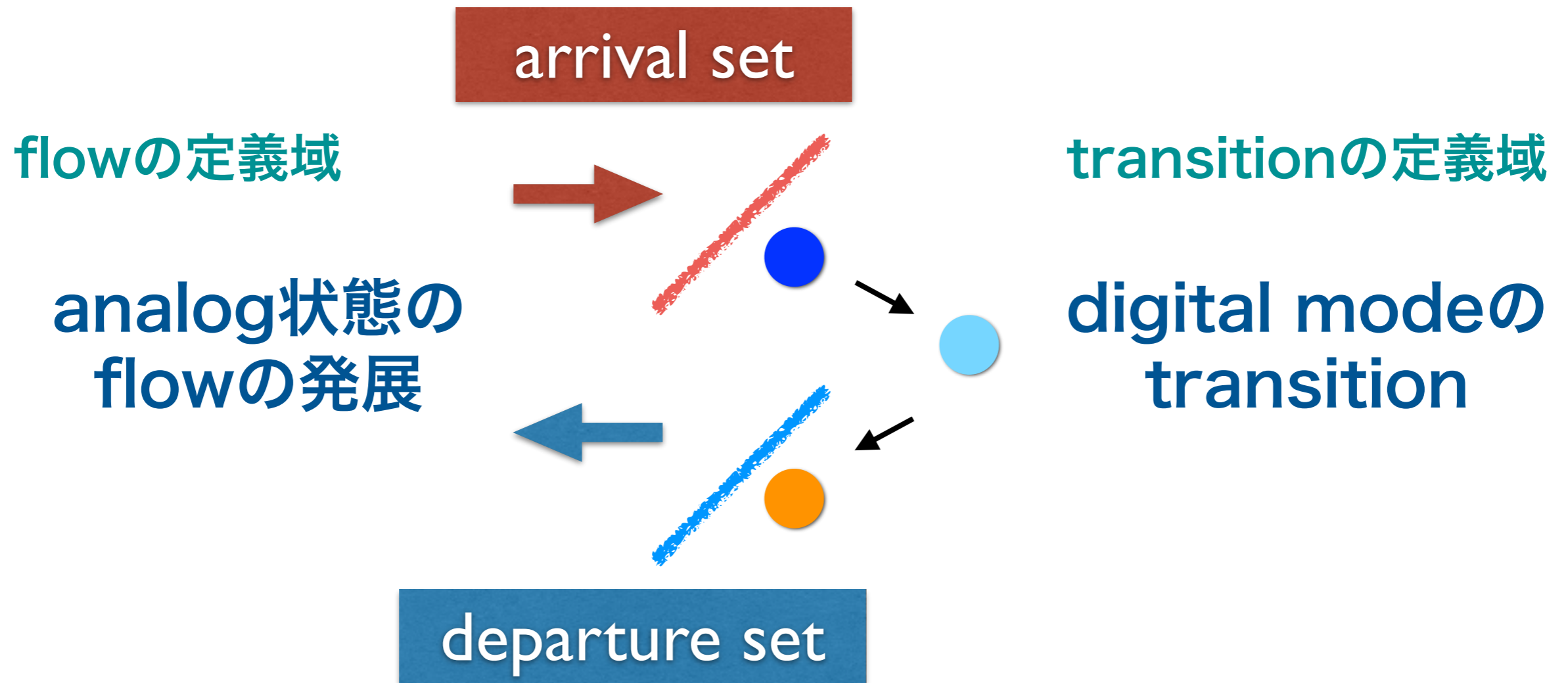
1 (01)	0 (00)
2	3
0	1
3 (11)	2 (10)

nextModeNum(q, x)
 mode qとphase x
 を与えると
 transitionするmode
 番号を返す

next transition number = region number



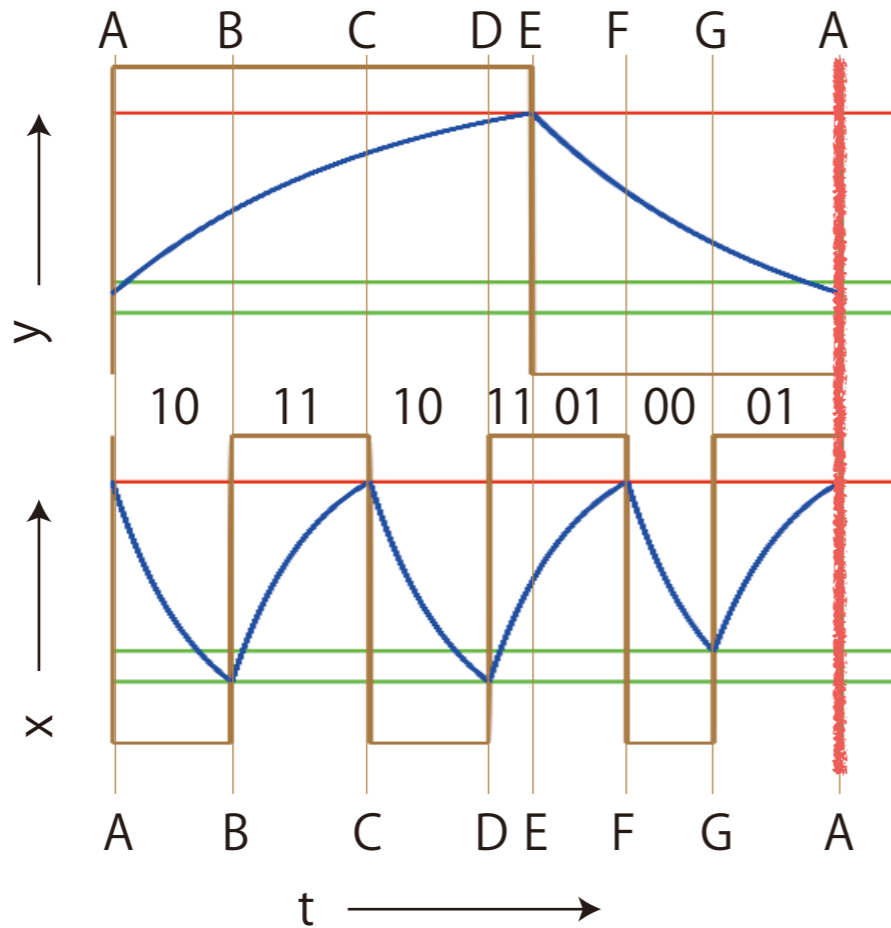
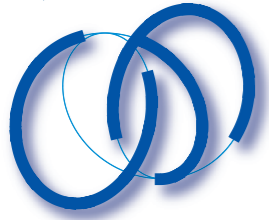
Hybridの運動=flowとtransitionの時系列



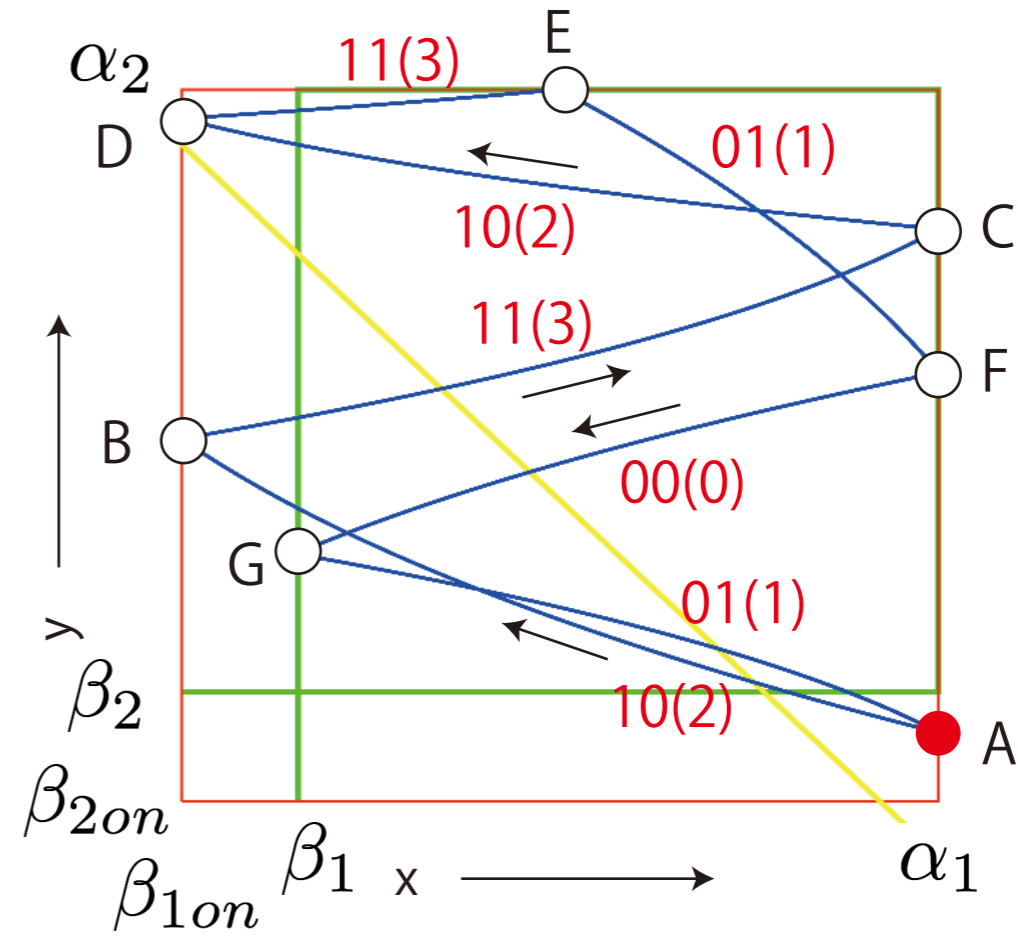
flow進展時は, modeが固定されている
mode遷移時は, flowが固定されている



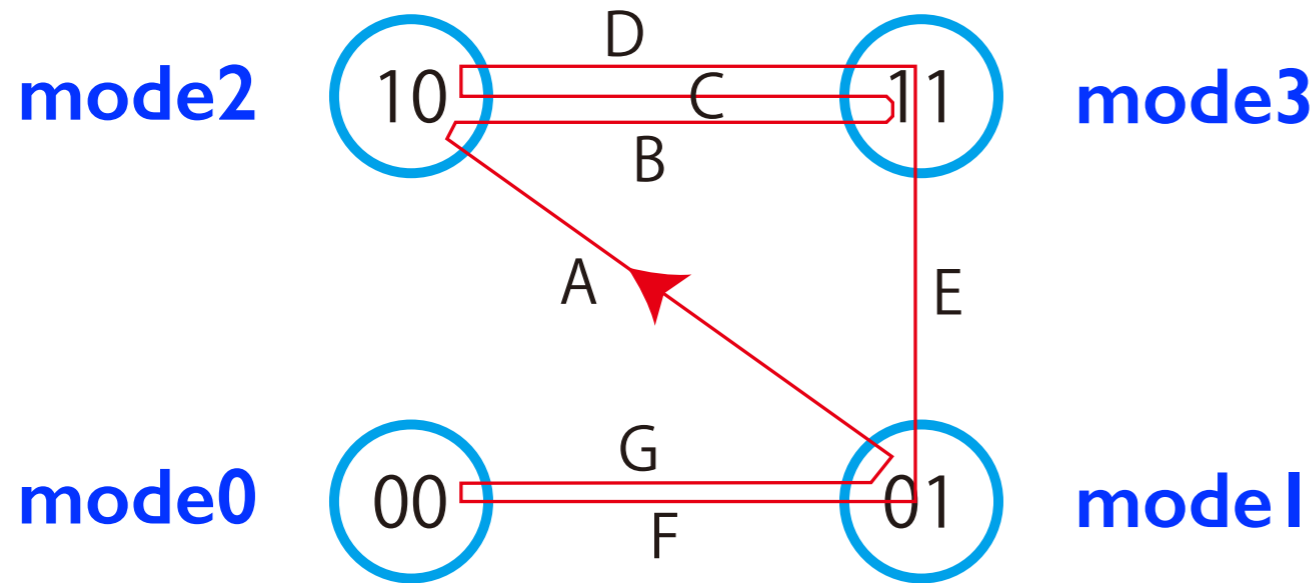
波形, モード線図, 相平面図



wave form



phase portrait



mode transition diagram

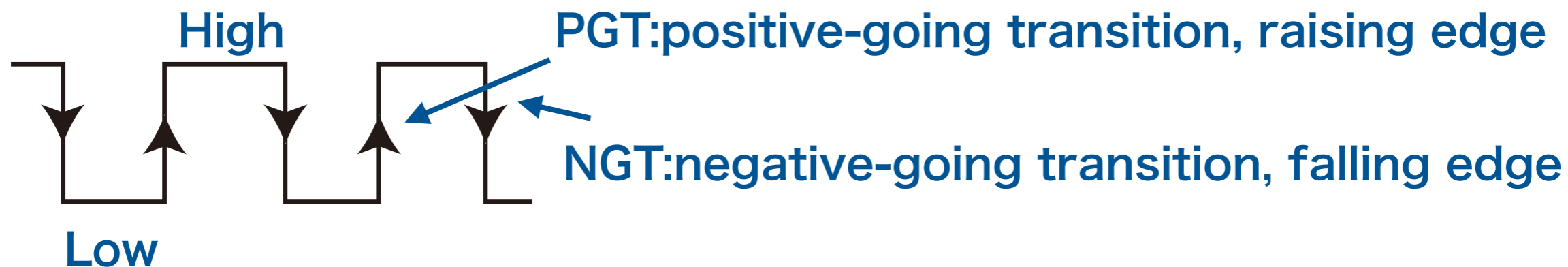
記号列 : (2323101)

