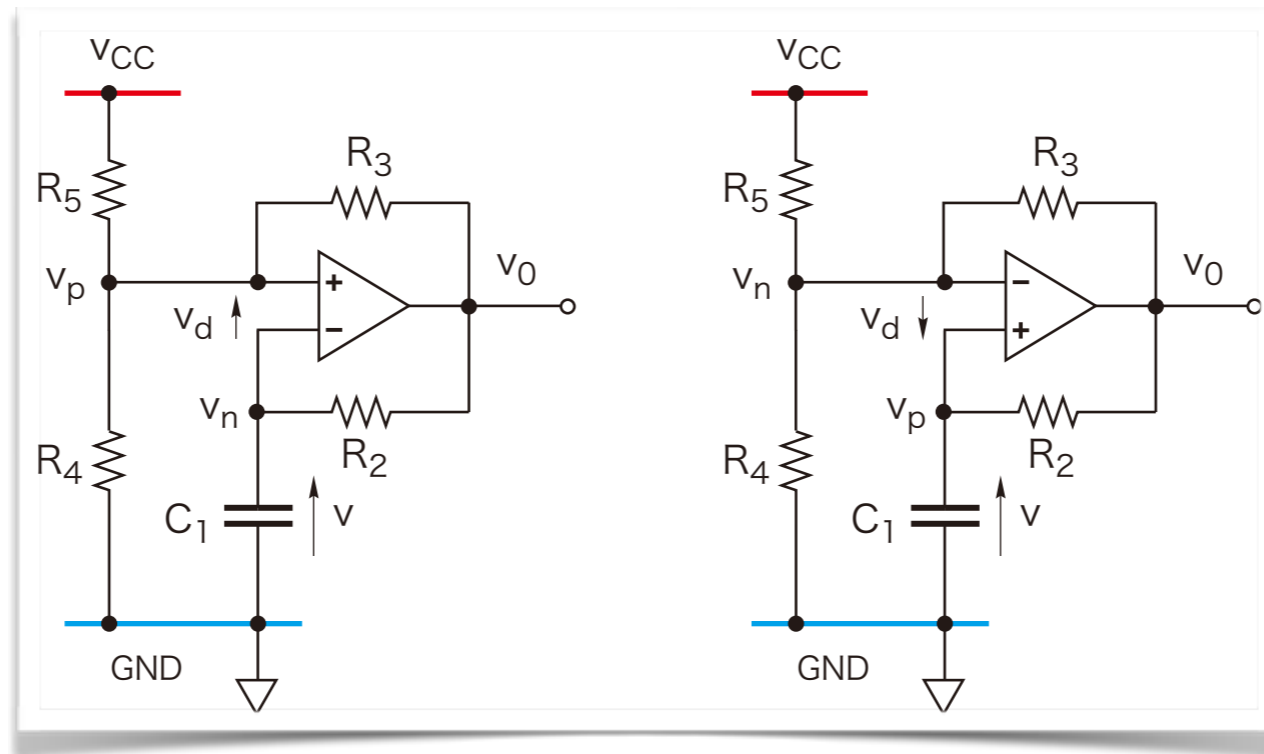




NBA 2015 in Kagawa

Op amp multivibrator

phase portraitで考えよう



<http://cms.db.tokushima-u.ac.jp/DAV/person/S10723/NBA2015/>

川上 博

2015(H27).12.19



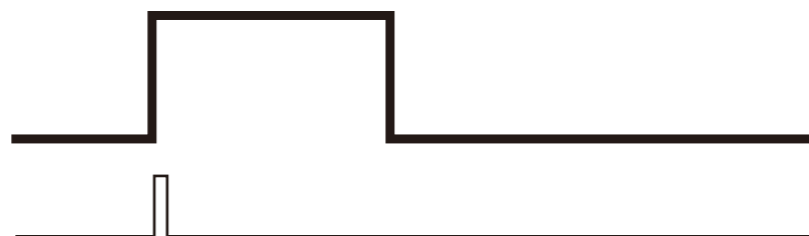
マルチバイブレータ

3つのタイプがある

- **astable multivibrator:** 方形波（弛張振動）発振器



- **monostable multivibrator:** 入力パルスがあると1度だけ出力方形波を出力する

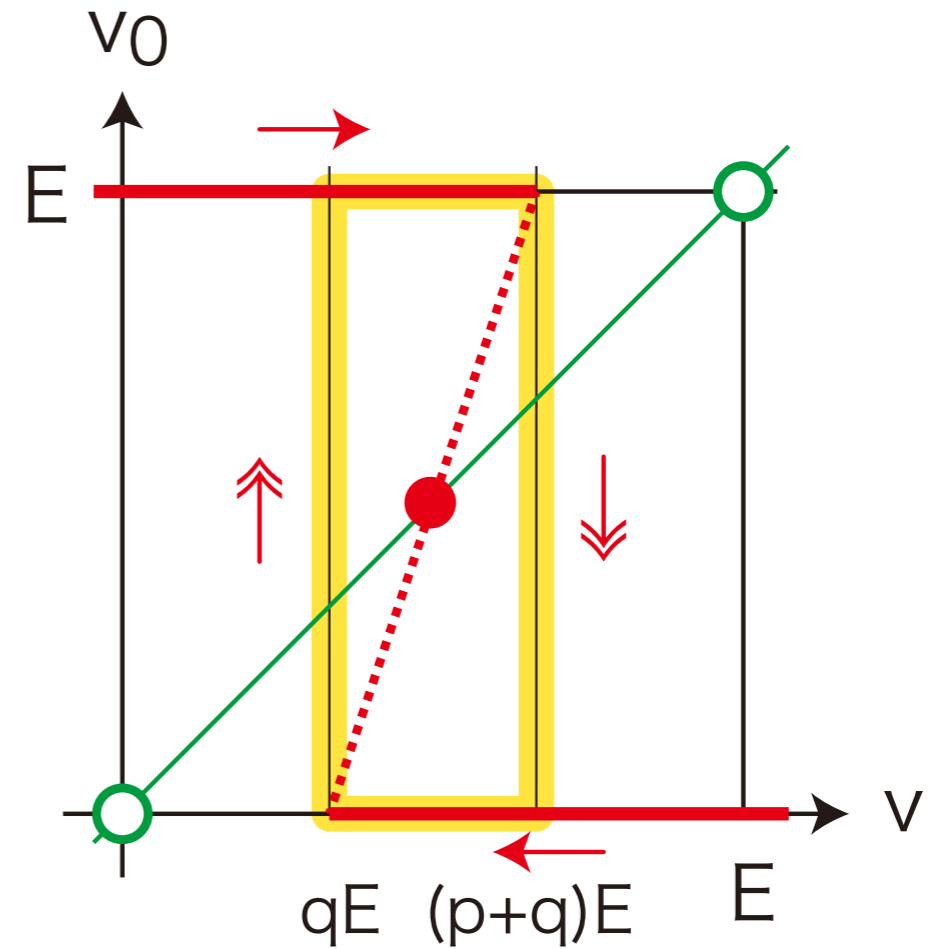
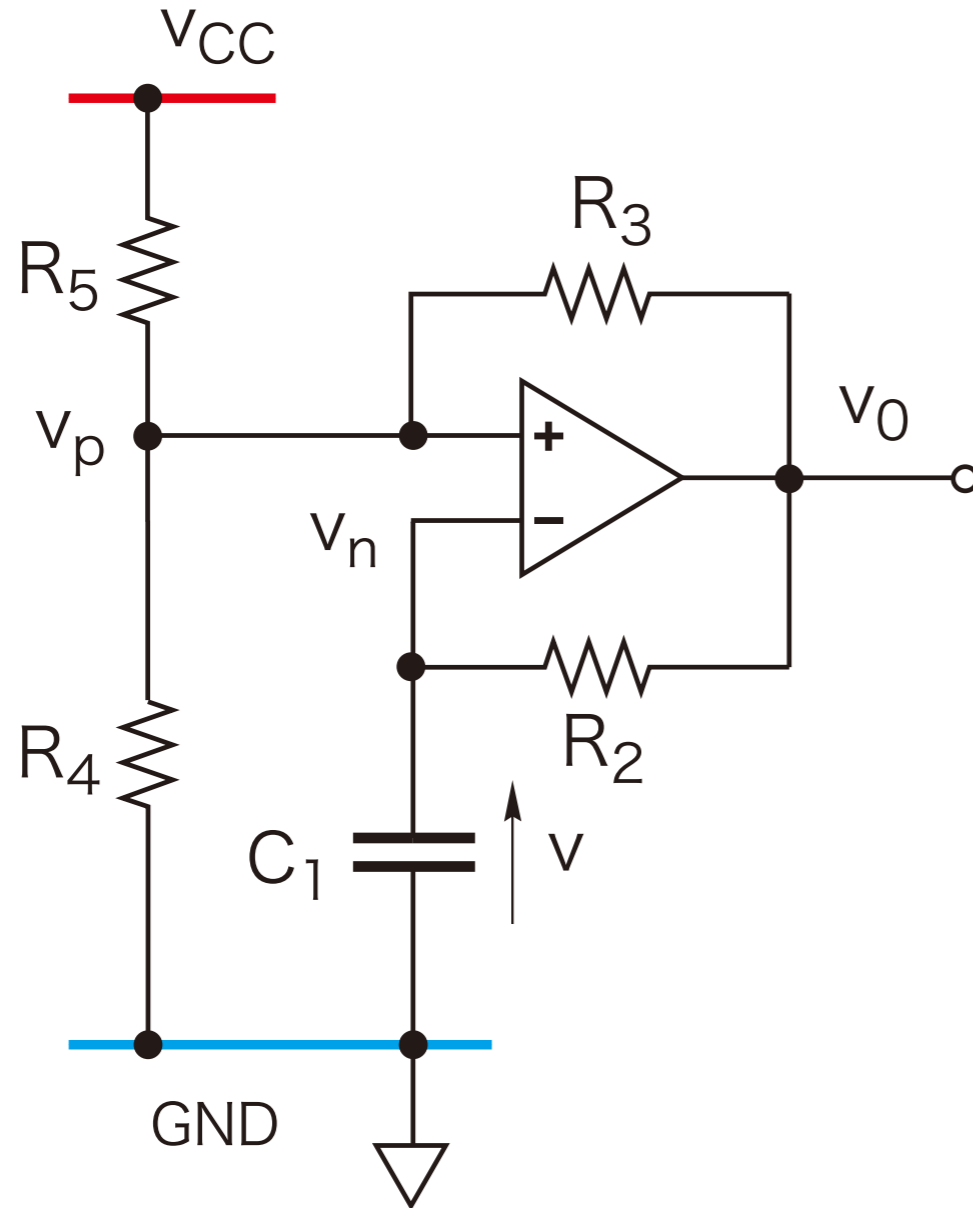


- **bistable multivibrator:** 2つの安定状態を持つ : flip-flop





Astable multivibrator

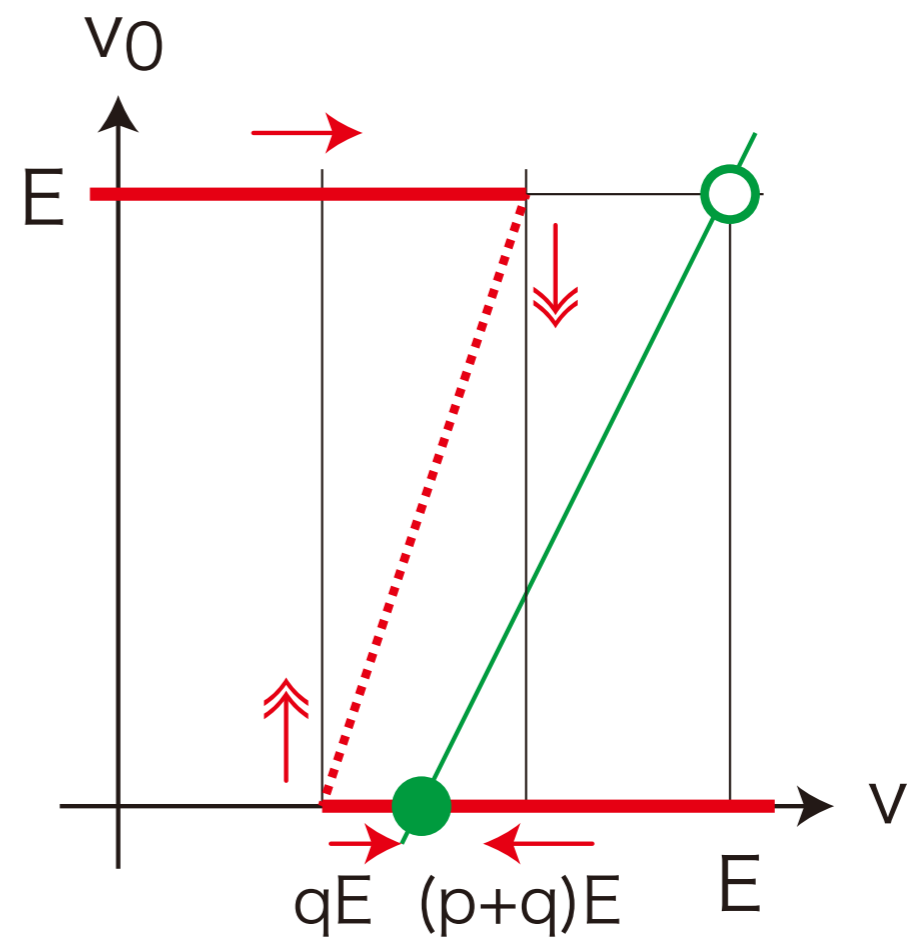
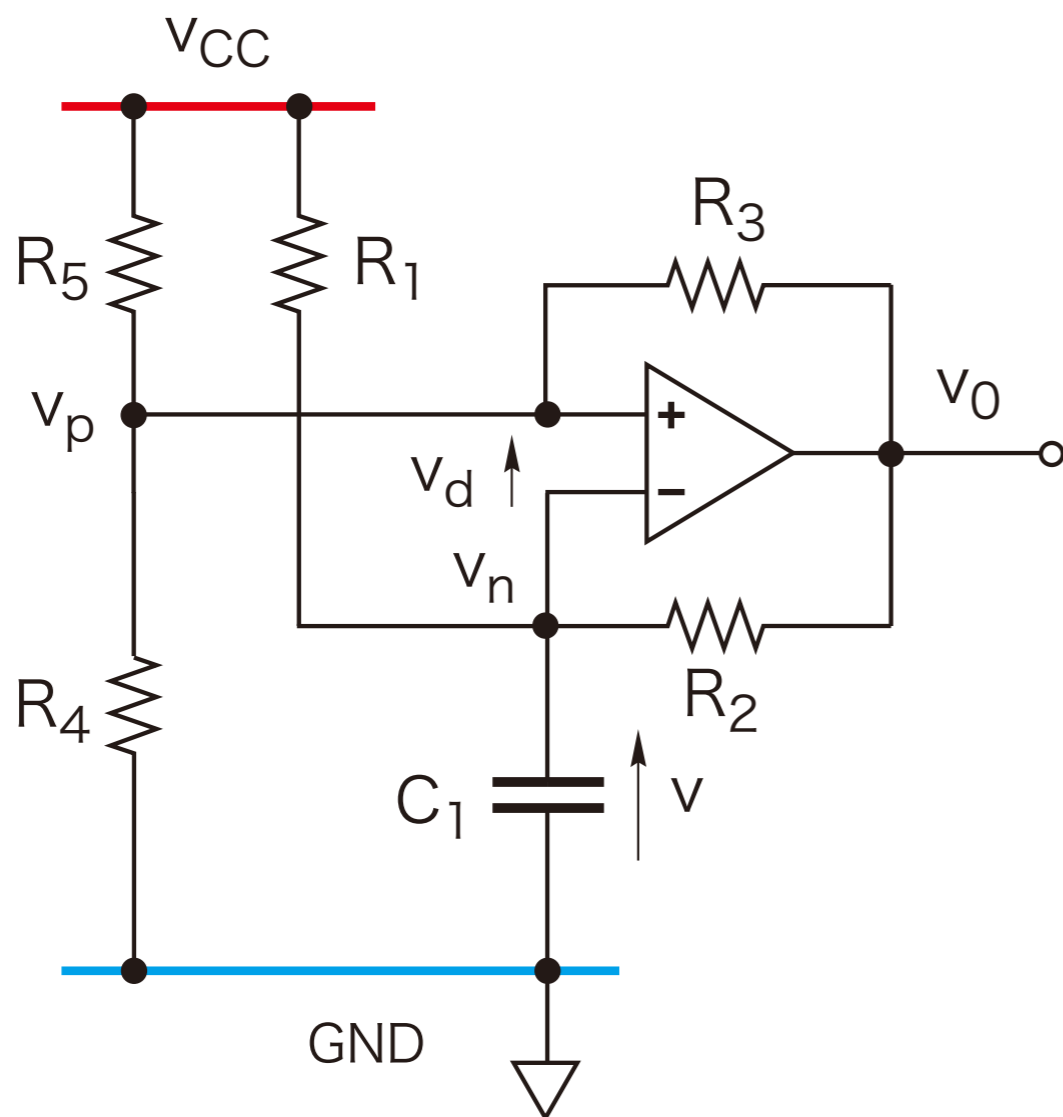