3 : 1 の同期振動

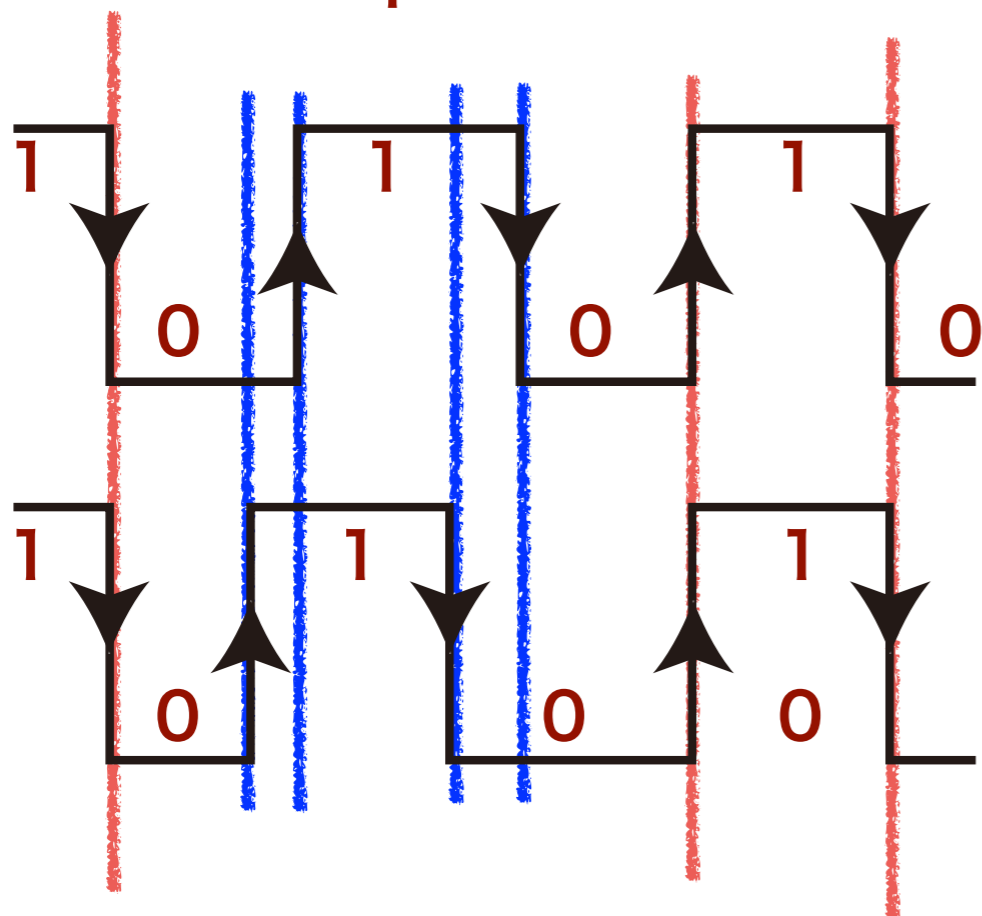
$$\gamma = \frac{2}{6}$$



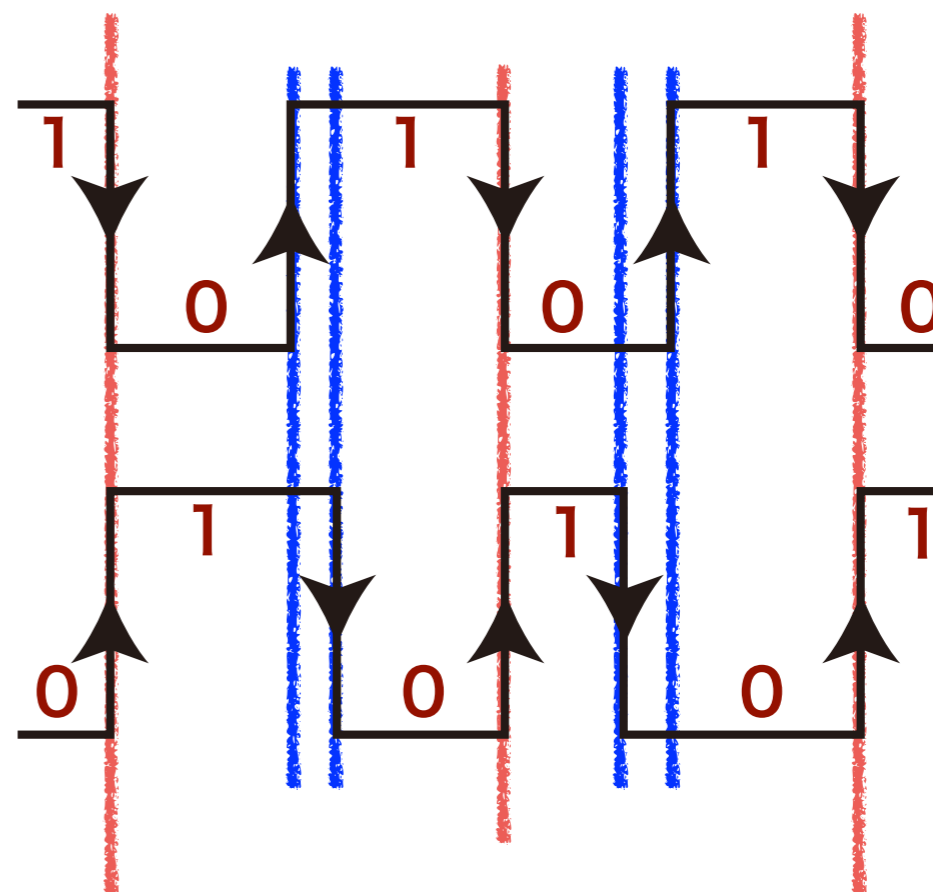
波形：同期率，同相率，逆相率



in-phase

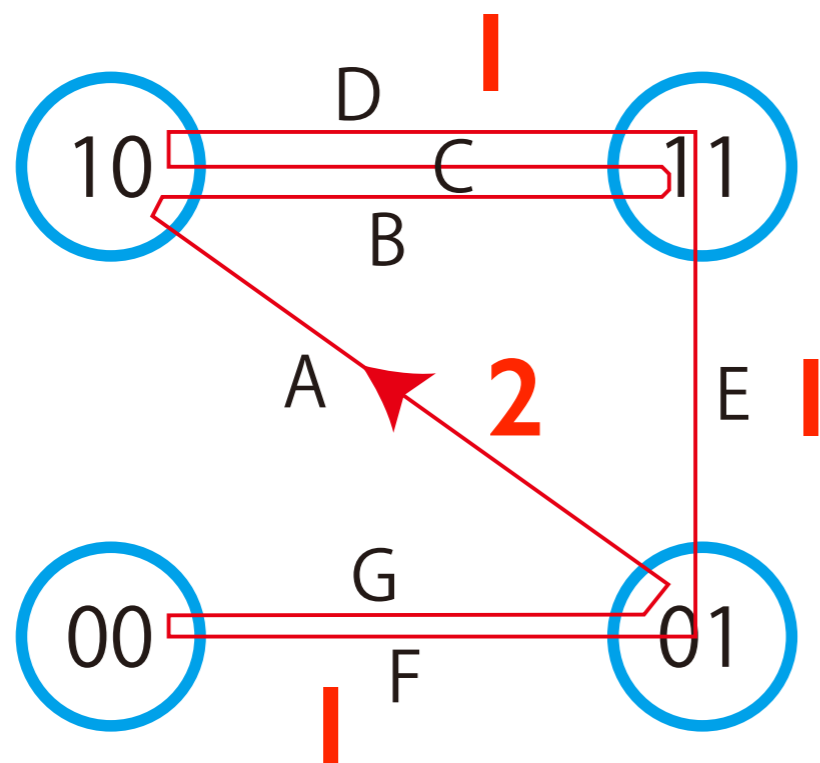


anti-phase





モード間のHamming距離

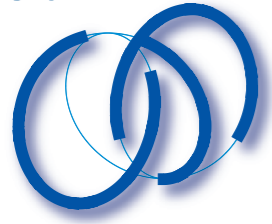


$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \quad 0bq_2q_1$$

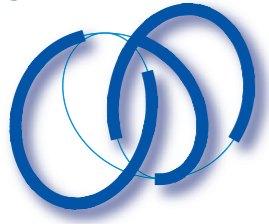
$$d(p, q) = \begin{matrix} (p_2 - 2 p_2) + 10 (p_1 - 2 q_1) \\ (p_2 + 2 p_2) + 10 (p_1 + 2 q_1) \end{matrix}$$

$$\begin{aligned} d(01, 00) &= 1, & d(00, 10) &= 1 \\ d(00, 11) &= 2, & d(01, 10) &= 2 \\ d(01, 11) &= 1, & d(10, 11) &= 1 \end{aligned}$$

- ◎周期解にはHamming距離2の遷移が少なくとも1つある
- ◎距離2の遷移は余次元2の性質をもつarrival borderでおこる
- ◎周期解は同期する



関川先生 Q&A



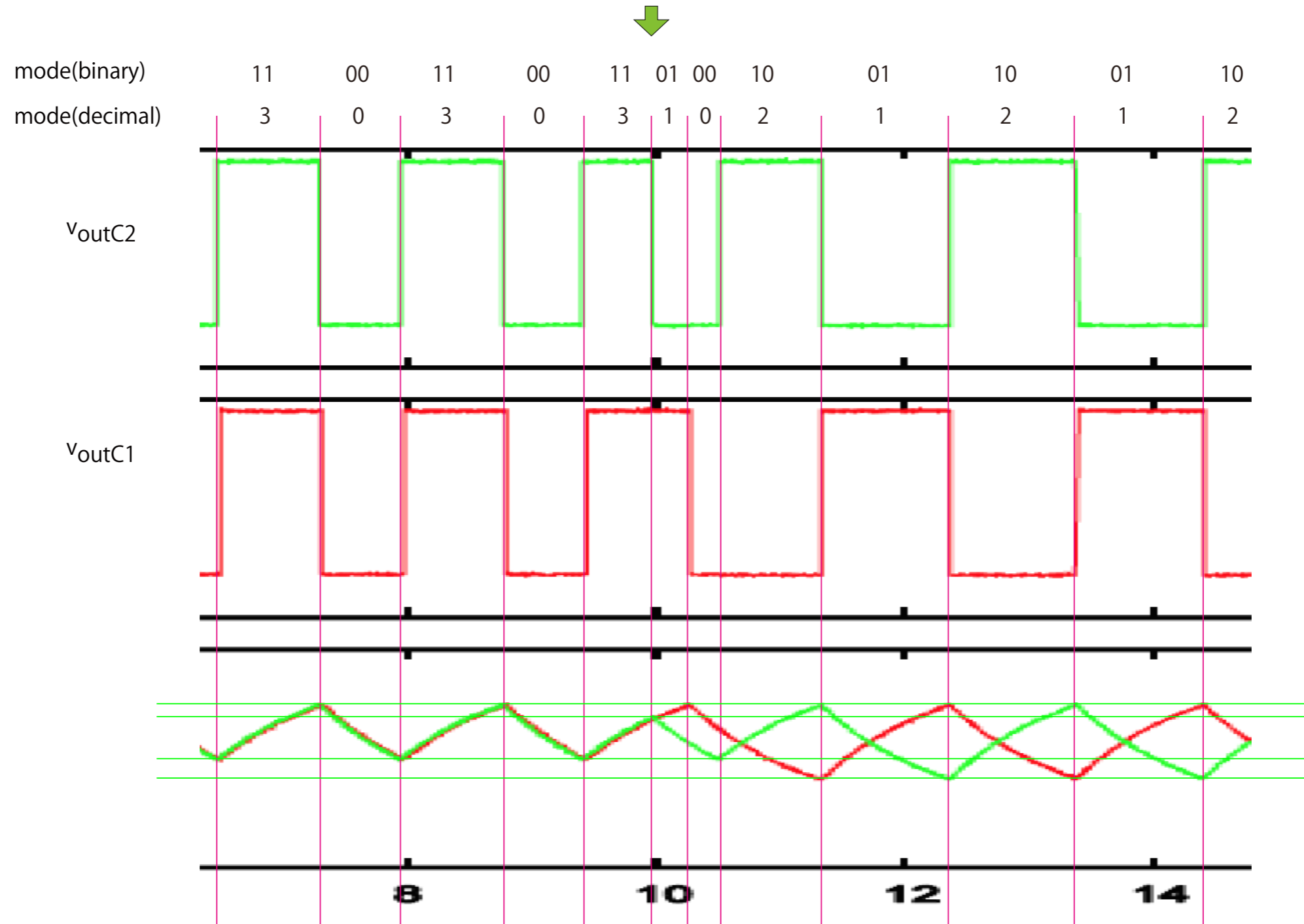
質問 1 : 回路実験と数値解析で波形に一致しない箇所がある

data 1: CI-CI coupling

data 2: AI-CI coupling



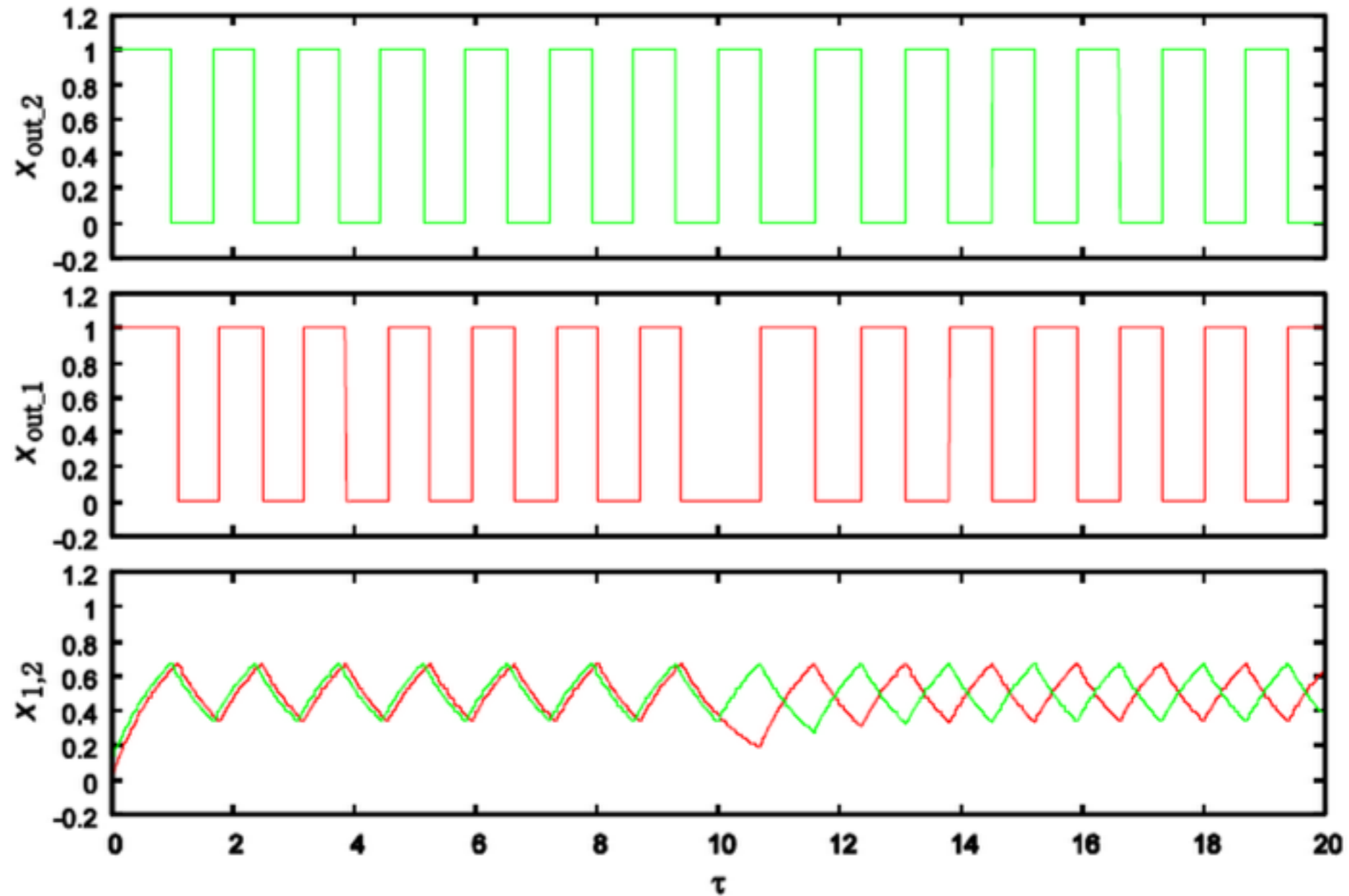
typeC-typeC coupled system(Experimental results) p. 14
in ExperimentalResults_20120402.pdf

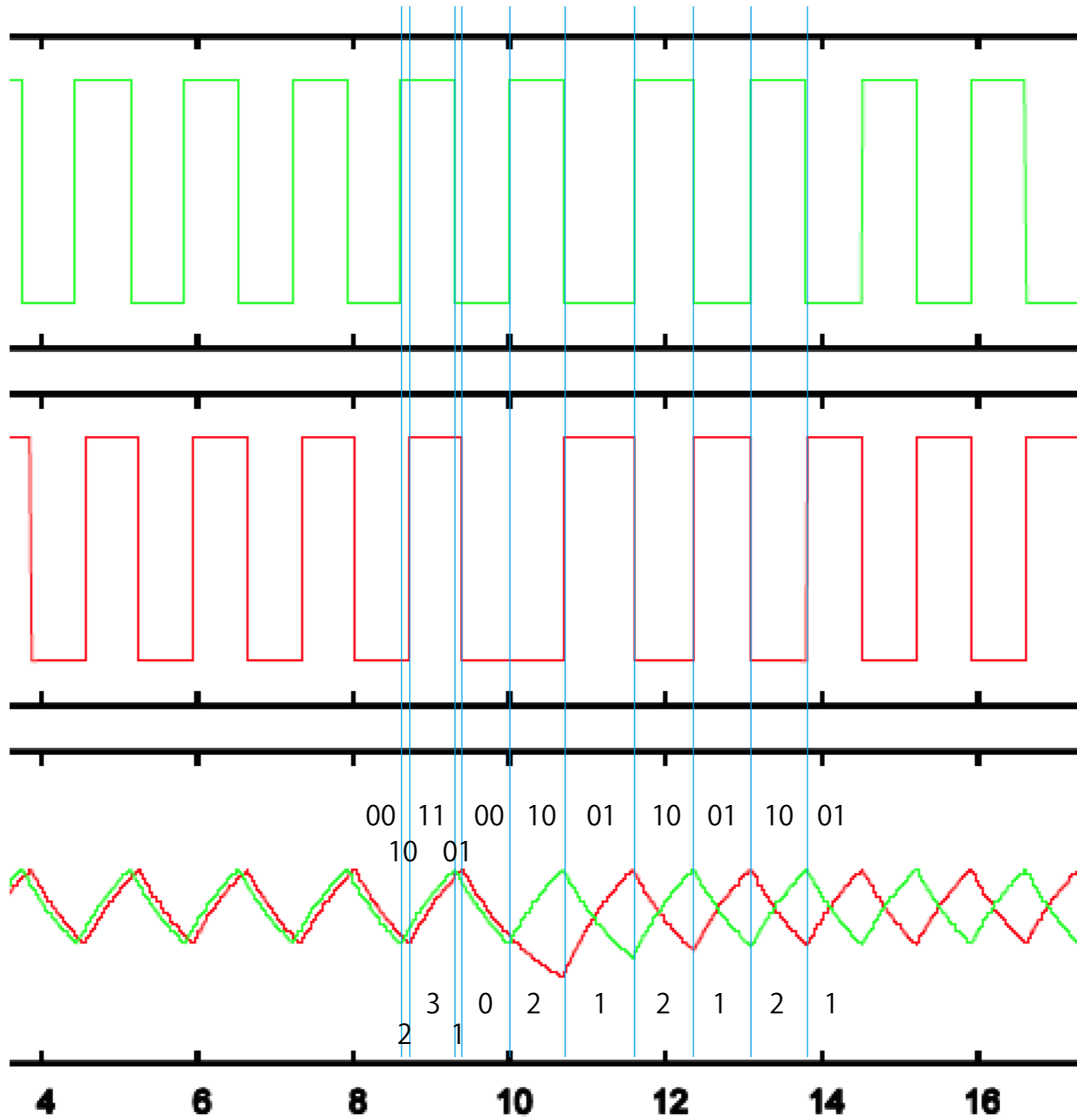
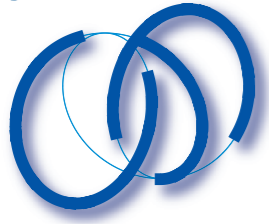


310212121...



typeC-typeC coupled system (Numerical simulation)



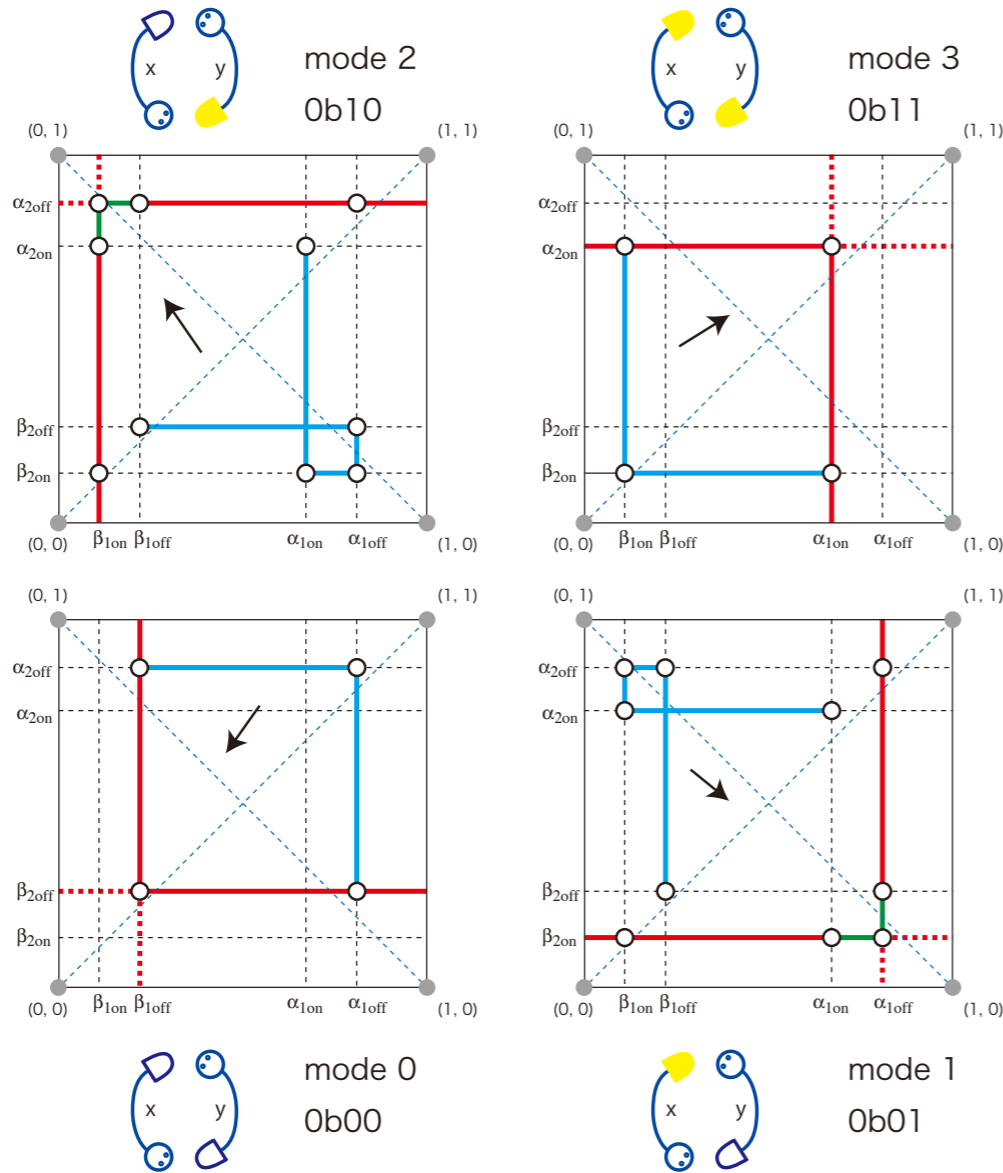


310212121...