stable limit cycle

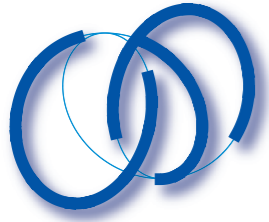
square wave generator



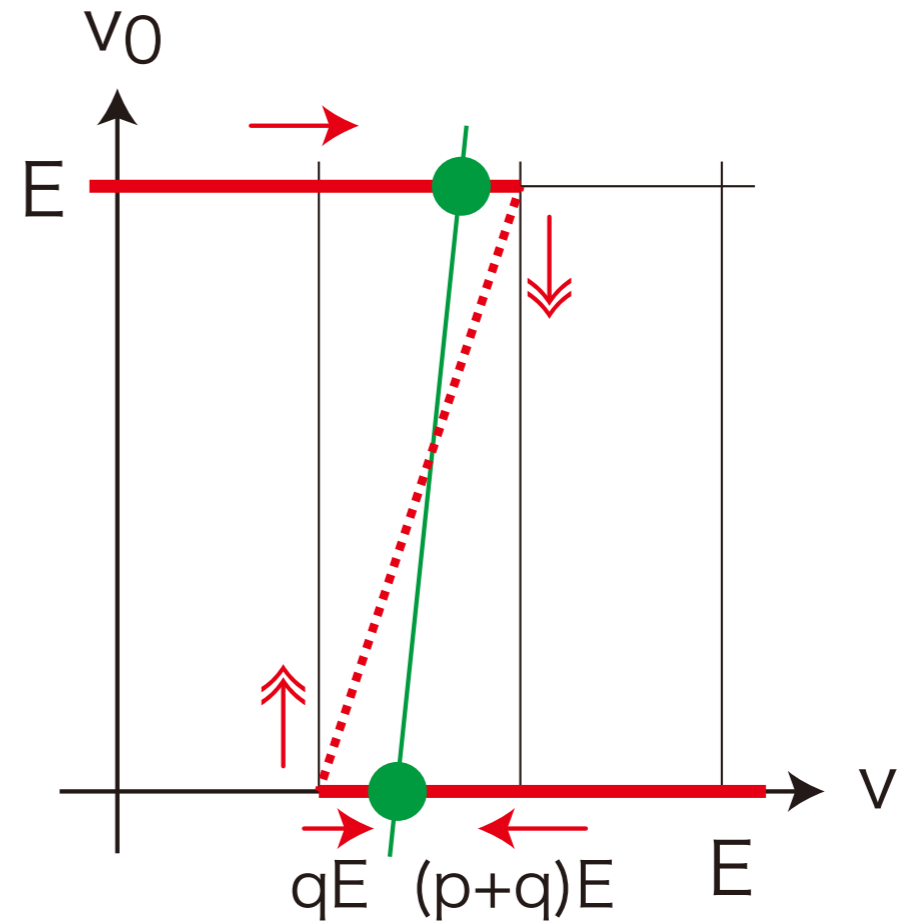
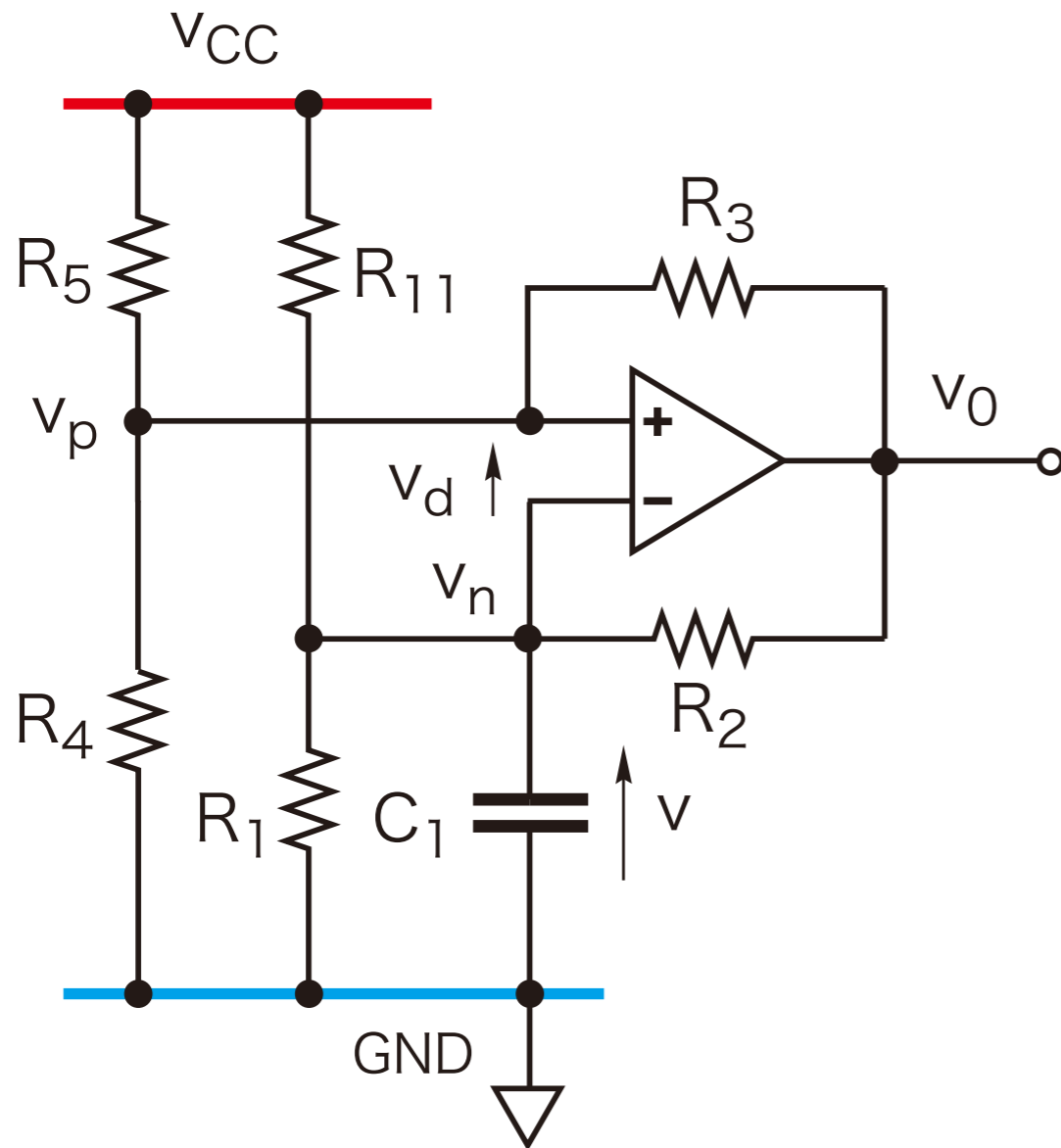
Monostable multivibrator



安定平衡点：stable low

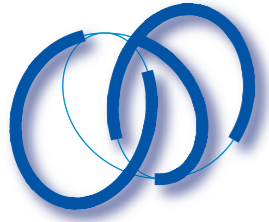


Bistable multivibrator : flip-flop

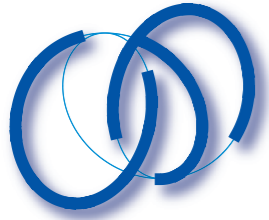


2個の安定平衡点

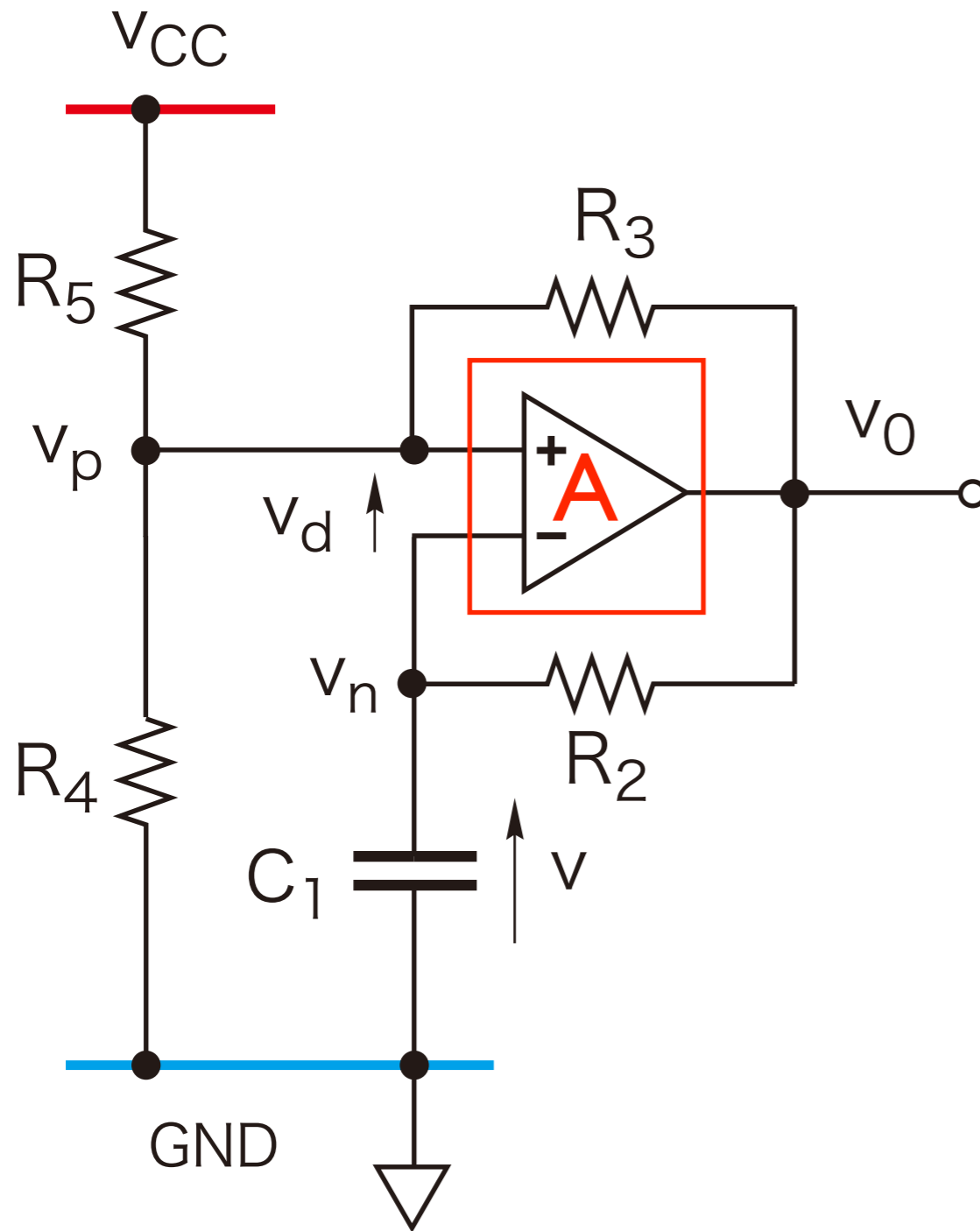
6



Astable multivibrator



Astable multivibratorの解析



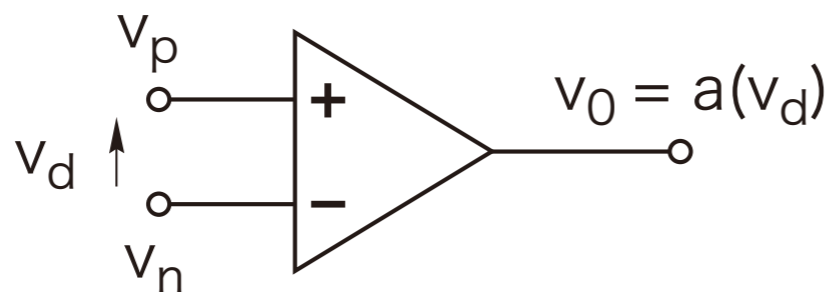
解析の手順

1. オペアンプの静特性
2. feedback 回路方程式
3. 総合して解析
4. オペアンプの動特性
5. slow-fast model
6. hybrid model

赤の正方形の内外を別々に解析する

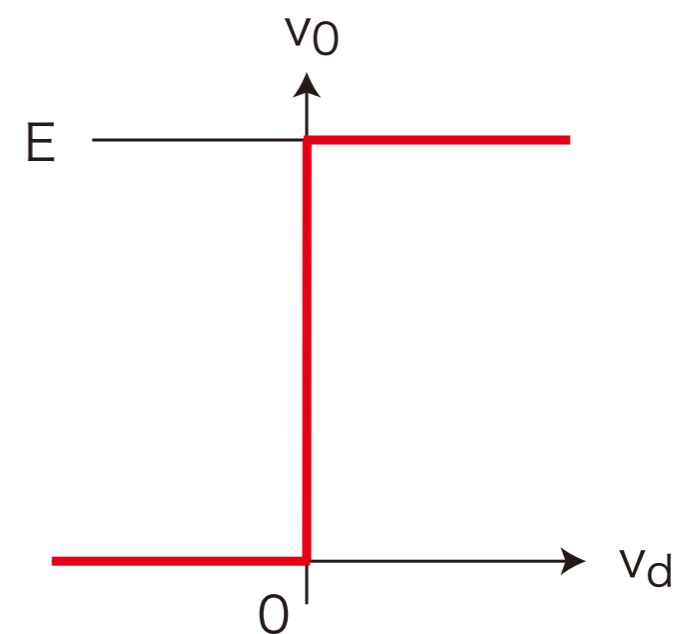
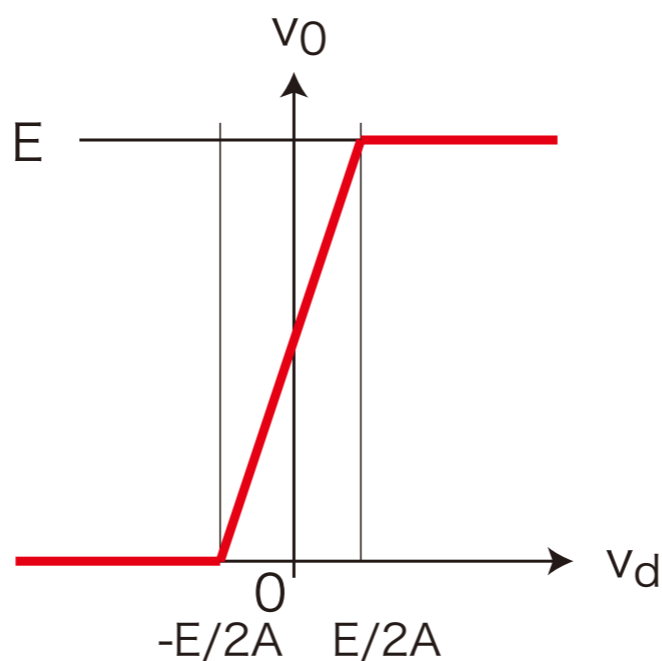
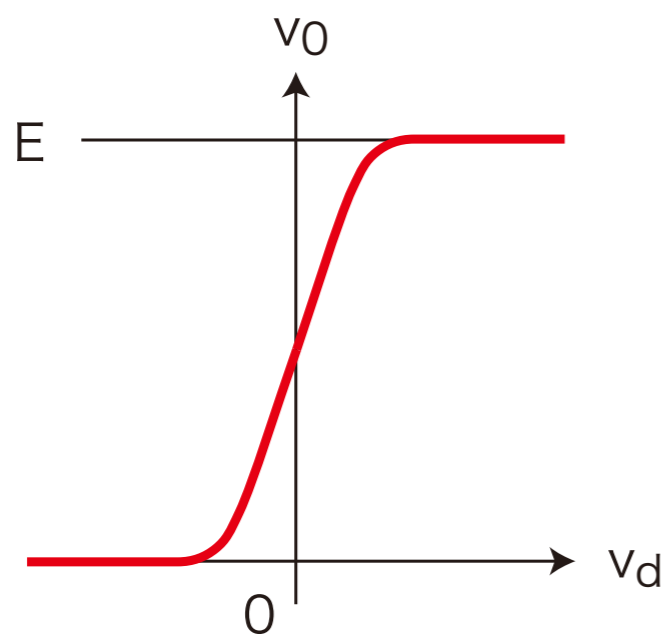


オペアンプの静特性 (内部)



$$v_d = v_p - v_n$$

$$v_0 = a(v_d)$$

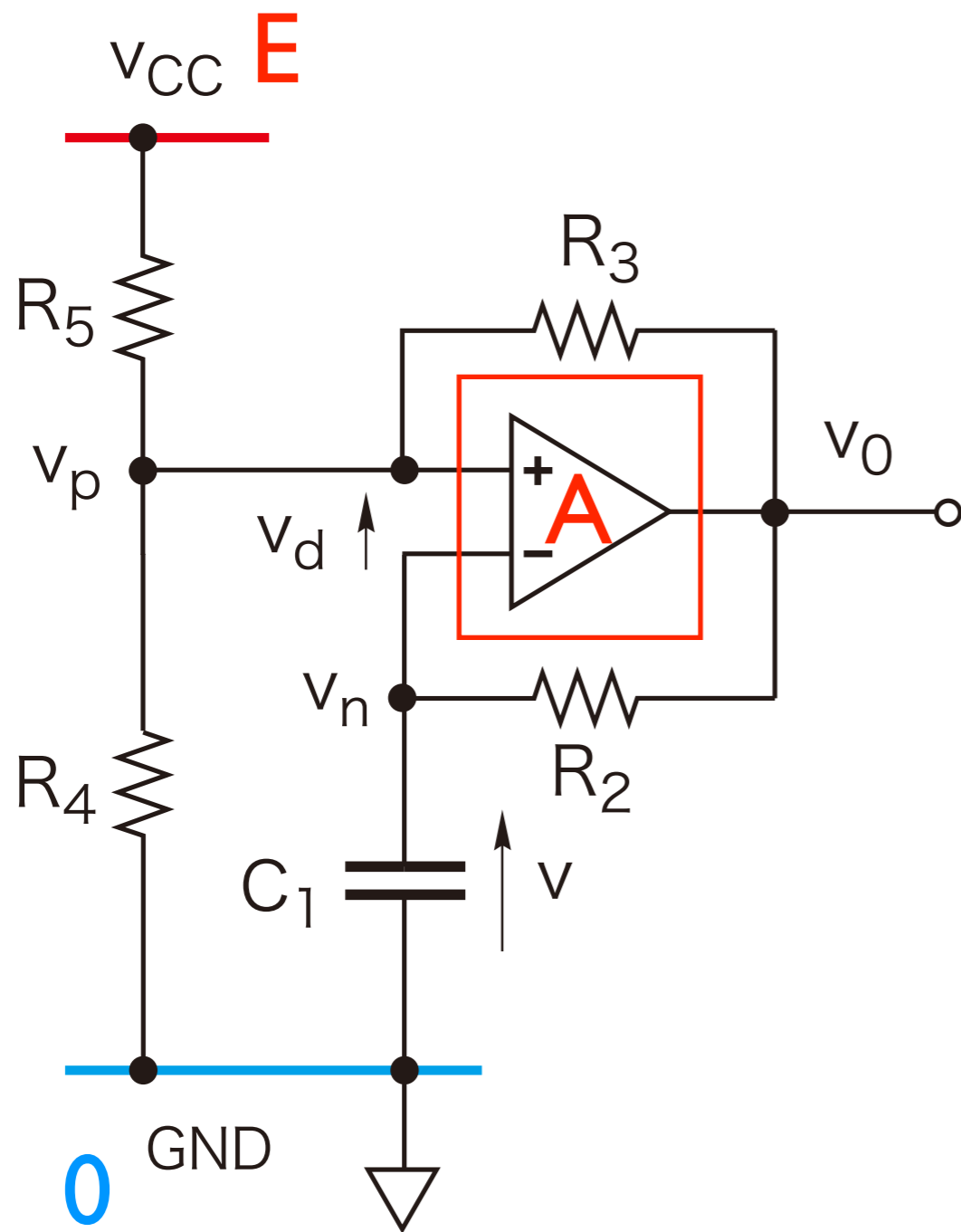


$$v_0 = \frac{1}{2}E \left(1 + \tanh\left(\frac{v_d}{2V_T}\right) \right)$$

$$v_0 = \begin{cases} 0 & v_d < -\frac{E}{2A} \\ Av_d + \frac{E}{2} & v_d \leq \left| \frac{E}{2A} \right| \\ E & v_d > \frac{E}{2A} \end{cases}$$

$$v_0 = Eu(v_d)$$

feedback 回路 (外部)



$$\frac{v_0 - v_p}{R_3} + \frac{-v_p}{R_4} + \frac{E - v_p}{R_5} = 0$$

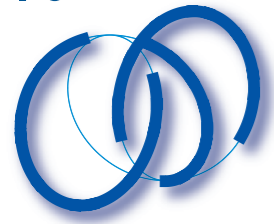
$$v_p = pv_0 + qE$$

$$p, q = \frac{\frac{1}{R_3}, \frac{1}{R_5}}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}}$$

$$v_n = v$$

$$v_d = pv_0 + qE - v$$

$$R_2 C_1 \frac{dv}{dt} + v = v_0$$

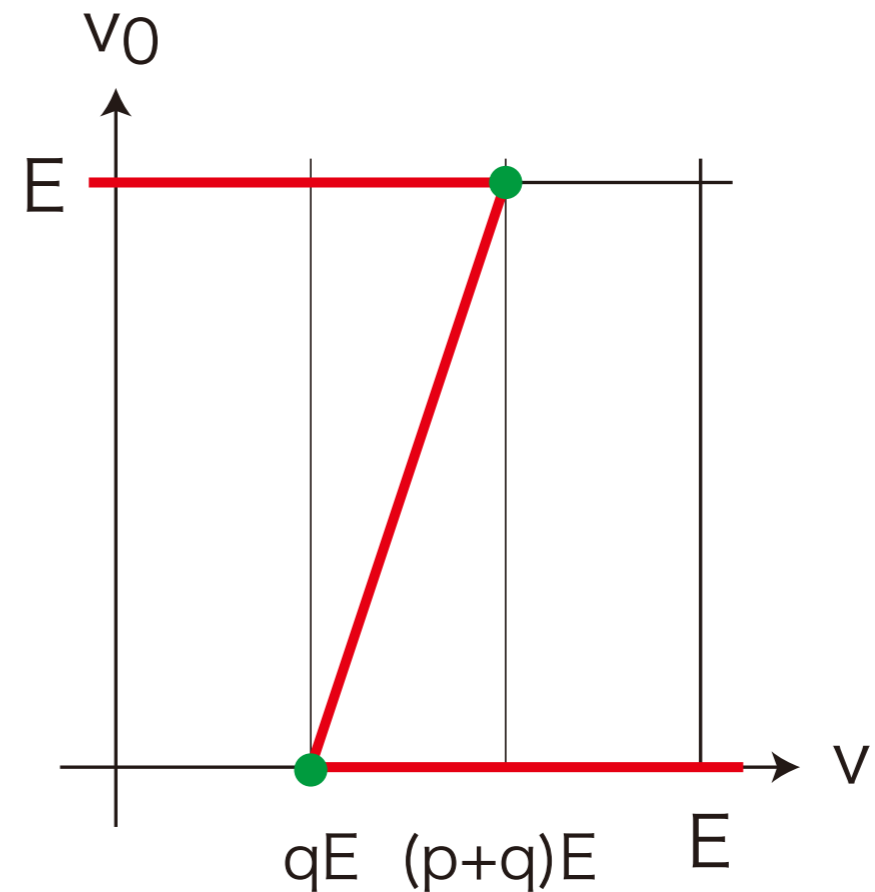
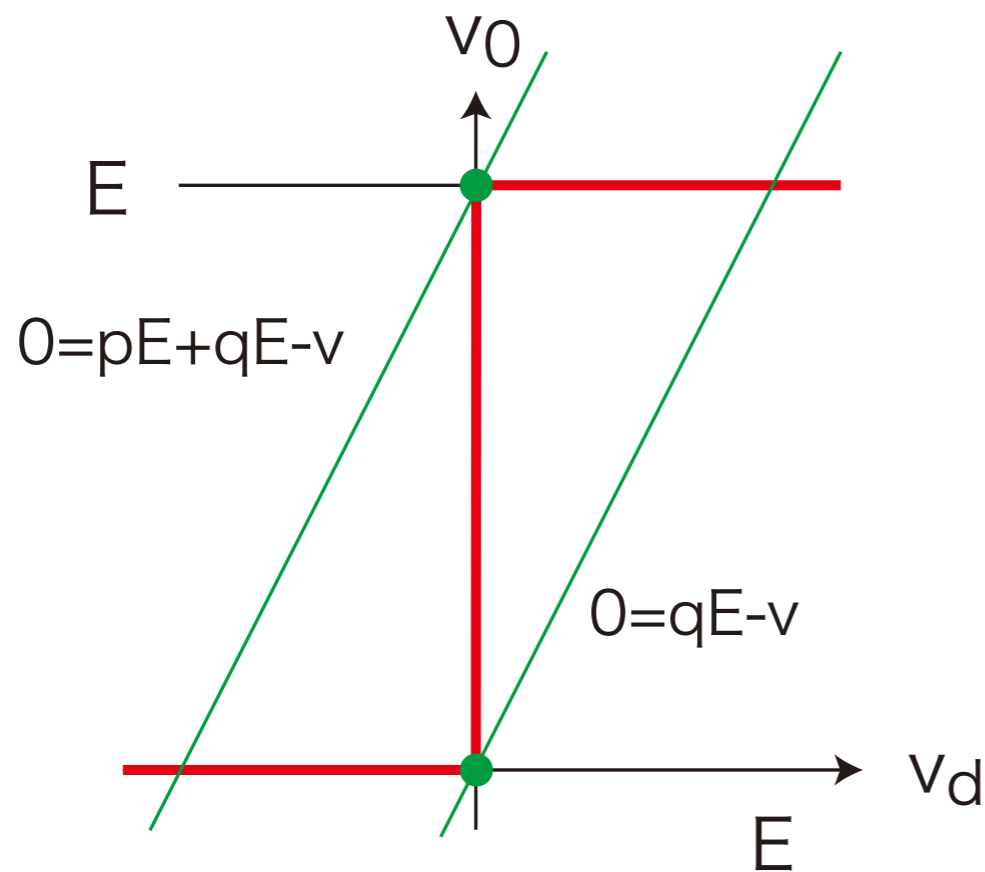