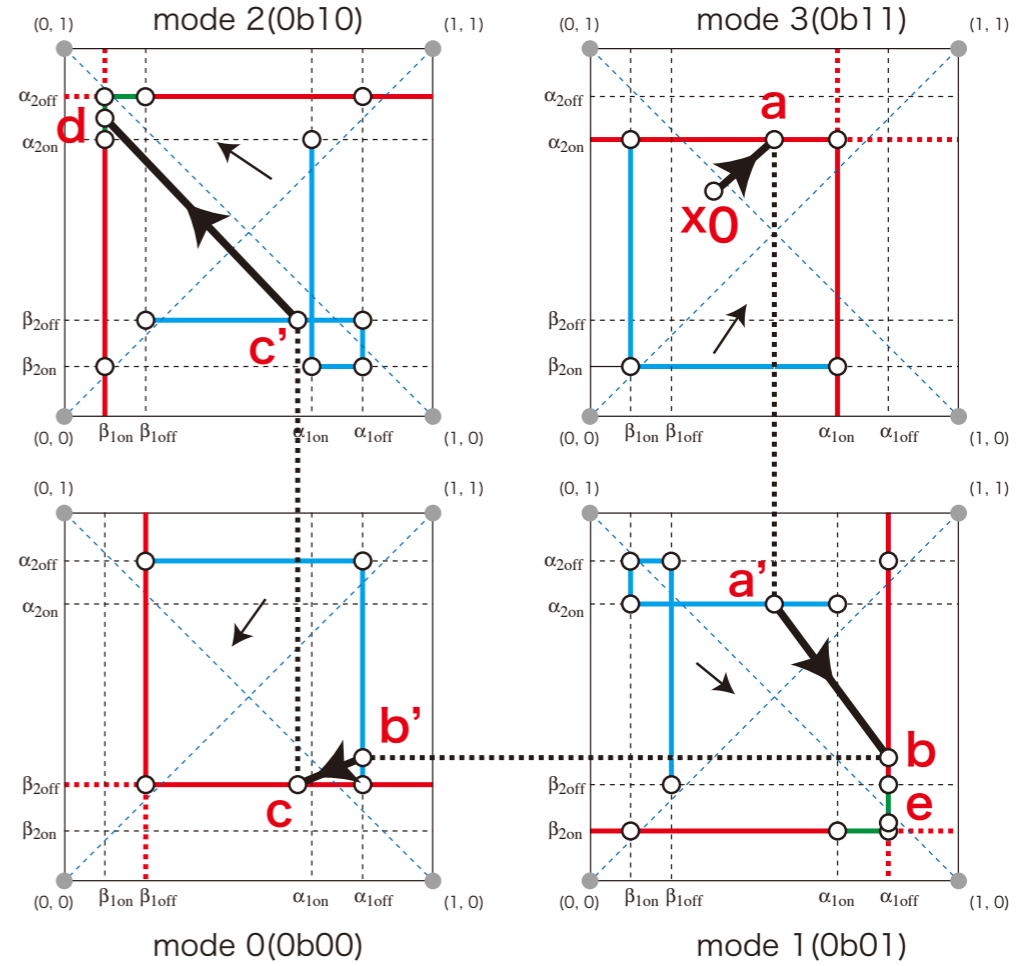


- arrival set
- departure set
- codimension 2 property

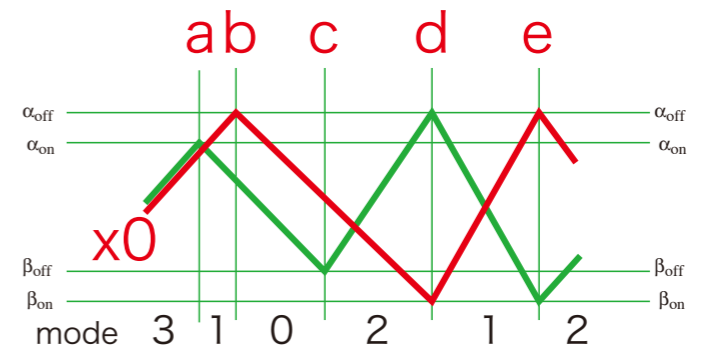
type C1-C1 circuit : phase portrait

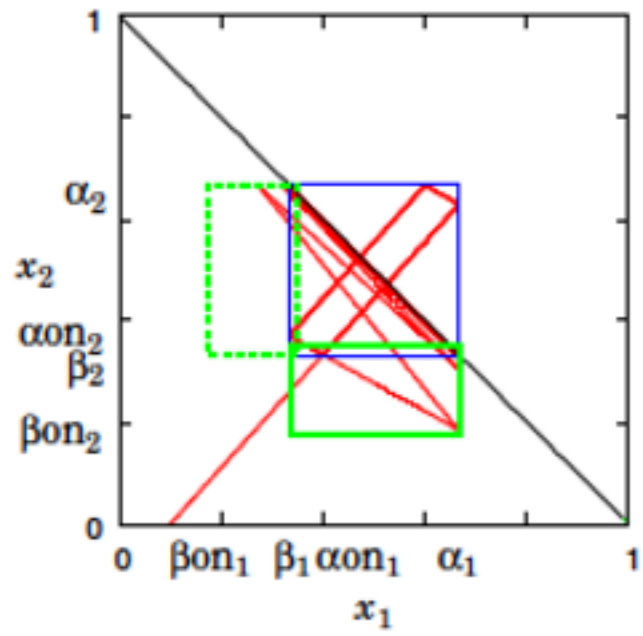


template for vector fields

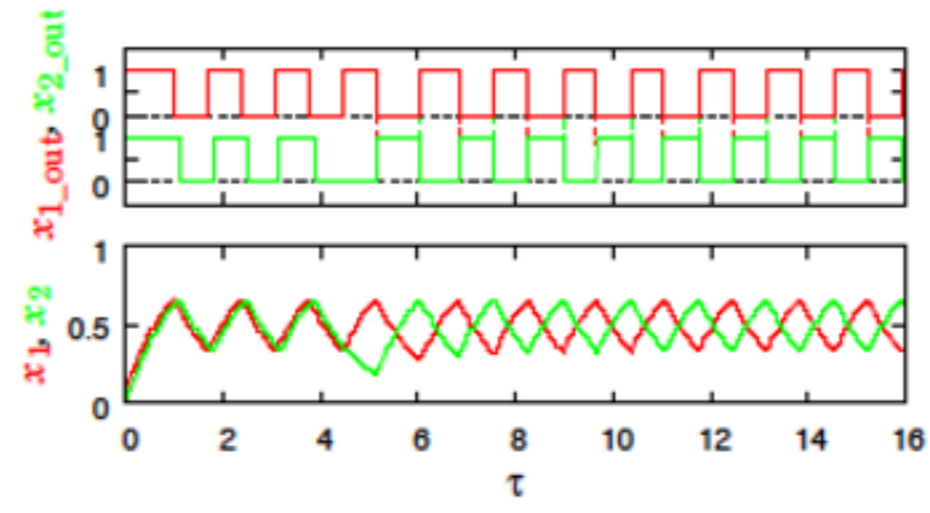


waveforms

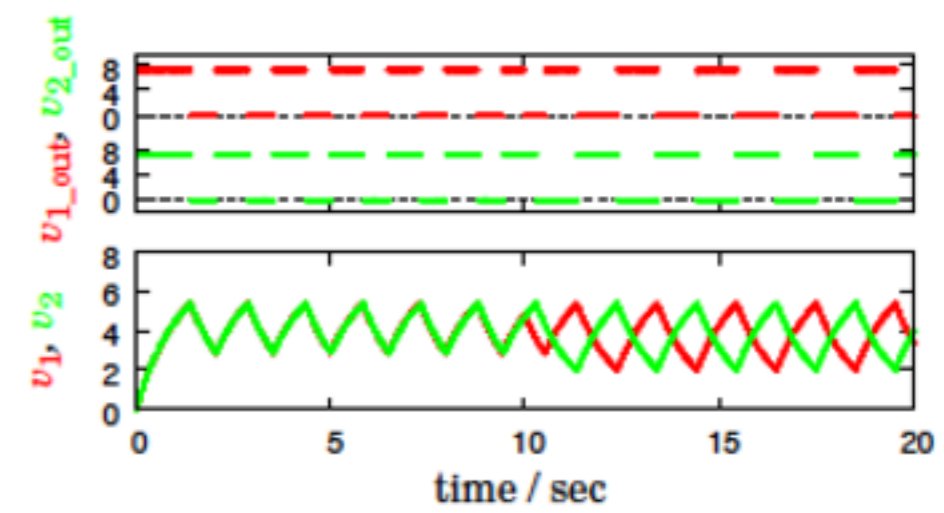




(c1)

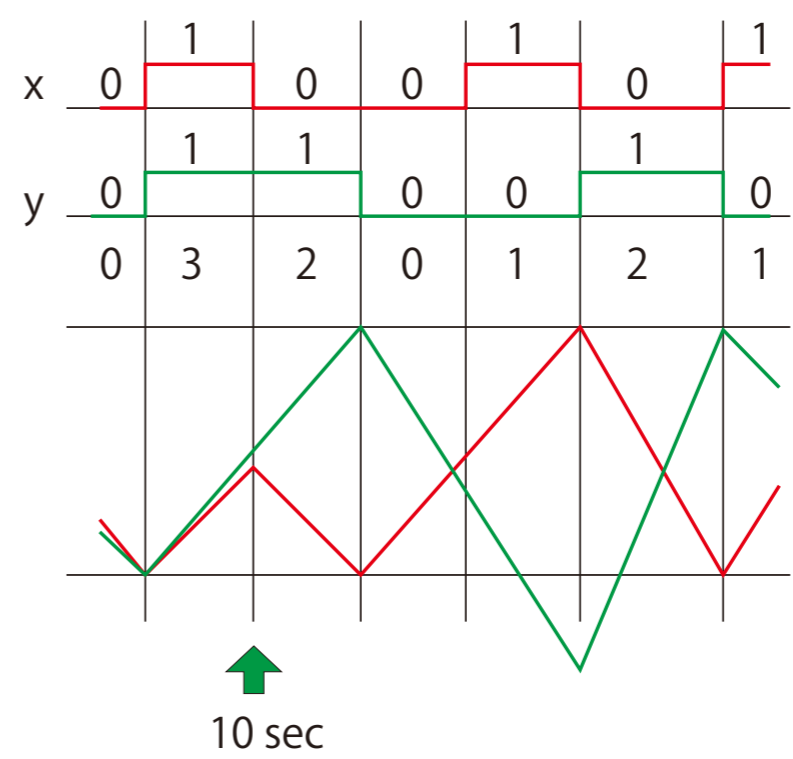
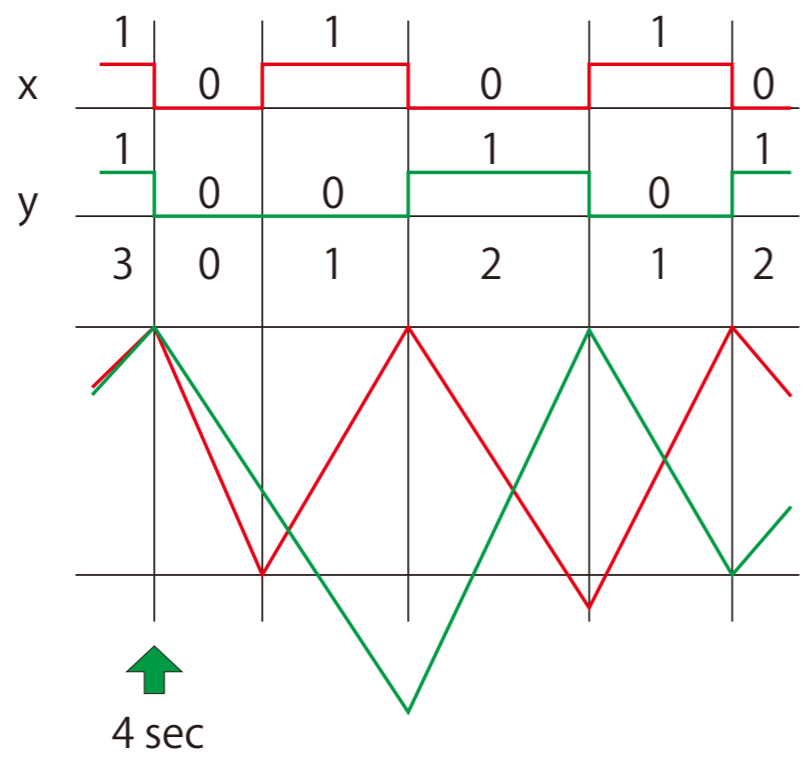