回路の $v - v_0$ 動作点集合

$$v_0 = a(v_d)$$

$$v_d = pv_0 + qE - v$$

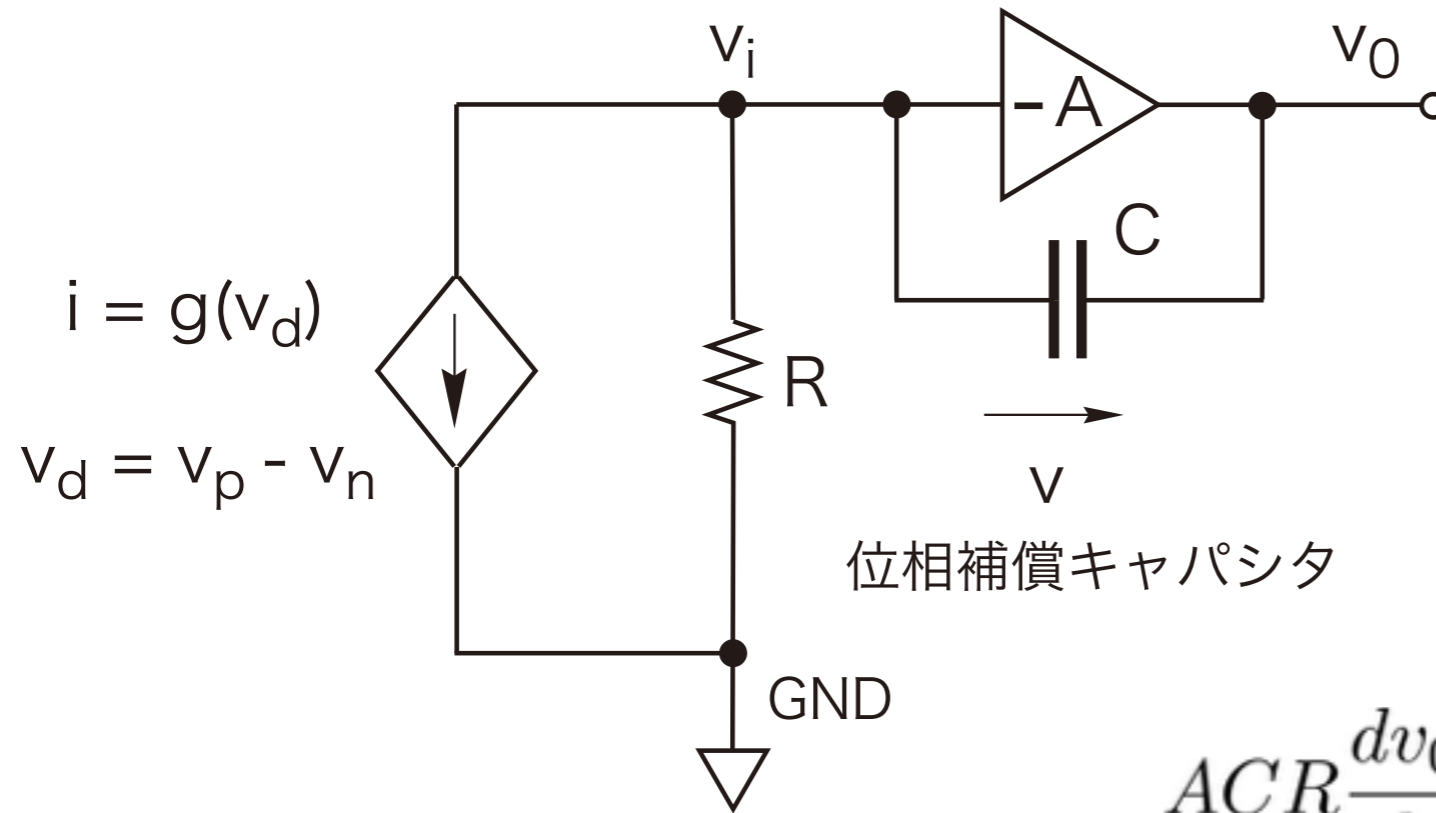
$$p, q = \frac{\frac{1}{R_3}, \frac{1}{R_5}}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}}$$