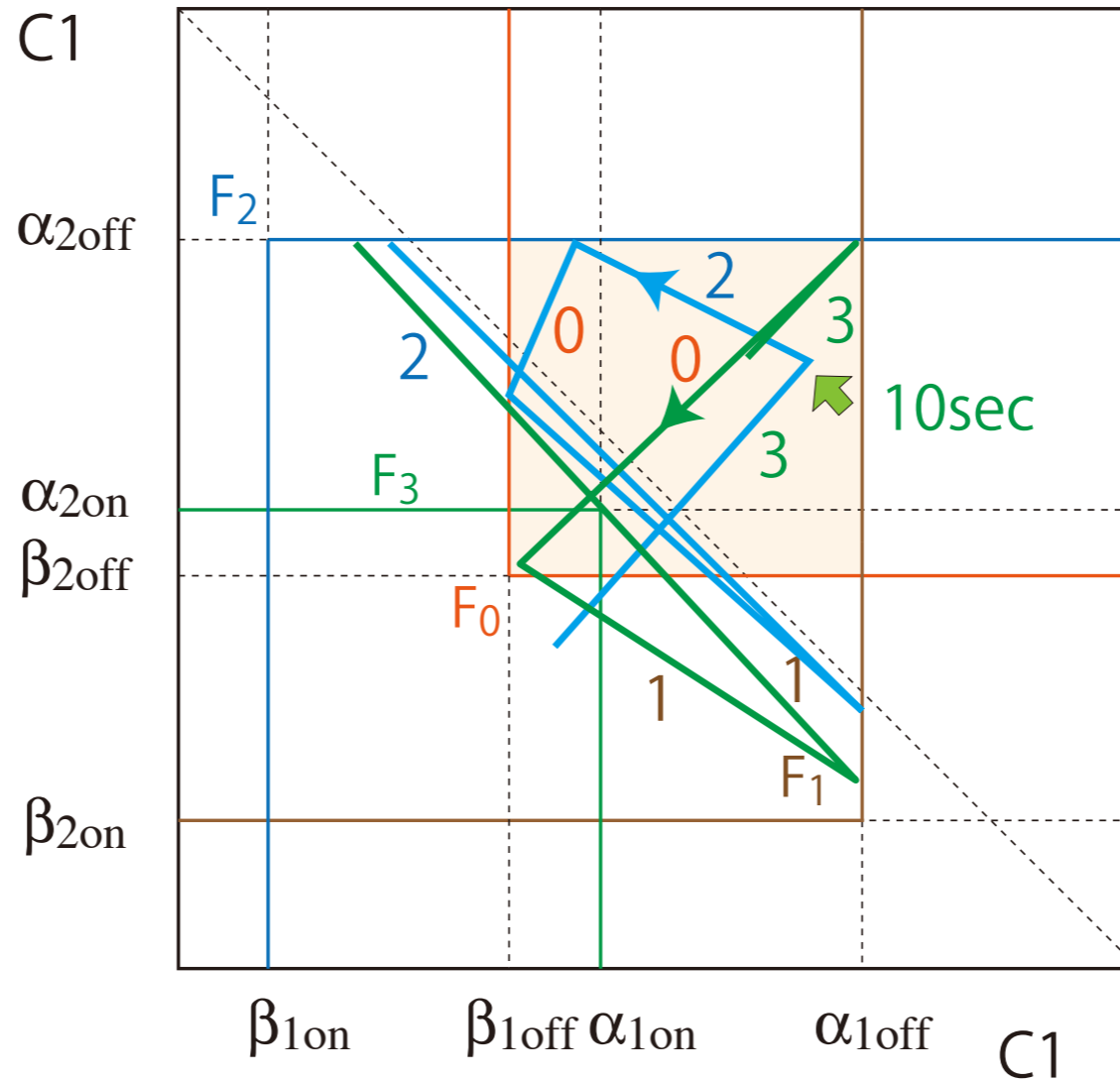
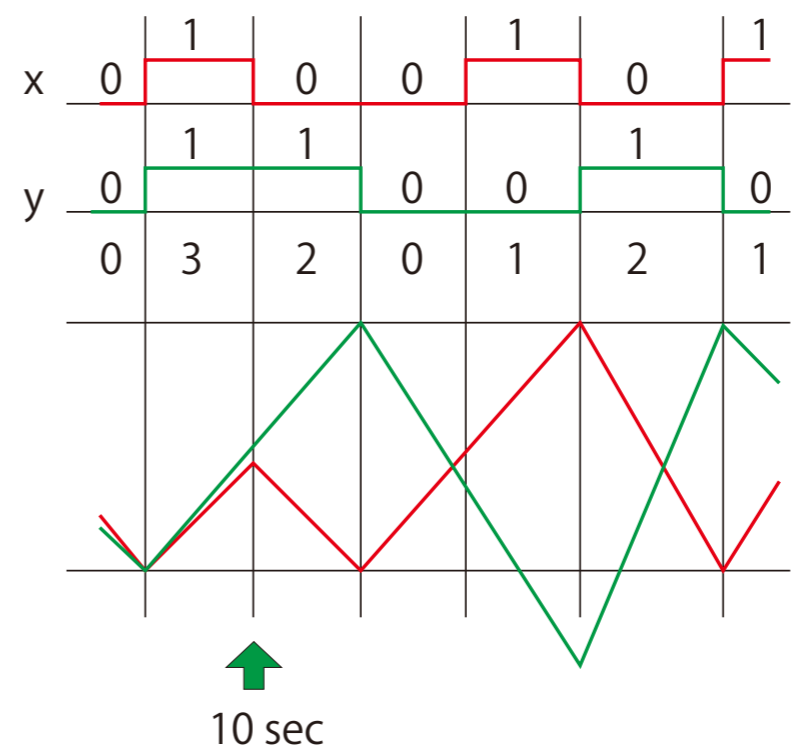
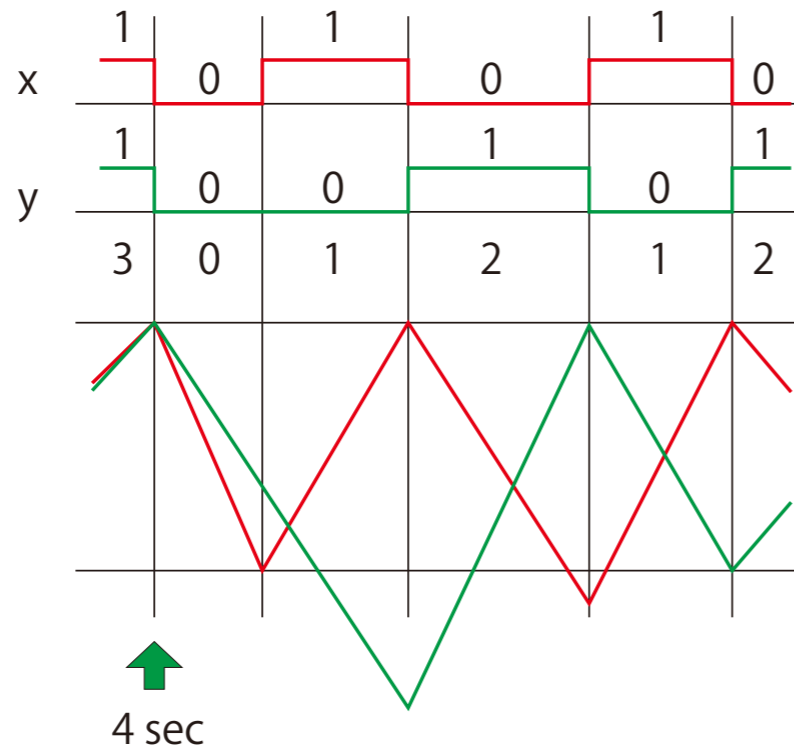
(c2)



(c3)

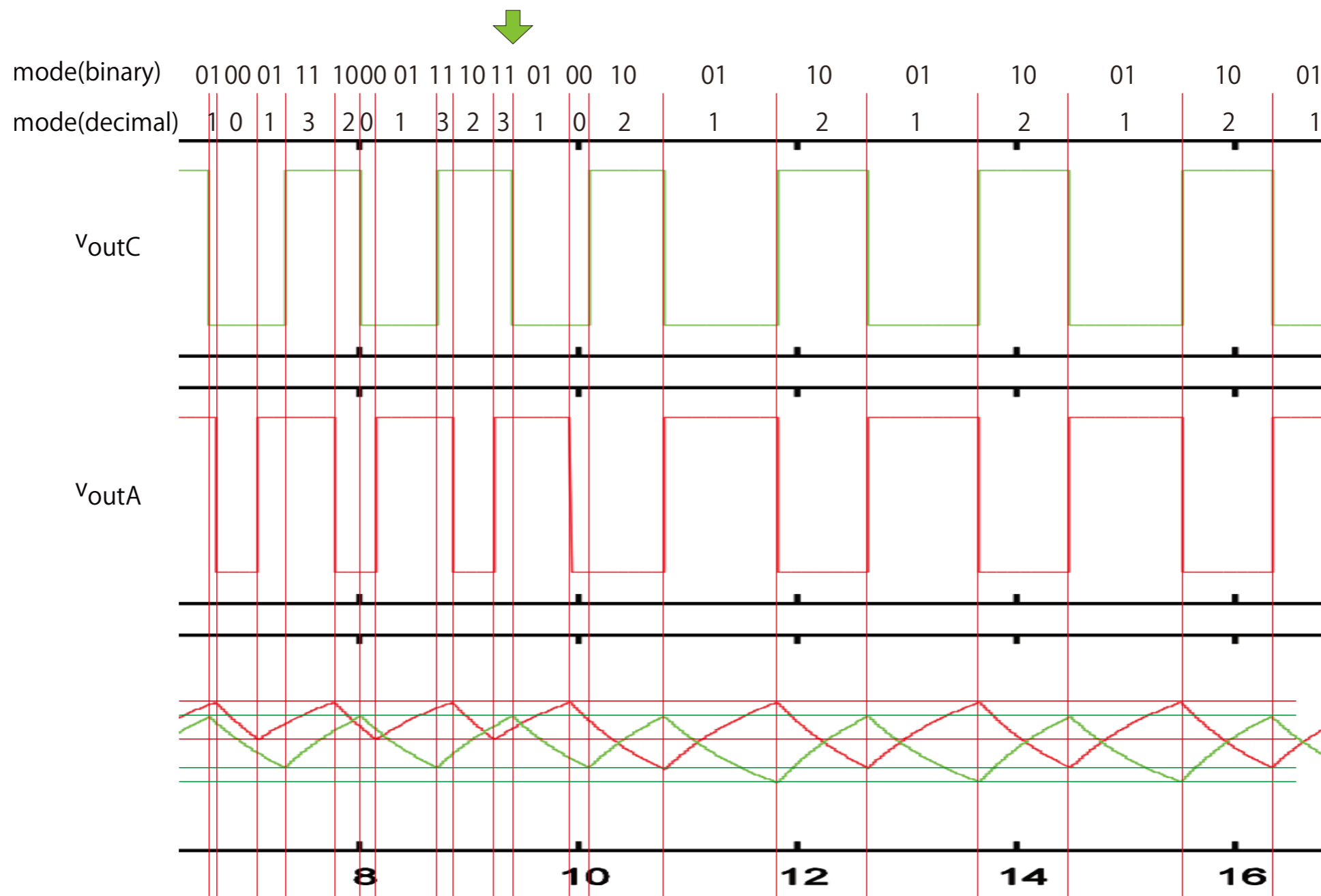
Fig8 (c) from Sekikawa's note.