オペアンプの動特性 *

差動増幅器 電圧増幅器



$$v_0 = (-A)v_i$$

$$v_0 = v + v_i$$

$$C \frac{dv_0}{dt} = i + \frac{v_i}{R}$$

$$ACR \frac{dv_0}{dt} + v_0 = ARi = ARg(v_d) = a(v_d)$$

$$\mu \frac{dv_0}{dt} + v_0 = a(v_d), \quad \mu = ACR \ll 1$$

* 北野正雄：電子回路の基礎，第10章，培風館，2000



slow-fast系としての回路の動的挙動

遅い運動 $R_2C_1 \frac{dv}{dt} + v = v_0$

速い運動 $\mu \frac{dv_0}{dt} + v_0 = a(v_d) \quad \mu \ll 1$

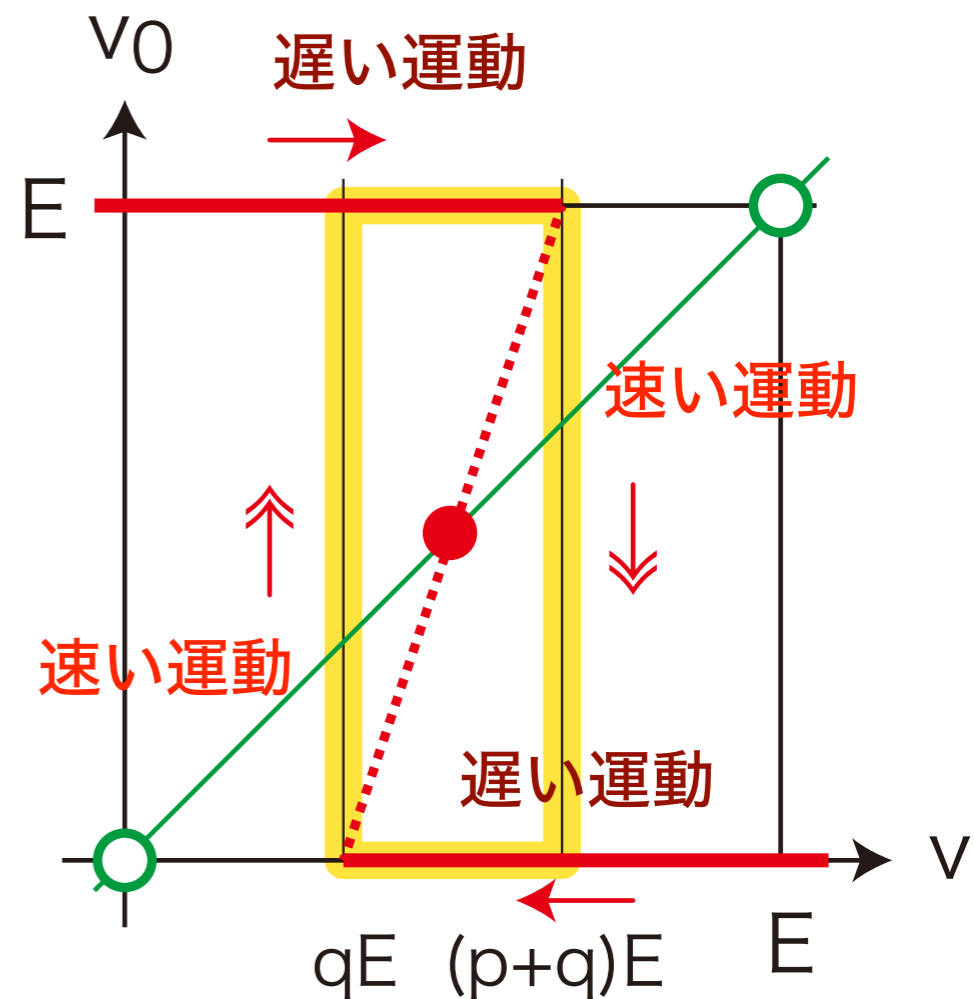
Op ampの特性 $v_0 = a(v_d)$

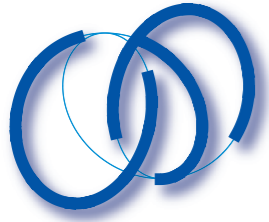
速い運動の平衡点

遅い運動の乗る多様体

遷移条件 $v_d = pv_0 + qE - v$

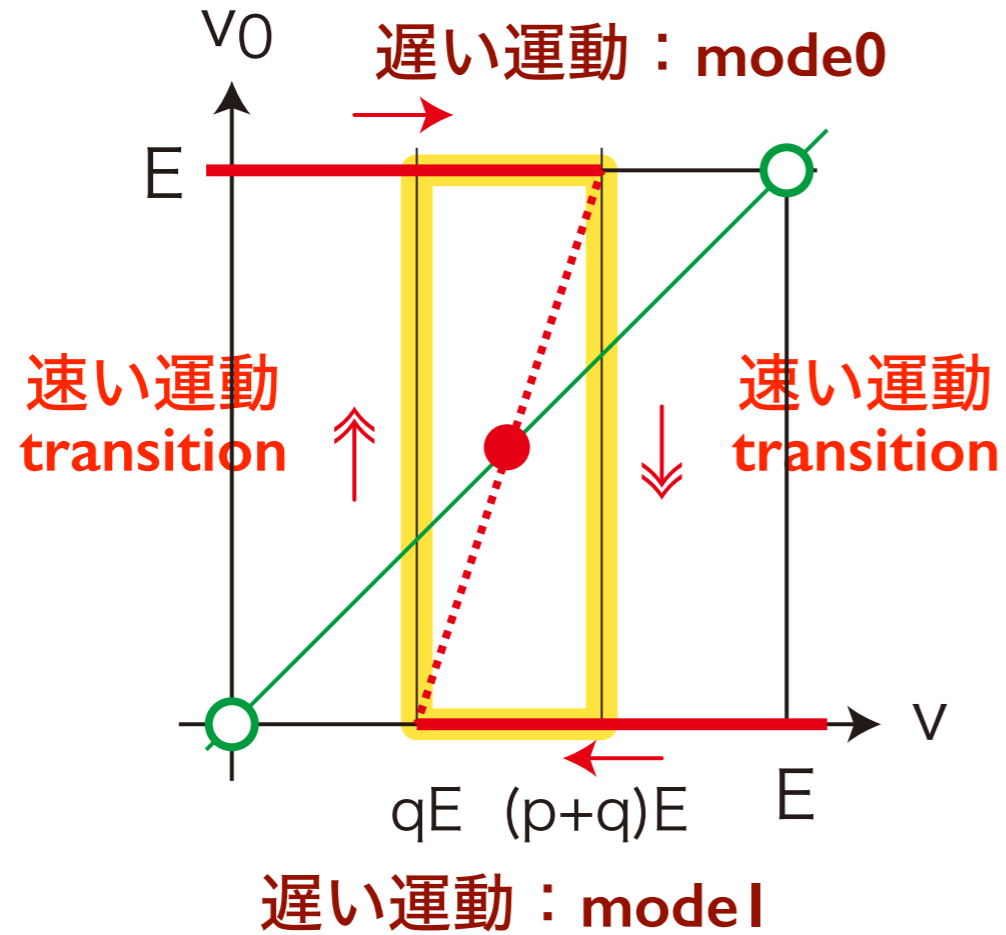
速い運動の平衡点の分岐点



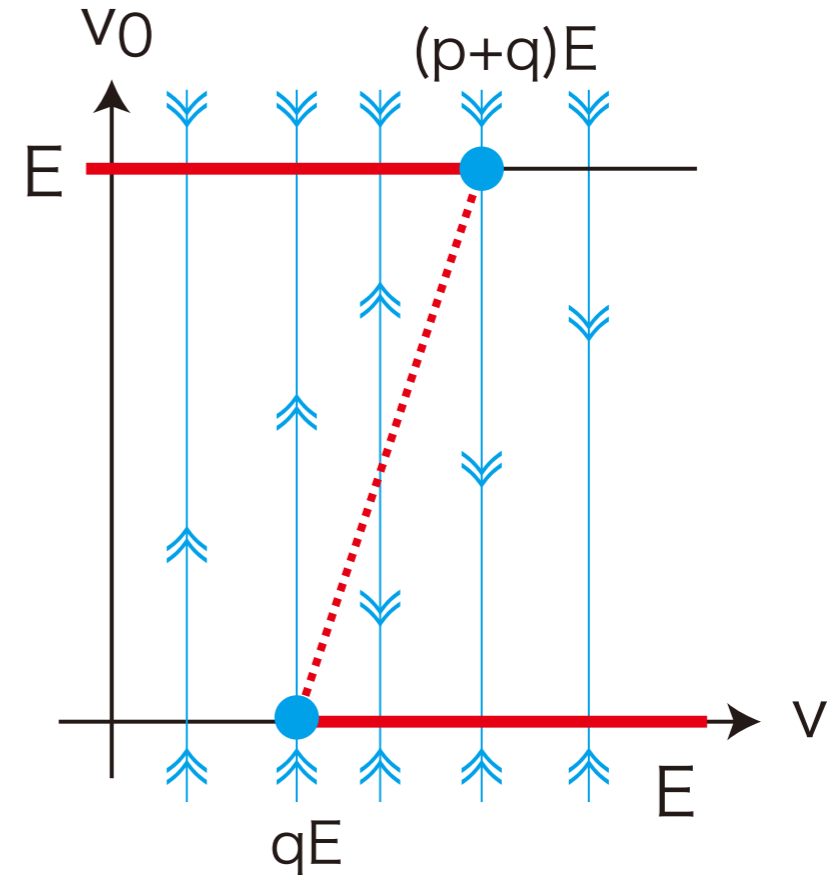


fast系の運動でみる：分岐図

phase portrait



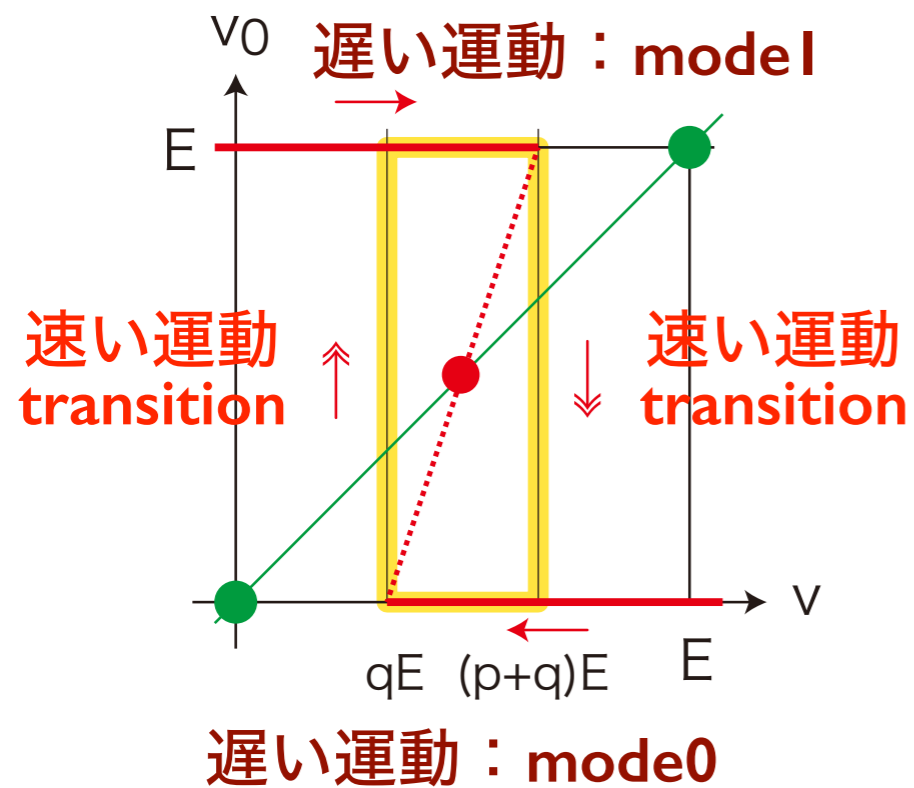
bifurcation diagram



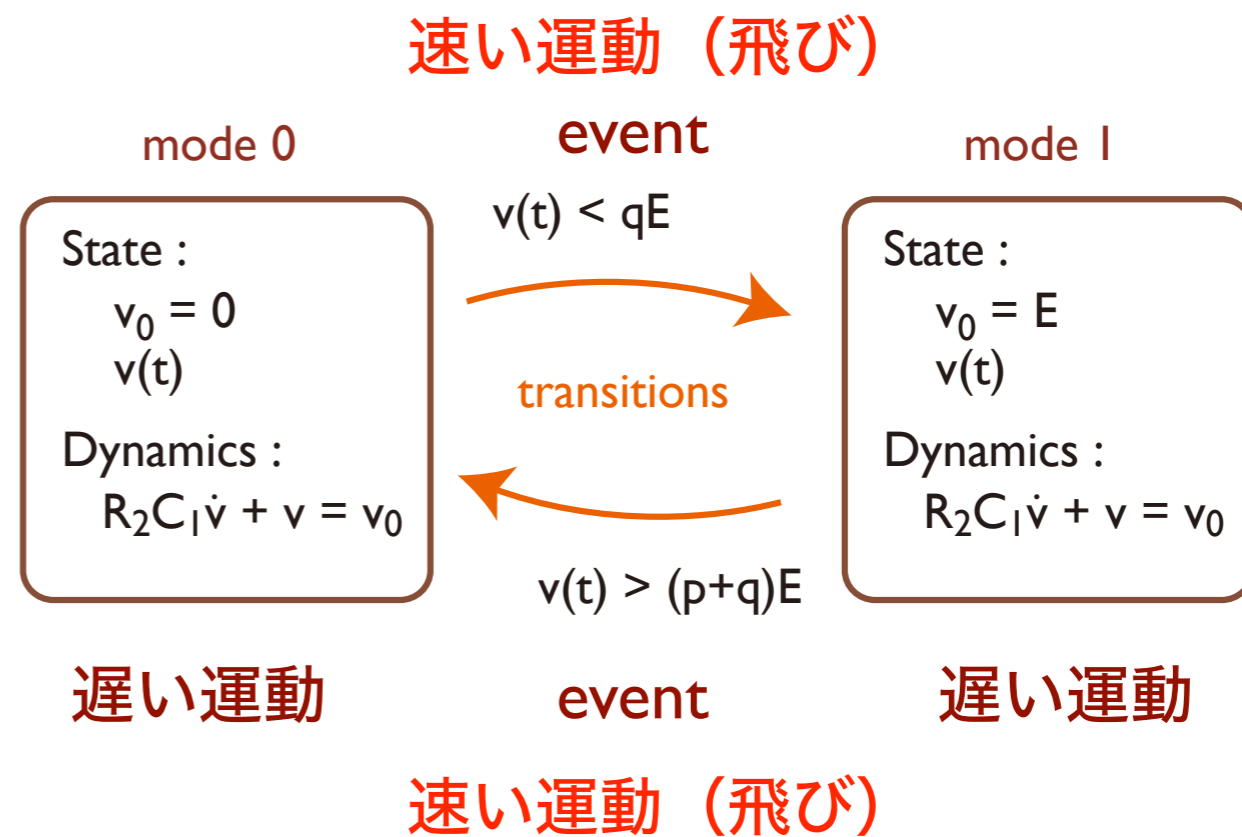


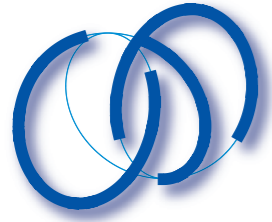
slow系でみる : FSM

phase portrait

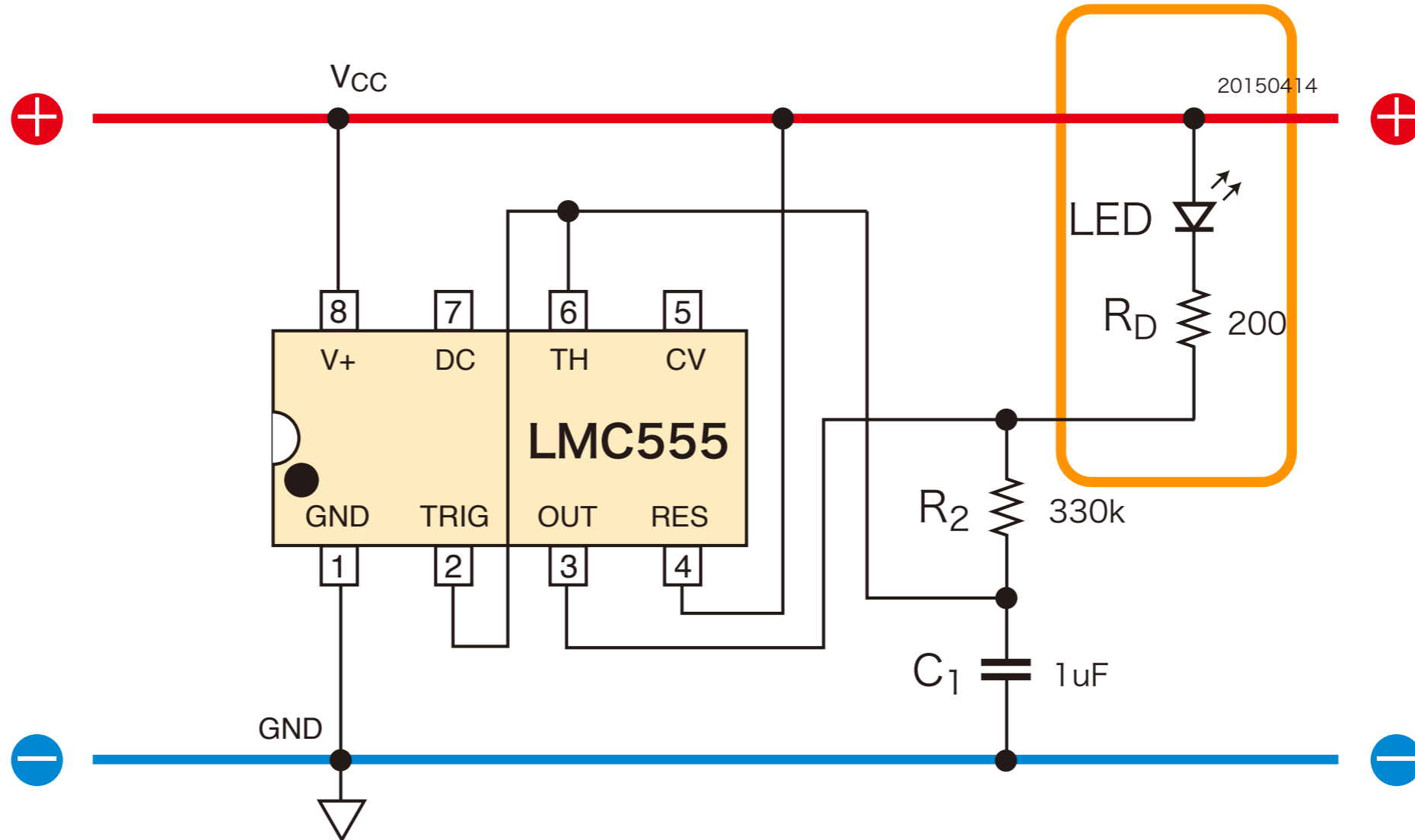


Finite State Machine

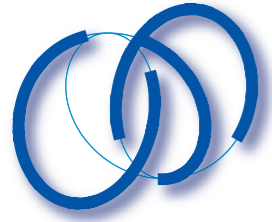




実験 1 : RC方形波発振器

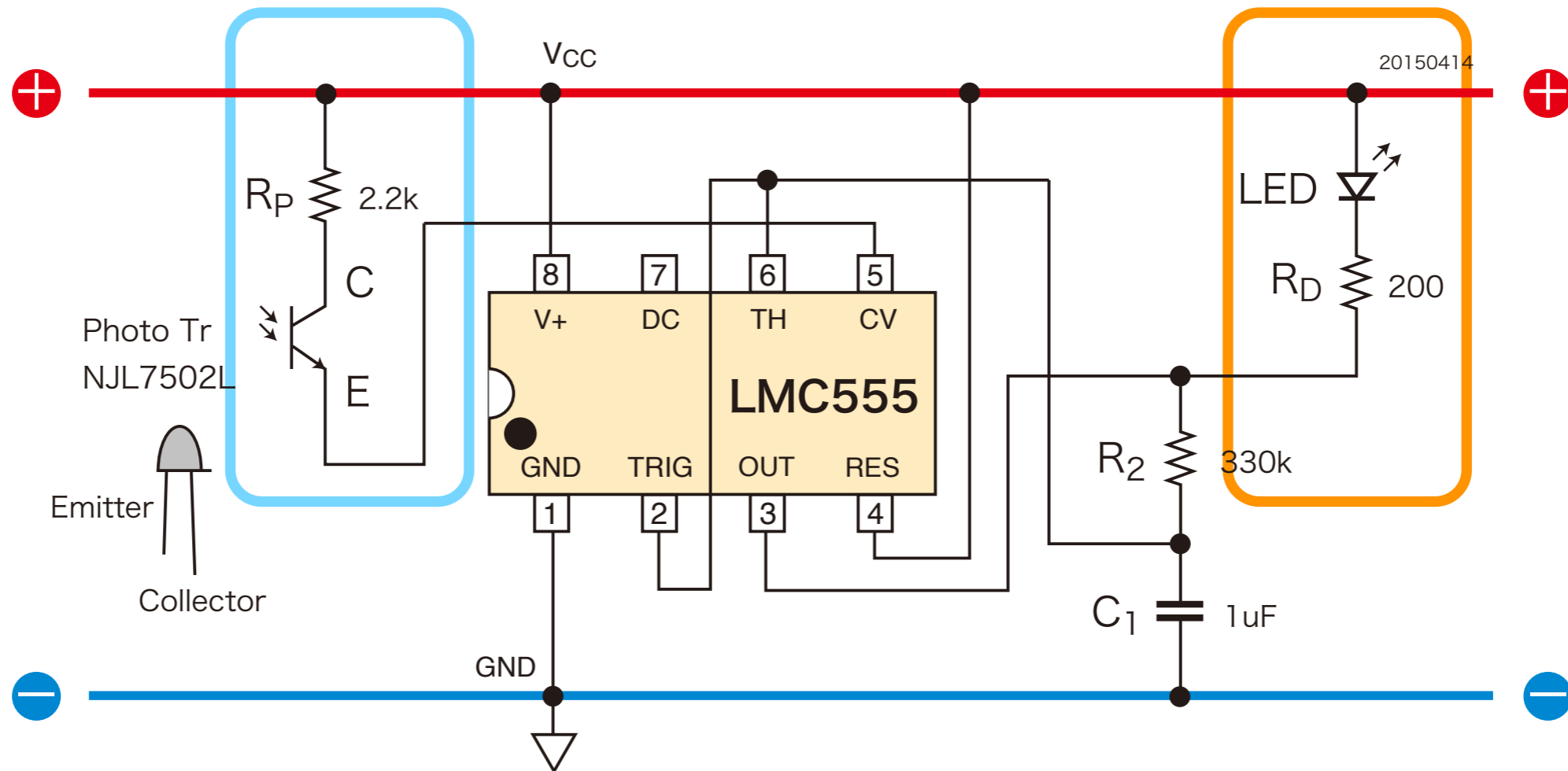


この後すぐ体験できます

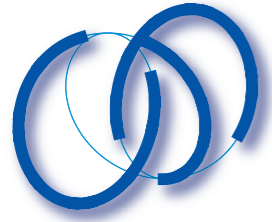


実験 2 : LED fire fly

sensor + oscillator + led



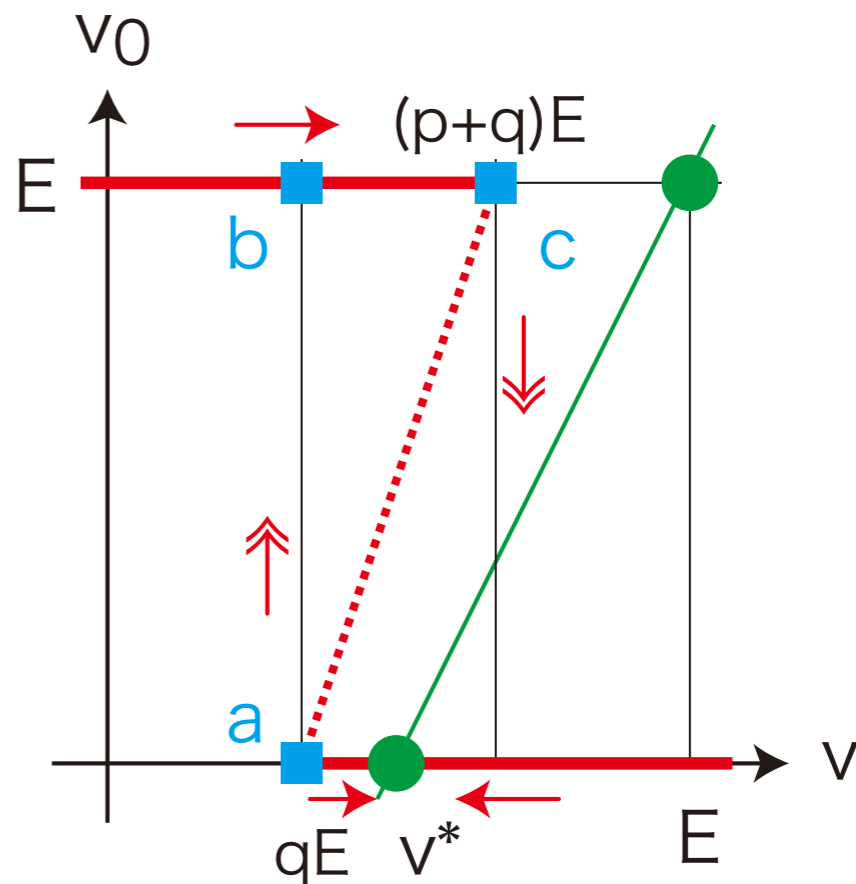
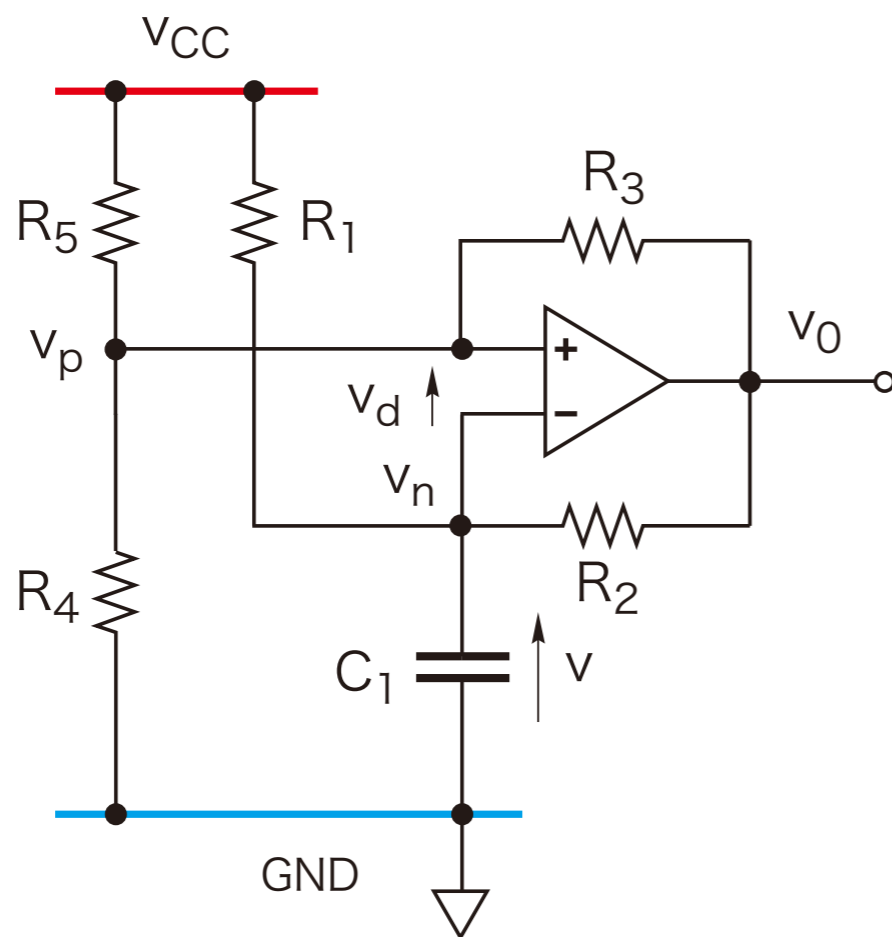
この後すぐ体験できます



Monostable multivibrator



Monostable multivibrator(SL)



$$v^* = \frac{R_2}{R_1 + R_2} E = \gamma E, \quad \gamma = \frac{R_2}{R_1 + R_2}$$

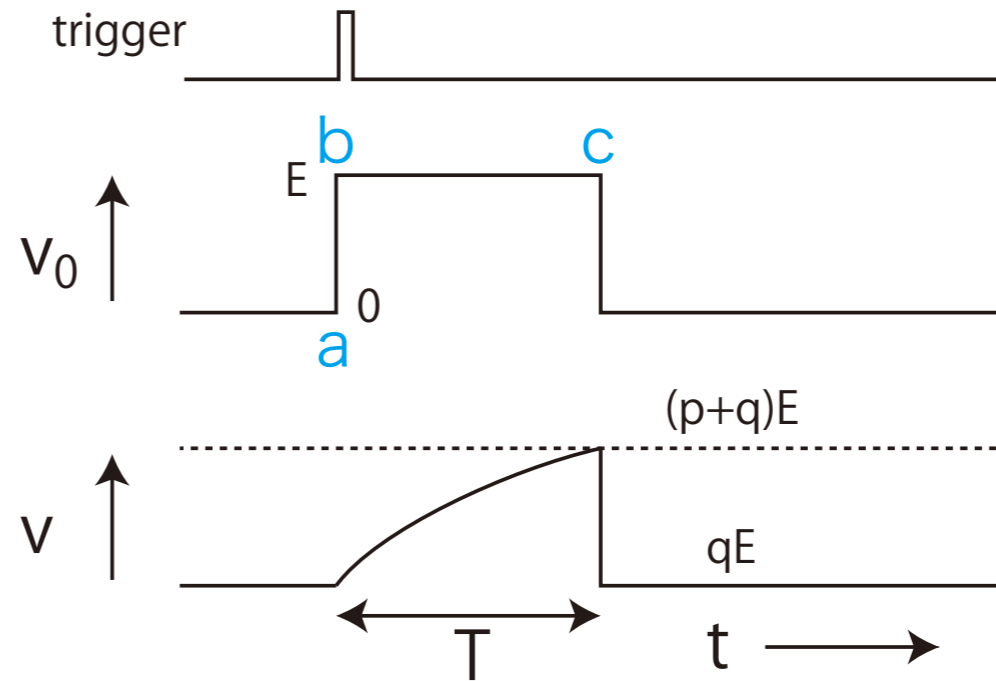
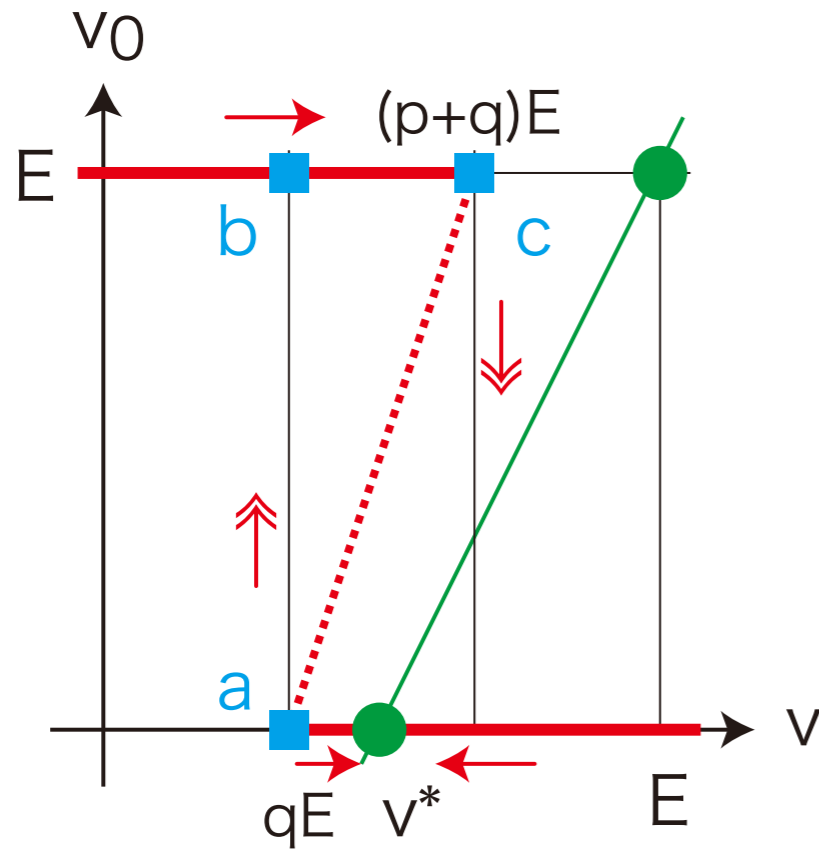
monostable multivibratorとなる条件

$$q < \gamma$$





Monostable multivibrator(SL)



遅い運動bcの微分方程式

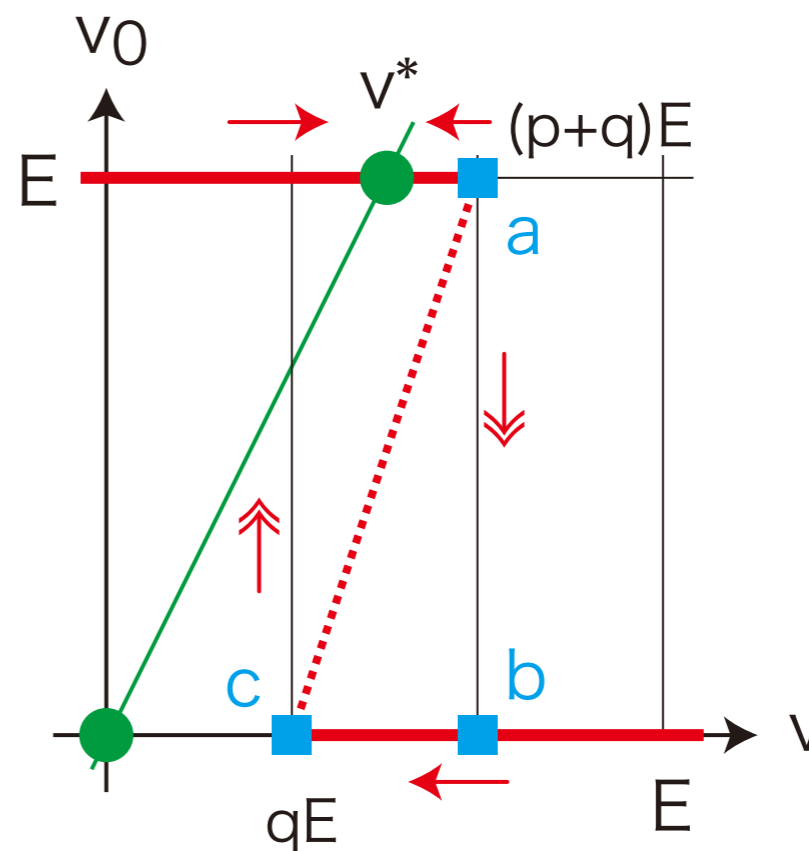
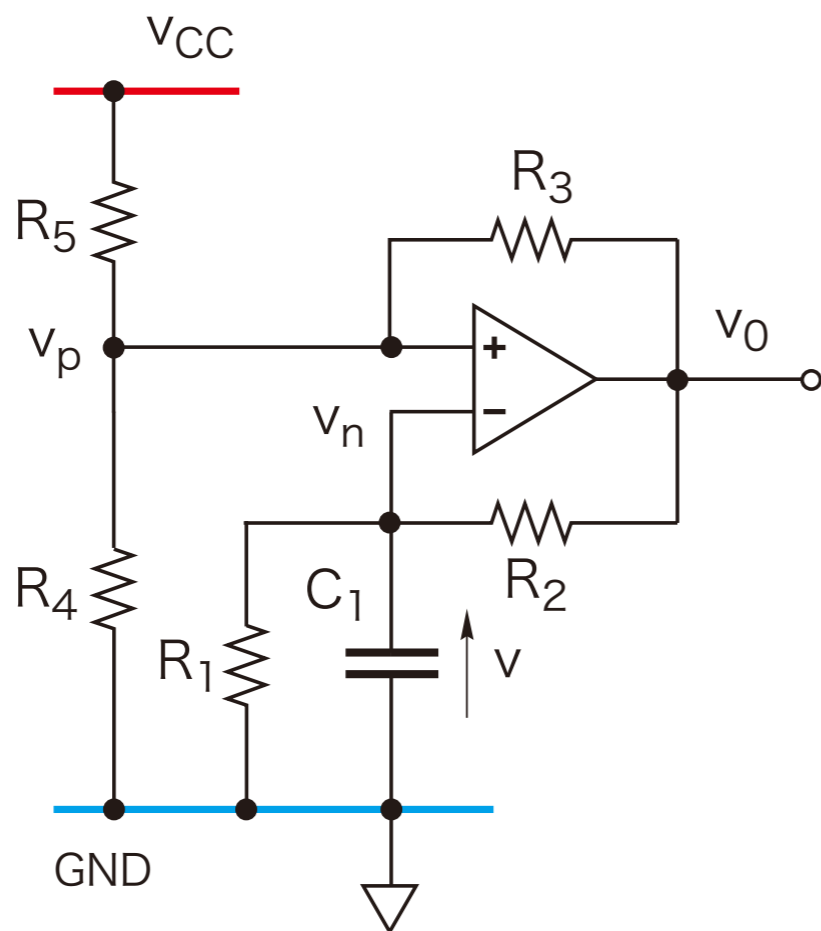
$$C_1 \frac{dv}{dt} + \left(\frac{1}{R_1} + \frac{1}{R_2} \right) v = \frac{E}{R_1} + \frac{v_0}{R_2}$$

遅い運動bcの所要時間T

$$T = \tau \ln \frac{q-1}{p+q-1}, \quad \frac{1}{\tau} = \frac{1}{C_1} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$



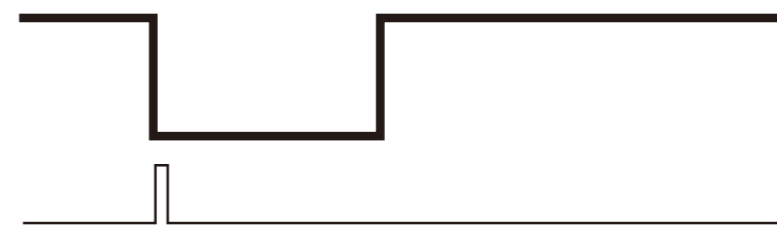
Monostable multivibrator(SH)



$$v^* = \frac{R_1}{R_1 + R_2} E = \gamma E, \quad \gamma = \frac{R_1}{R_1 + R_2}$$