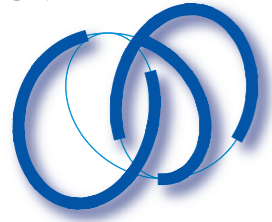




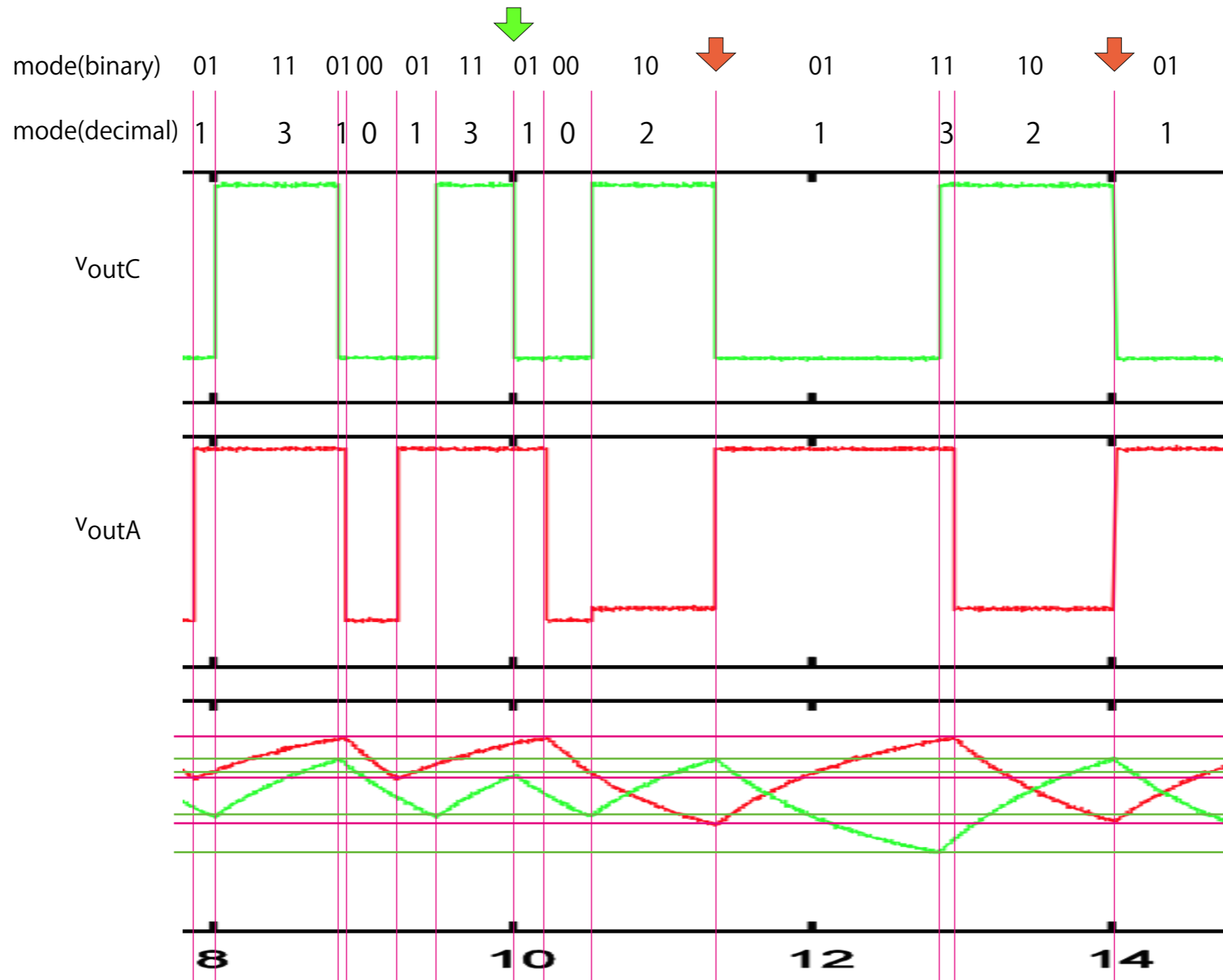
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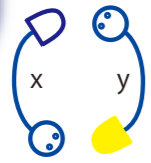
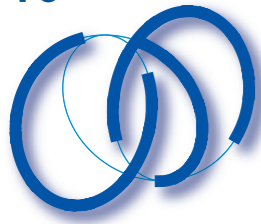
310212121...



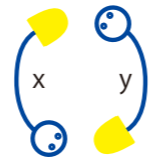
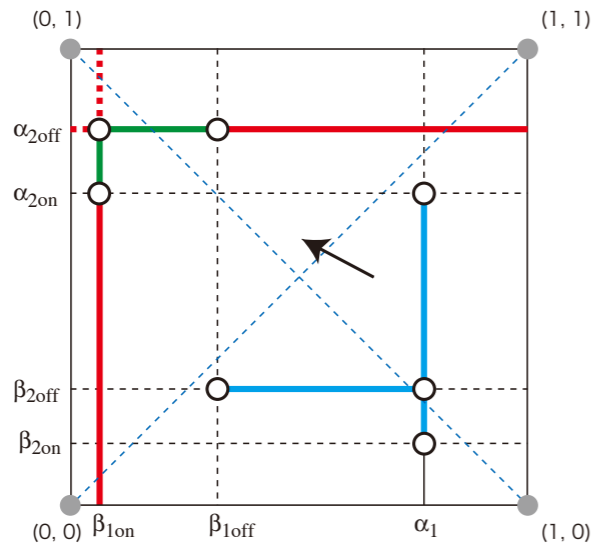
typeA-typeC coupled system(Experimental results) p. 21
in ExperimentalResults_20120402.pdf



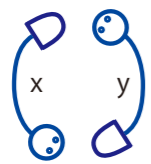
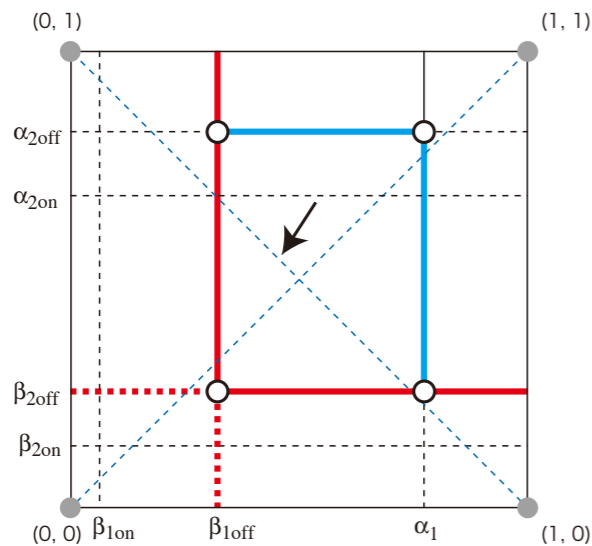
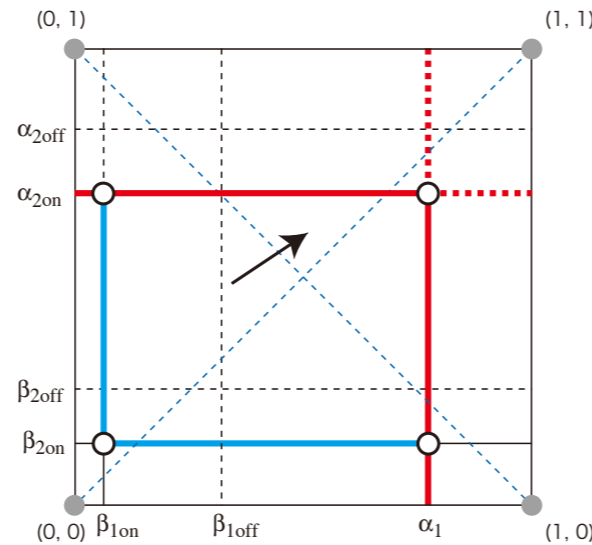
31021321321...



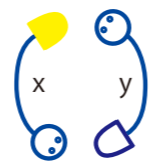
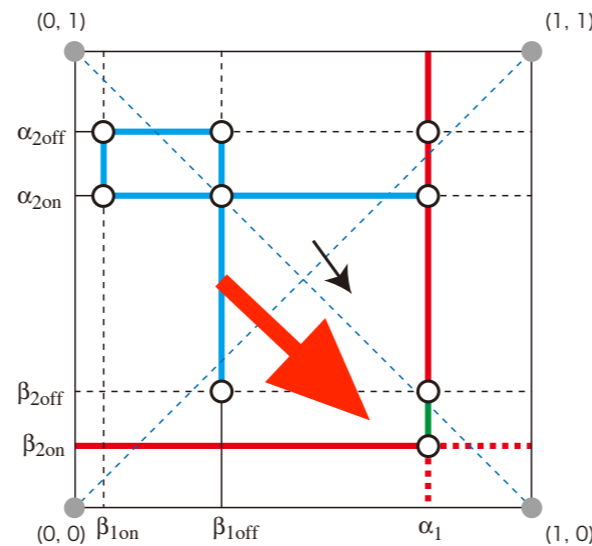
mode 2
0b10



mode 3
0b11



mode 0
0b00



mode 1
0b01

type A1-C1 circuit : phase portrait

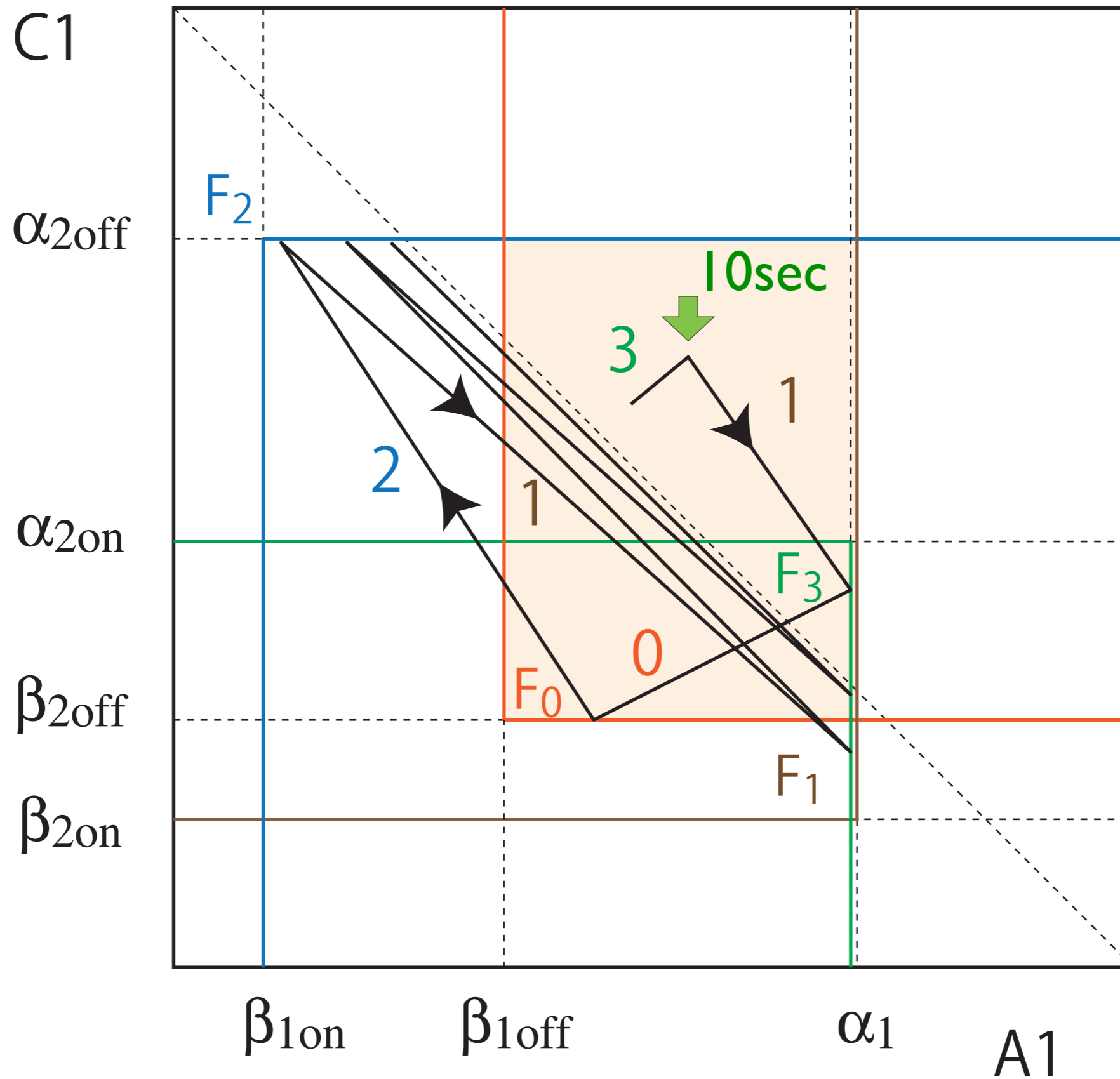
- arrival set
- departure set
- codimension 2 property

- 1) 横軸 x (type A) の閾値 α_1 はモード 1, 3 で同じ
- 2) 縦軸 y (type C) の閾値 α はモード 2 で α_{2off} , モード 3 で α_{2on} となり, 違った閾値となる.
これが波形がモード 3 で上の閾値が 1 回小さくなる場合に対応している. experimental results は正常といえる.
- 3) 定常状態では, $-45deg$ の対角線に漸近する運動となる.
したがって, 左右両端で codim2 の arrivale set に到達し, 同時切り換えが起こり, 逆相同期解がみられる.
- 4) 関川氏の numerical simulation ではモードの遷移 3 から 1 へのときの v の閾値が低くなっていない. 回路の抵抗の値を変えて閾値の変化量をうまく選ぶと, 実験データと同じ結果が得られるかも知れない.