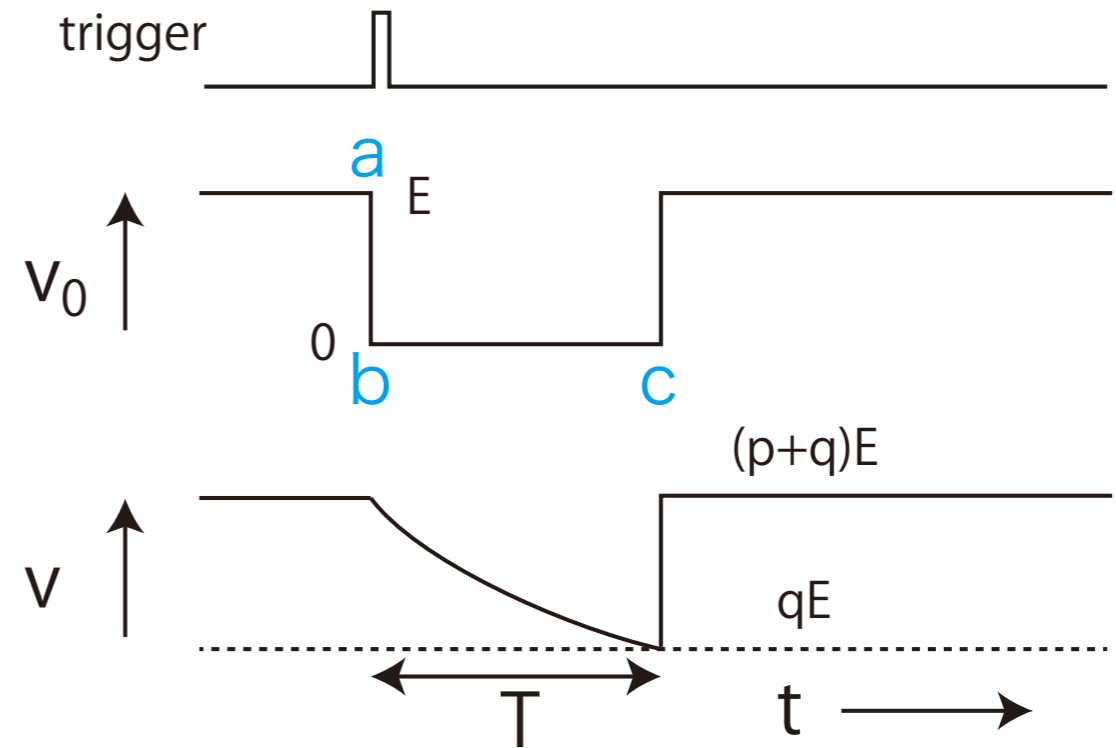
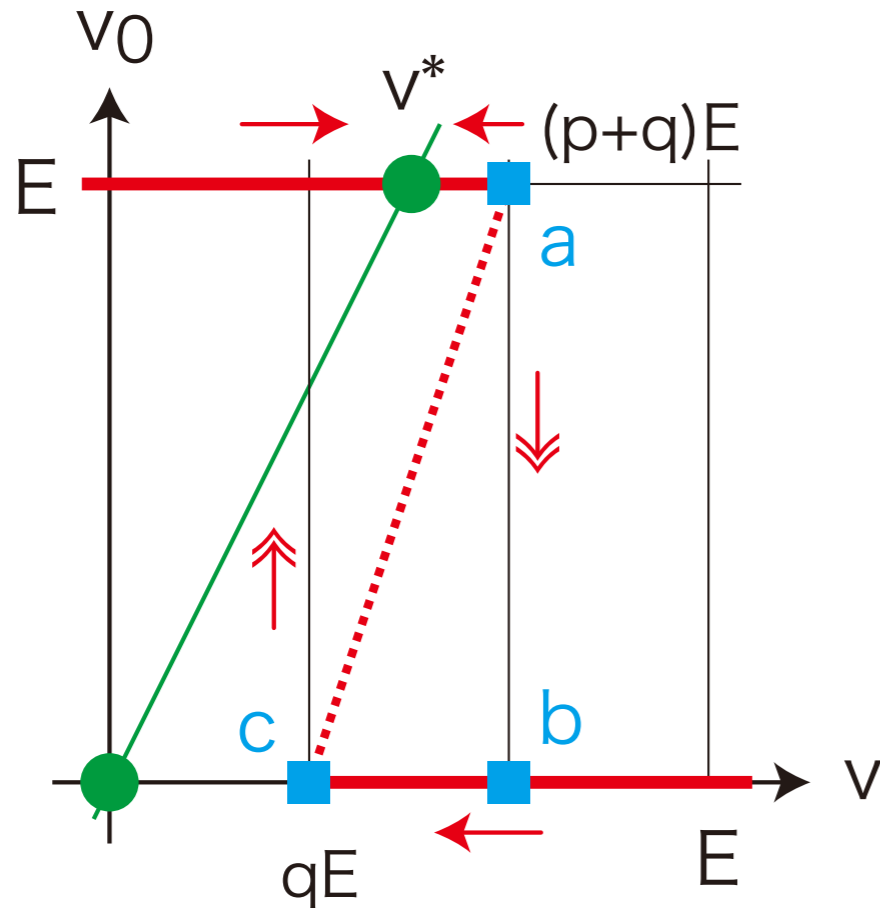
monostable multivibratorとなる条件

$$\gamma < p + q$$





Monostable multivibrator(SH)

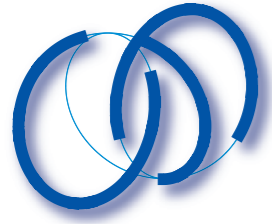


遅い運動bcの微分方程式

$$C_1 \frac{dv}{dt} + \left(\frac{1}{R_1} + \frac{1}{R_2} \right) v = \frac{v_0}{R_2}$$

遅い運動bcの所要時間T

$$T = \tau \ln \frac{p+q}{q}, \quad \frac{1}{\tau} = \frac{1}{C_1} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$



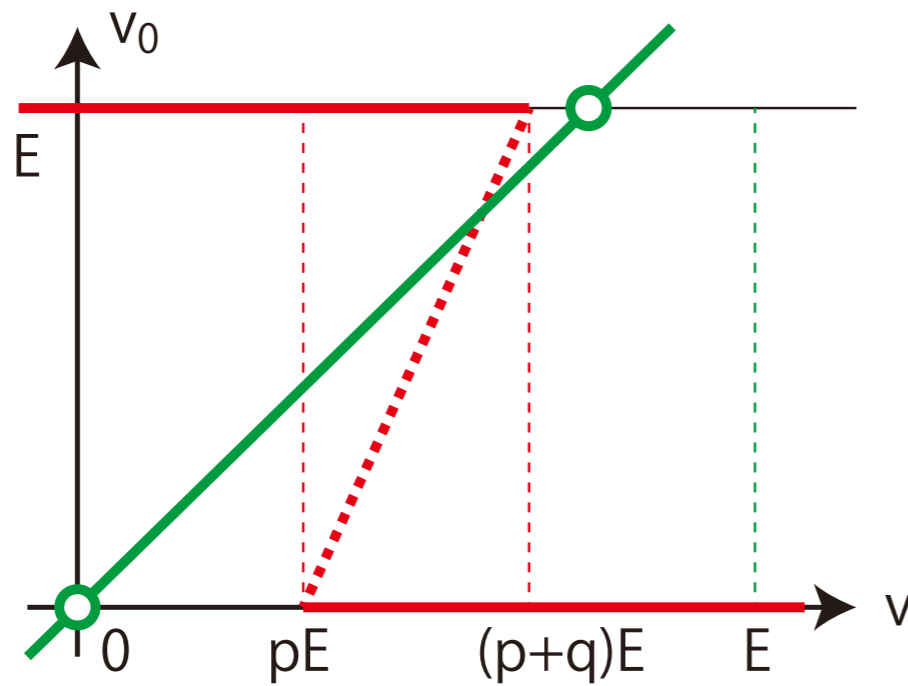
Excitable fire fly



Excitable fire fly

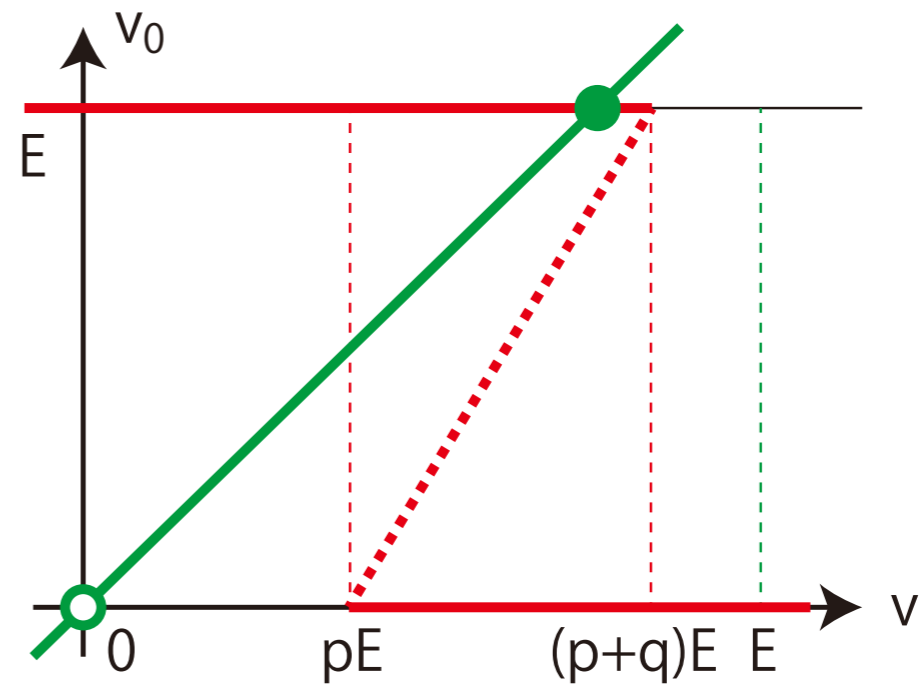
暗い場所ではastable, 明るい場所ではmonostableとなるmultivibrator

暗い環境



astable

明るい環境



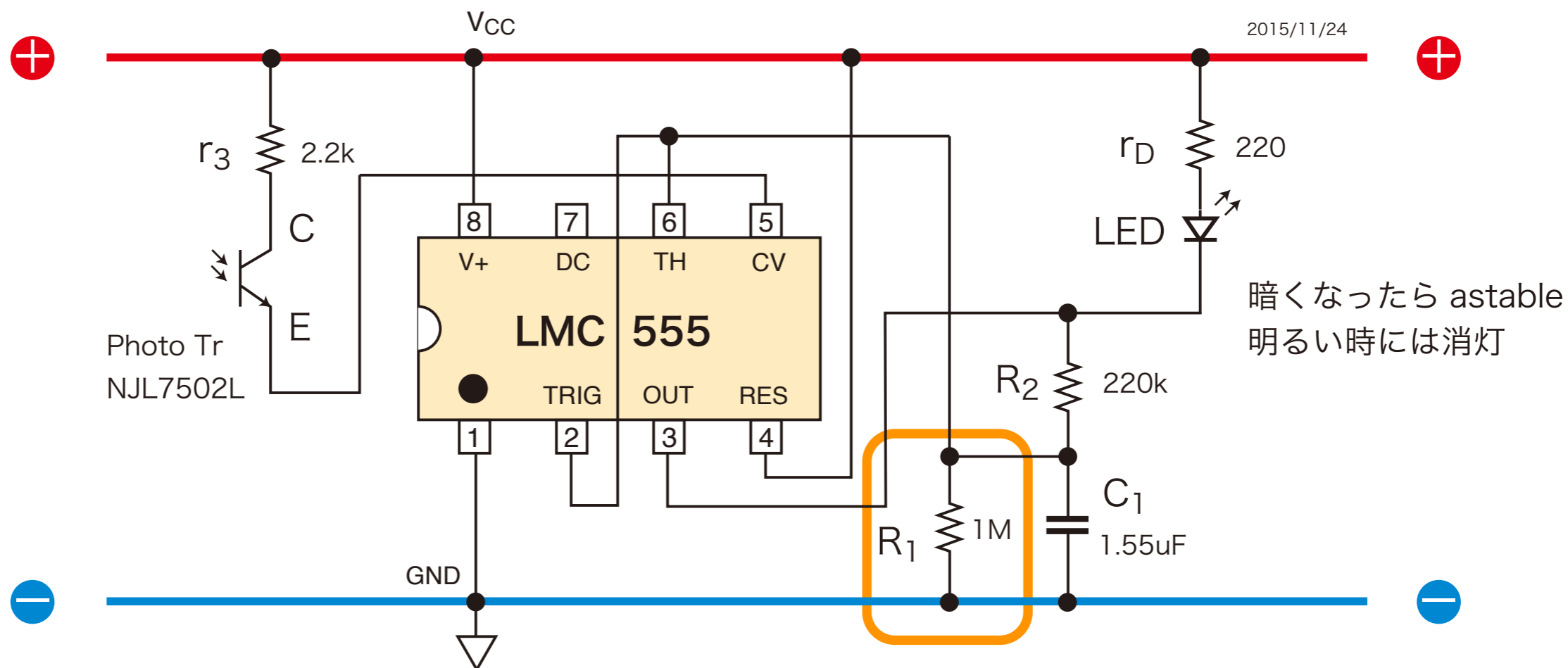
monostable

active lowで使う



実験3. Excitable fire fly

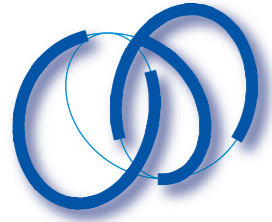
暗い場所ではastable, 明るい場所ではmonostableとなるmultivibrator



この後すぐ体験できます

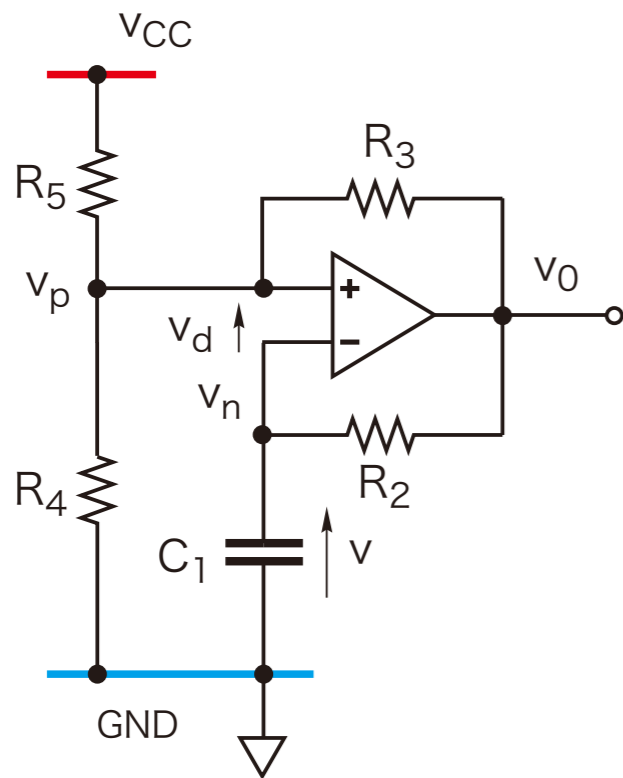


まとめと発展問題

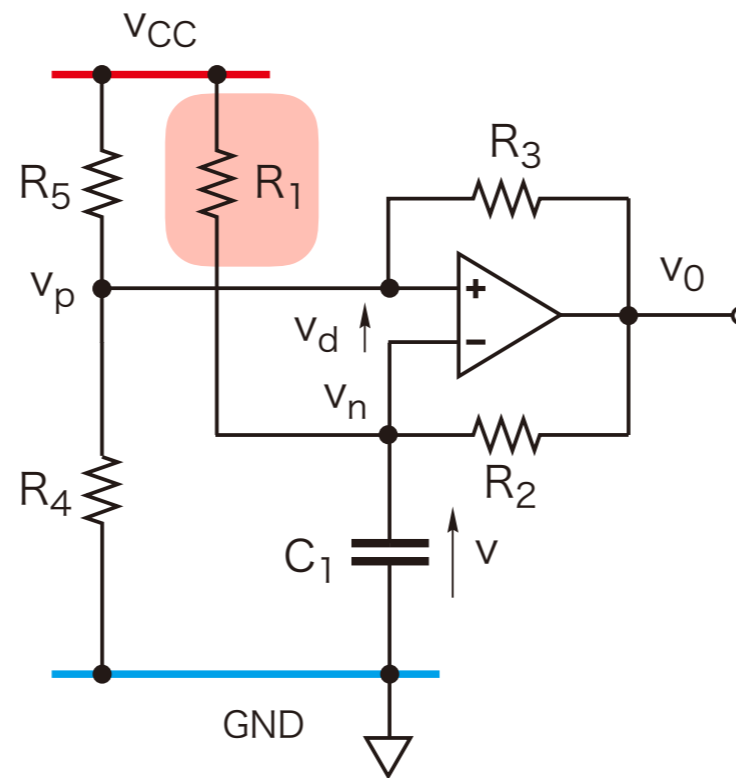


multivibrator回路族に共通の性質は何か

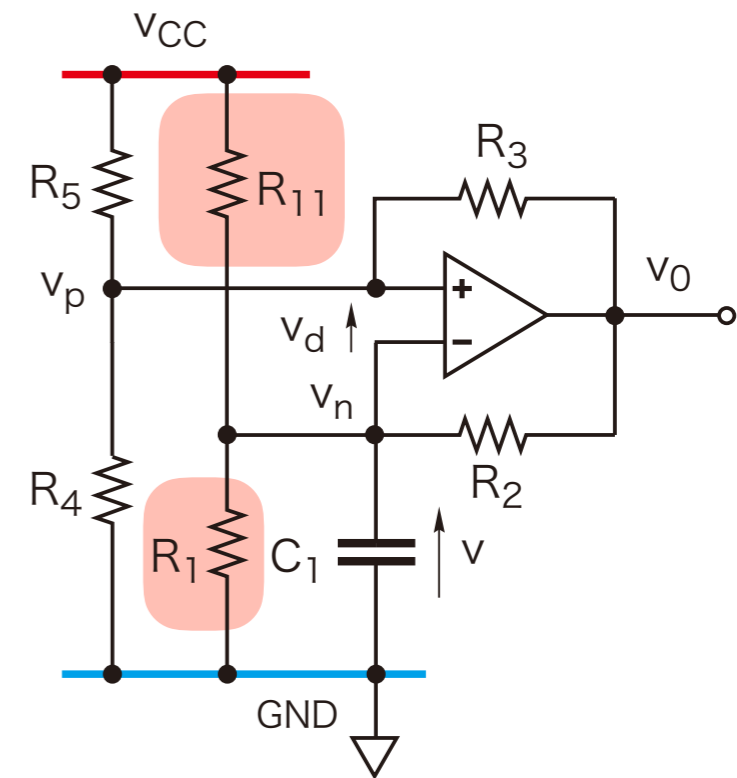
astable



monostable

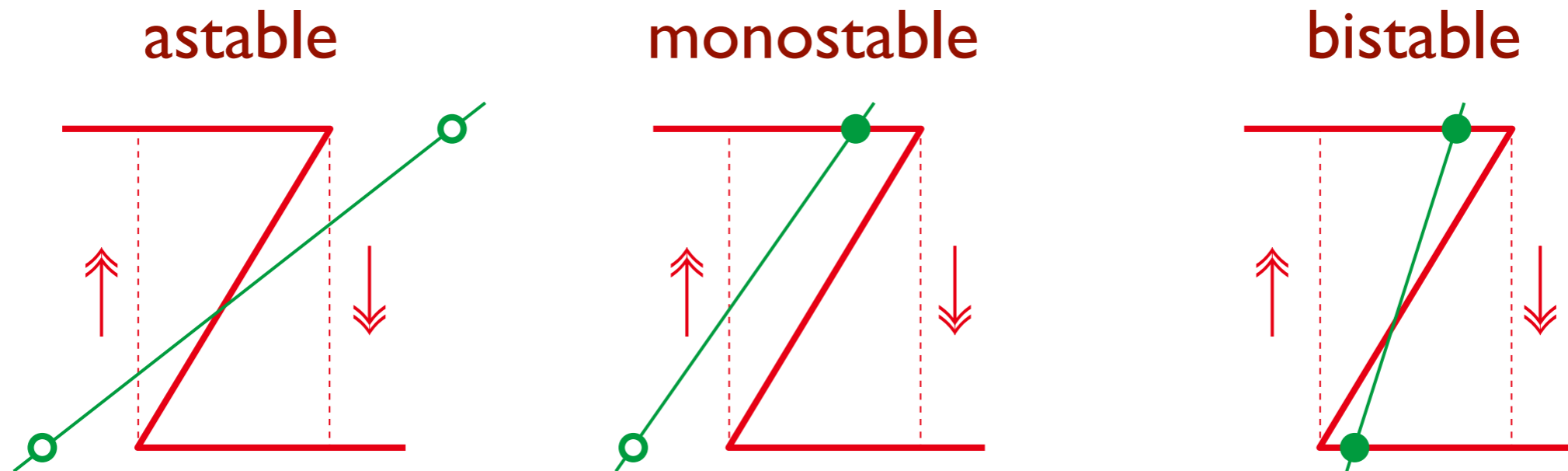


bistable





multivibrator回路族に共通の性質は何か



1. hysteresis特性 : positive feedbackで決まる
=> v_0 方向の速いvector場はどれも同じである
2. 平衡点 : negative feedbackで決まる
=> 遅いvector場で3つのタイプを作る

common property = excitability



共通の性質を生かした解析方法は何か

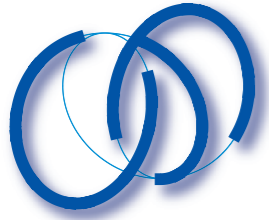
1. phase portraitを用いた状態の解析

波形を見るだけではvector場の様子は見えてこない

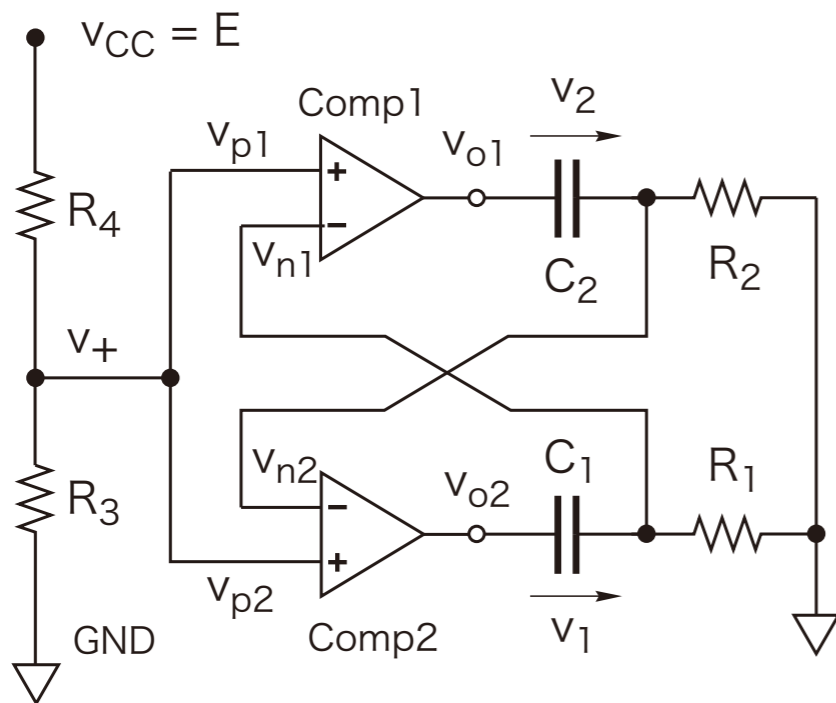
2. slow-fast系として動的挙動

- ① fast vector 場の平衡点集合 (slow 多様体上) の動的挙動
- ② slow 多様体をeventで貼り合わせる (貼り合わせ多様体) を決定
- ③ 貼り合わせ多様体上の遅いvector場を解析する

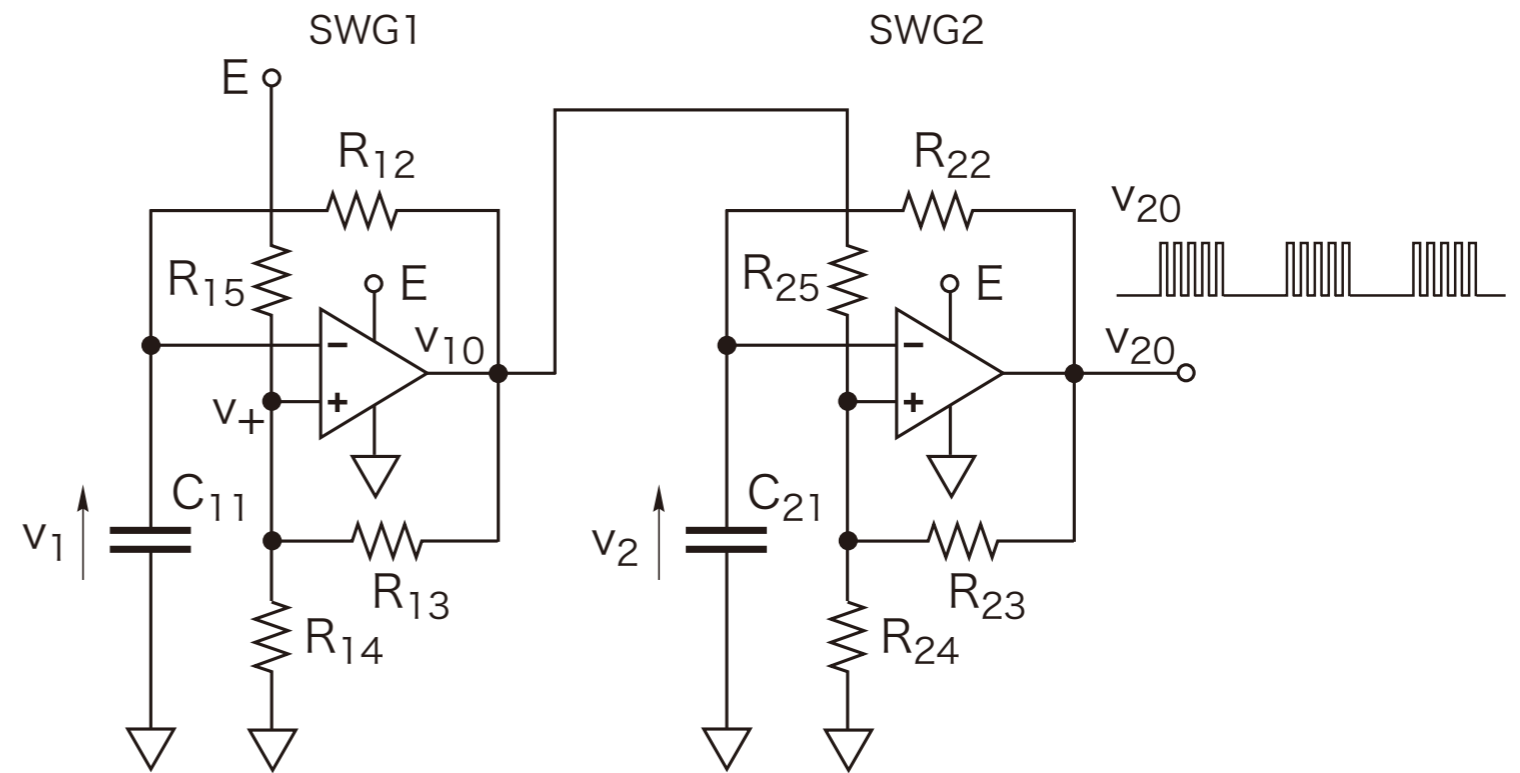
3. 力学系+分岐パラメータをslow-fast系として考察



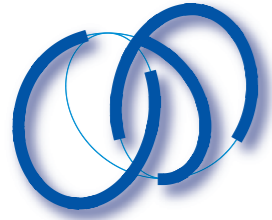
発展問題



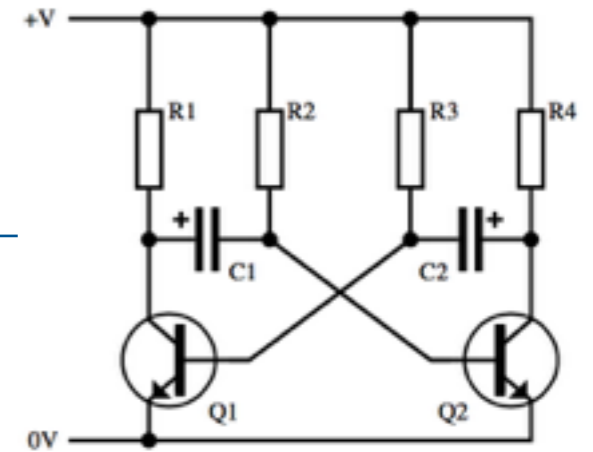
(a) symmetrical multivibrator



(b) burster



文献



- [1] A.A. Andropov, A.A. Vitt and S.E. Khaikin: **Theory of Oscillators, Chap. 10 Discontinuous Oscillations**, Pergamon Press, 1966.

通常multivibratorと言え、上図のような対称性をもつ回路が例示される。
この本では、3極真空管を使ったmultivibratorの解析例が示されている。
不連続状態を解析するために、phase portraitが多用されている。

- [2] E. M. Izhikevich: **Dynamical Systems in Neuroscience — The Geometry of Excitability and Bursting —**, MIT Press, 2007.

漠然とexcitabilityの定義を探していた。この本ではneuronの解析を分らせるために繰り返し、phase plane上の解の様子（幾何学）が語られている。まるで、excitable vector fieldとでも呼ぶべき力学系を提唱しているようだ。