310212121...

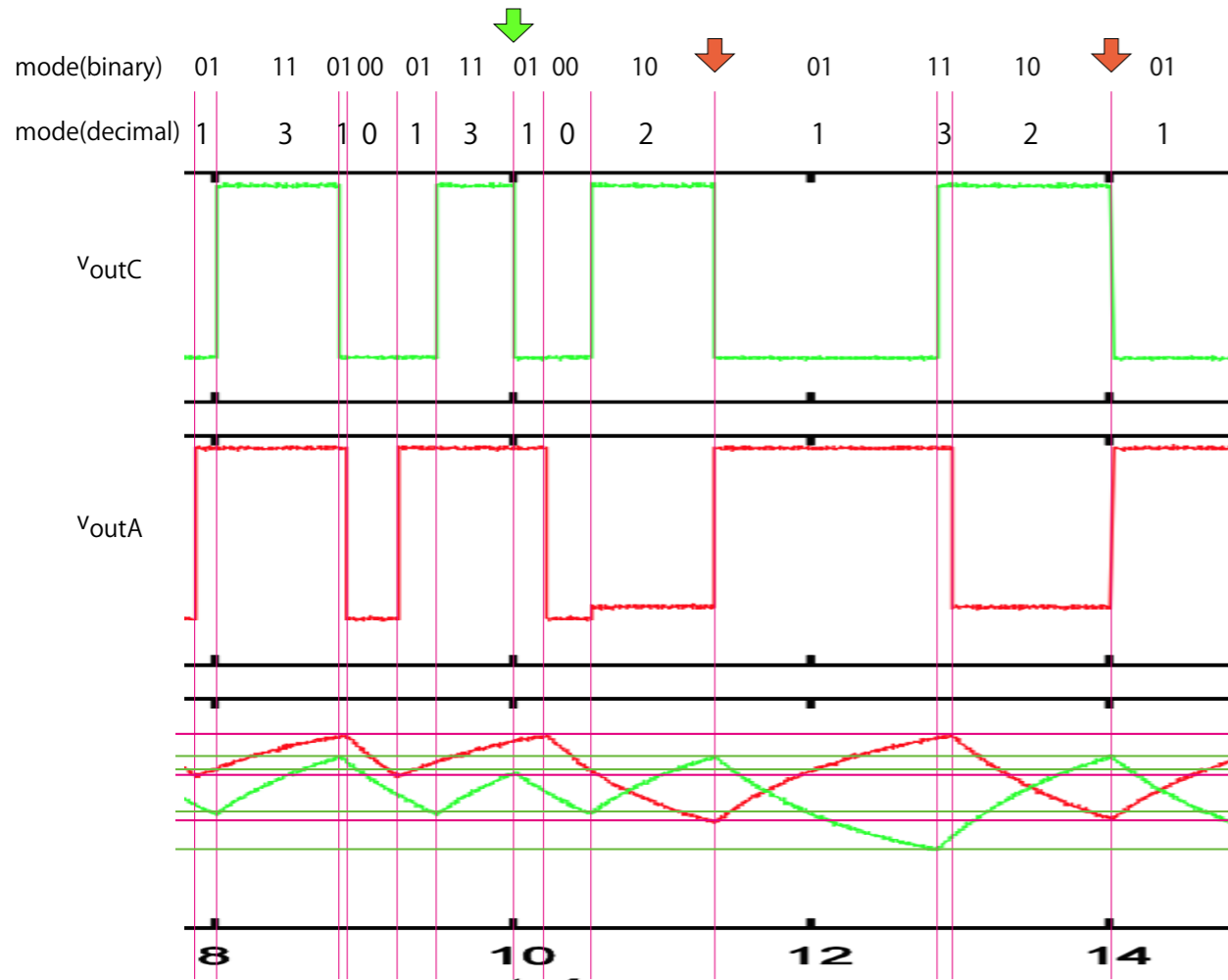
31021321321...

2121...と213213...の違いは？



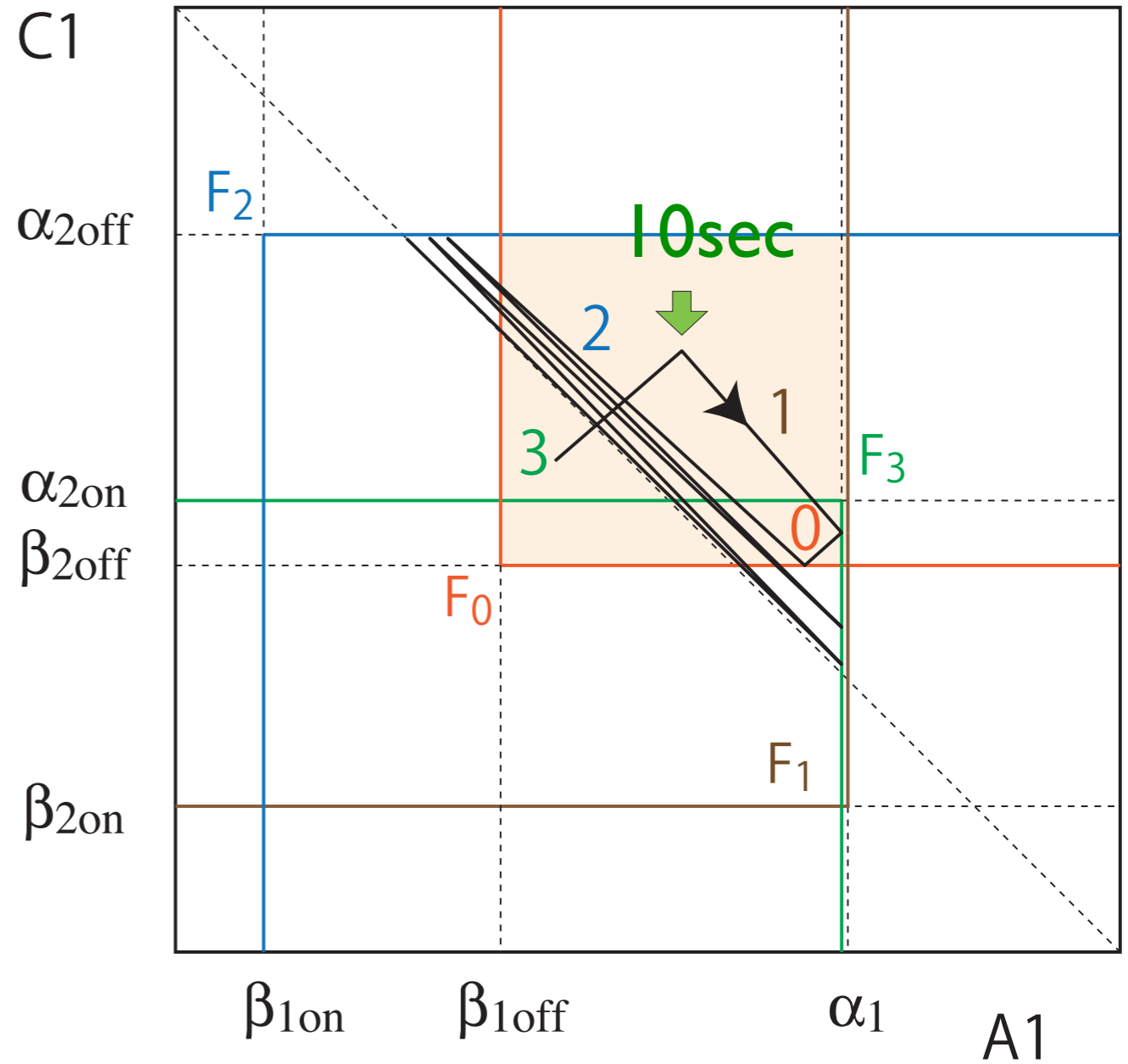


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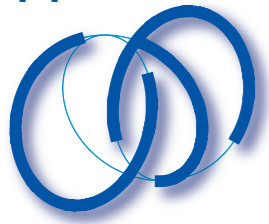
31021321321...

C1

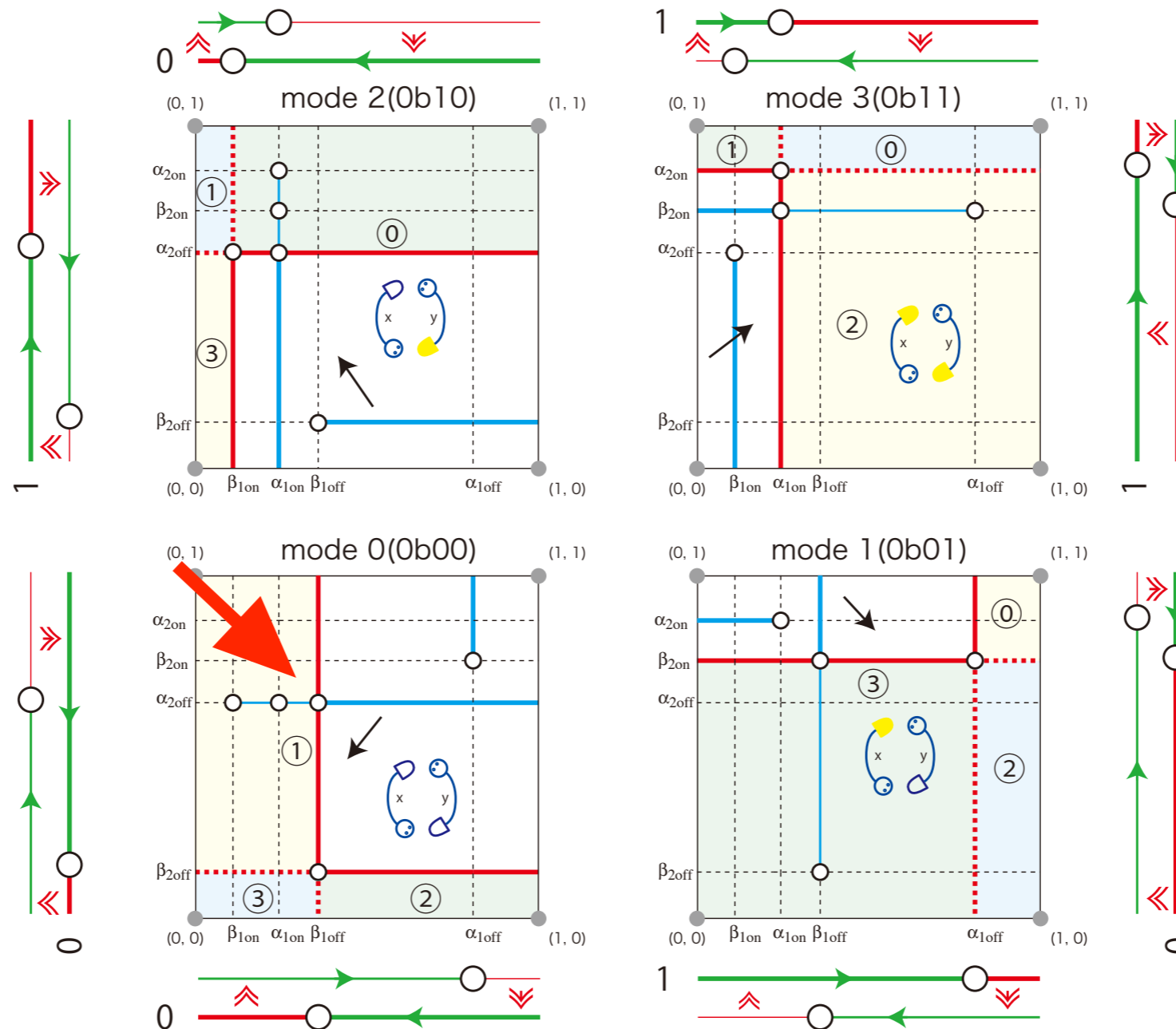




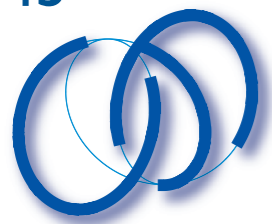
**質問2 : CI-DI 結合では, oscillation deathがおこる.
なぜ?**



type C1- D1 circuit : phase portrait

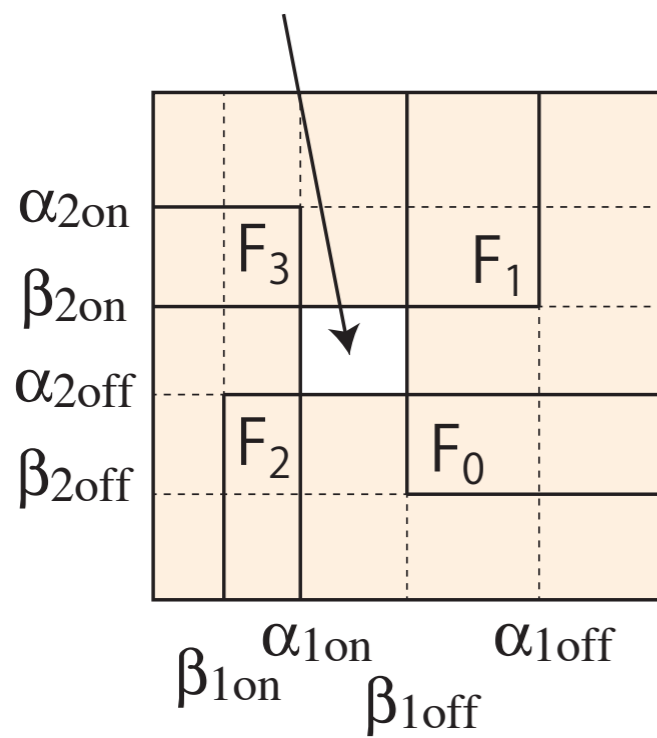


- 1) あるinvariant set 上で transition cycleがおこる.
- 2) このinvariant set はattractiveである.

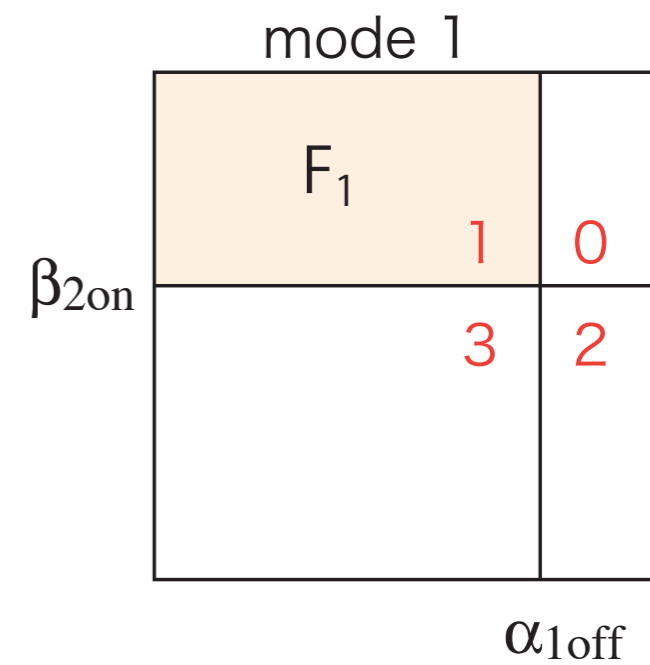
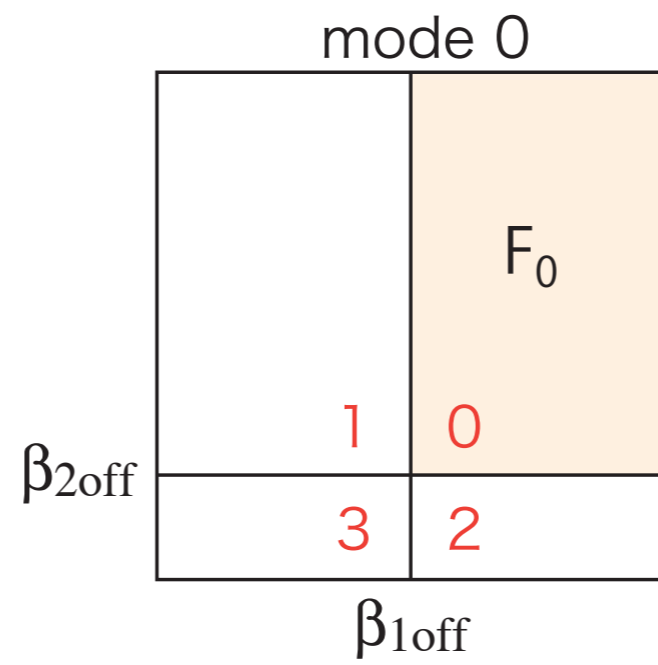
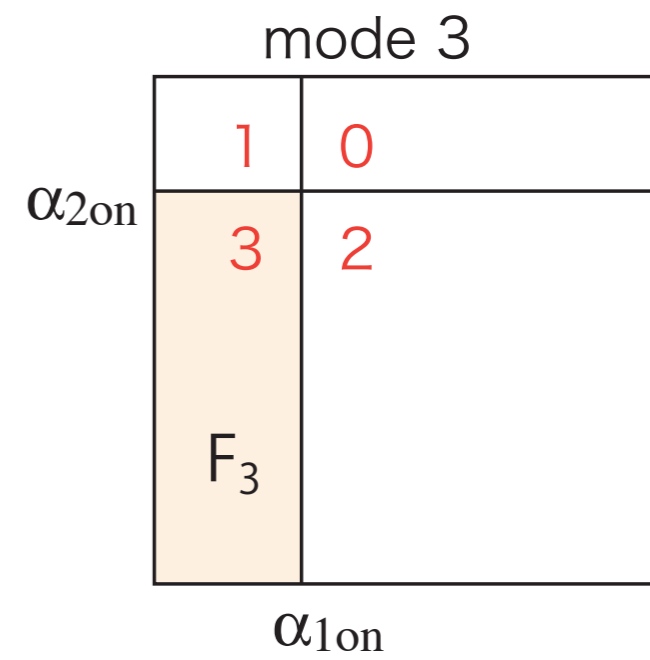
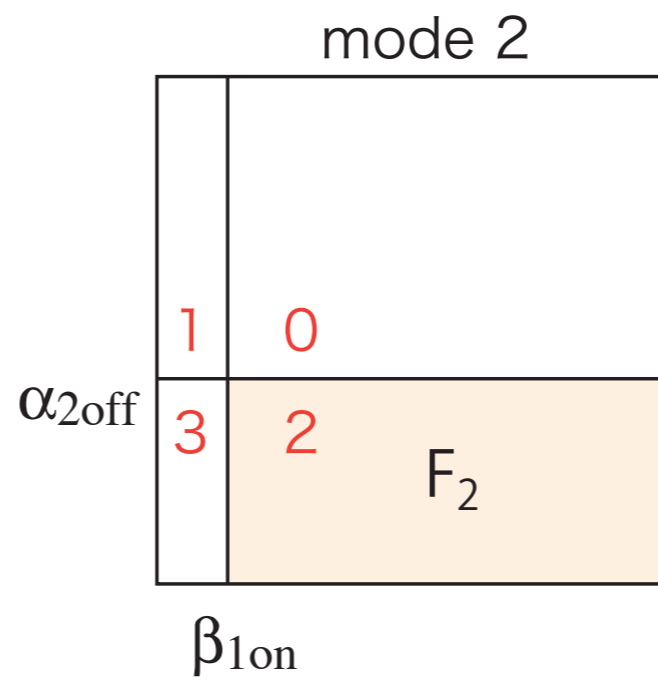


transition cycle:

$0 > 1 > 3 > 2 > 0$



(a)

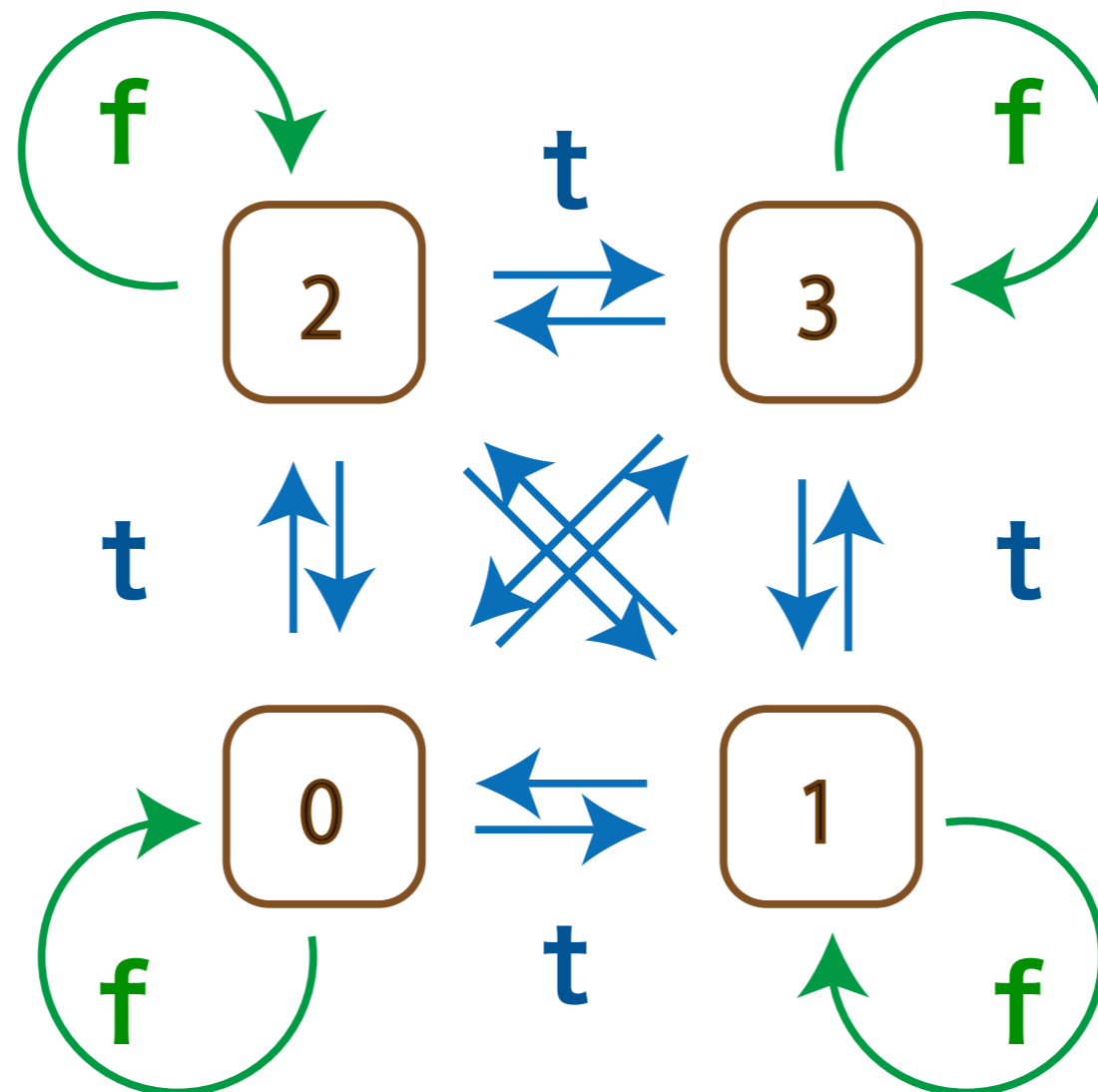


(b)



Hybrid系：状態の進展と遷移

flowが進展している時は、modeは固定されている
modeが遷移している時は、flowは固定されている





	1	2	1
α_2			
	2	4	2
β_2			
	1	2	1
	β_1		α_1

C1	1	1	2	2	1
α_{2off}		F_2			
	1	2	3	3	2
α_{2on}				F_0	
β_{2off}	2	3	4	3	2
			F_3		
	2	3	3	2	1
β_{2on}				F_1	
	1	2	2	1	1
	β_{1on}	β_{1off}	α_{1on}	α_{1off}	C1



Hybrid系の定常状態

f: flow t: transition

- 1) fffffff... flowのattractorに収束
- 2) ftftftft... flowとtransitionの繰り返し
- 3) fttftttftt... flowと transitionⁿ の繰り返し
- 4) fttttfttt... flowと transitionⁿ の繰り返し
- 5) tttttt... transition cycle(loop)